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A COURSE OF
GENERAL MATHEMATICS.

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A COURSE OF GENERAL MATHEMATICS

FOR SECONDARY SCHOOLS

(Written in accordance with the Revised Syllabus in Elementary Mathematics for the S.S.L.C. Classes)

BY

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FIFTH EDITION
(Revised and considerably enlarged.)

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PREFACE TO THE FIRST EDITION.

THIS book is intended for use by pupils in Forms IV, V and VI preparing Group A Mathematics under the present S. S. L. C. Scheme. It is based on the notes that I prepared while working as a teacher in High Schools. The several method books were studied and in the treatment only such methods have been followed as I found successful in practice, especially in the observation lessons I had to give in the Model School, Teachers' College, Saidapet. As far as possible, exercises of a practical nature have been given. Many of them were suggested by my pupils when by way of home-work they were set to frame problems of their own in the light of models shown. It is presumed that such exercises would be taken to with interest by a large body of pupils. A few exercises of an advanced type are given in each Chapter to keep the more forward pupils engaged in the Elementary Mathematics Class. I have little claim to originality and I would feel amply remunerated if, as a result of my endeavours, a concise book has been produced helping the pupils to have an intelligent grasp of the subject. It is suggested that pages 1 to 158 may be done in Form IV, pages 159 to 261 in Form V and the rest in Form VI.

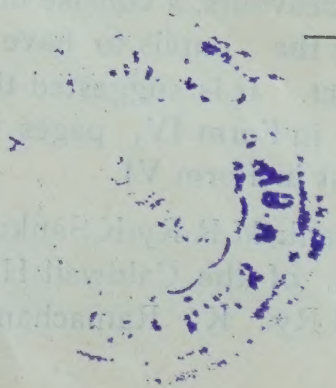
I take this opportunity to thank M.R.Ry.K.Sankarasubba Ayyar Avl., B.A., L.T., of the Caldwell High School, Tuticorin, and M.R.Ry. K. Ramachandra

Ayyar Avl., B.A., of the Ambasamudram High School, for the constructive criticism occasionally offered, and the University of Madras for having allowed me the use of some of its Examination questions. My special thanks are due to M.R.Ry. T. S. Krishnamurthi Ayyar Avl., B.A., L.T., Assistant Inspector of Schools, Ramnad, for the many useful suggestions kindly offered, to M.R. Ry. Visvanathier Avl., B.A., L.T., of the High School, Srivaikuntam, for preparing most of the answers, and to M.R.Ry.N. Sankara Ayyar Avl., M.A., L.T., for his valuable suggestions and for the trouble he took in passing most of the proofs.

In spite of precautions errors may have crept in and I would be highly grateful to those who would point them out to me for rectification.

SRIVAIKUNTAM,
15th July 1919.

V. RAGHUNATHA AIYAR.



PUBLISHERS' NOTE TO THE FIFTH EDITION.

WITH a view to make the book more useful, this edition has been considerably enlarged with the addition of Chapters on Discount and Exchange, Stocks and Shares, Time and Distance—Graphs, Partnership, Problems on Work and Time, Profit and Loss. The S. S. L. C. Papers have been brought up-to-date. The Publishers take this opportunity to thank Mr. R. Namby Aiyangar, B.A., L T., Mathematics Assistant, Hindu High School, Triplicane, Madras, for the additions and for passing the proofs of this edition promptly and carefully.

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May 1925.

THE PUBLISHERS.

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A COURSE OF
GENERAL MATHEMATICS
FOR
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CHAPTER I.
SECTION I.
THE FOUR RULES.

1. The digits used in Arithmetic are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0. With these ten digits many numbers could be formed. The decimal scale of notation is adopted in writing numbers and each digit has a *place-value* as well as an *intrinsic value*, *e. g.*, in 576, the place-values of 5, 7 and 6 are 500, 70 and 6 while their intrinsic values are 5, 7 and 6, and in 777 the place-value of each seven is ten times that of the seven to the right of it. '0' (zero) is a *non-significant* digit and the others are *significant* digits. In any number, the most significant digit is that of the highest place-value.

[Note.—A zero added at the middle of a number increases the place-value of each digit to the left of it ten times and leaves the other digits undisturbed.]

The Nos. 1, 2, 3, 4.....are called *natural Nos.* Numbers written in the natural order are *consecutive Nos.* 1, 3, 5.....are *odd Nos.* and 2, 4, 6.....are *even Nos.*

The signs of addition, subtraction, multiplication and division are respectively + (plus), — (minus), × (into), and ÷ (divided by). Read '=' as 'is equal to.'

2. ADDITION AND SUBTRACTION.

Only like quantities could be added,

$$e.g., \text{Rs. } 3 + \text{Rs. } 5 = \text{Rs. } 8.$$
$$3a + 5a = 8a.$$

In subtracting 6 from 8, that number is found which, added to 6, gives 8. Hence subtraction is called complementary addition.

3. MULTIPLICATION.

$$\text{Rs. } 6 \times 5 = \text{Rs. } 6 + \text{Rs. } 6 + \text{Rs. } 6 + \text{Rs. } 6 + \text{Rs. } 6.$$

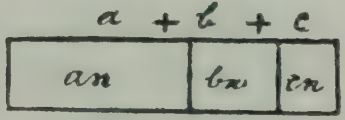

Fig. 1.

"In this diagram and diagrams 2 and 3, an means $a \times n$, pq means $p \times q$, a^2 means $a \times a$, &c."

The diagram shows that

$$(a + b + c)n = a \times n + b \times n + c \times n.$$
$$\therefore 756 \times 8 = (6 \times 8) + (50 \times 8) + (700 \times 8).$$

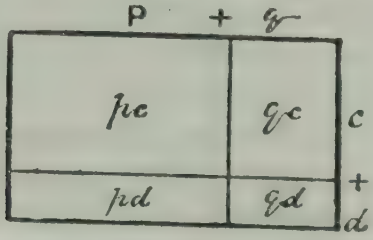

Fig. 2.

The diagram shows

$$\begin{array}{l} p + q \\ c + d \\ \hline q \times c + p \times c \\ + q \times d + p \times d. \end{array}$$

$$\begin{array}{l} \therefore 28 \times 12 \\ \quad = 20 + 8 \\ \quad\quad 10 + 2 \\ \hline (10 \times 8) + (10 \times 20) \\ \quad + (2 \times 8) + (2 \times 20) \end{array}$$

Similarly, $165 \times 78 = 165$

$$\begin{array}{rl} 78 & \\ \hline 1155 & = 165 \times 7 \text{ tens} \\ 1320 & = \quad " \quad \times 8 \text{ units} \\ \hline = 12870. & \end{array}$$

165 is the *multiplicand*, 78 is the *multiplier*, 12870 is the *product* and 165 and 78 are the *factors* of 12870.

A rough estimate of the product may be obtained beforehand:—$165 \times 78 = 150 \times 80$ or 12000 (roughly). The product may hence have 5 digits.

Artifices.—

1. $820 \times 997 = 820 \times 1000 - 820 \times 3.$
2. $88 \times 25 = 88 \times \frac{100}{4} = \frac{8800}{4} = 2200.$
3. $884 \times 125 = 884 \times \frac{1000}{8} = \frac{884000}{8} = 110500.$
4. The diagram shows that

$$(a + b) \times (a + b) \text{ or } (a + b)^2$$
$$= a \times a + 2a \times b + b \times b.$$

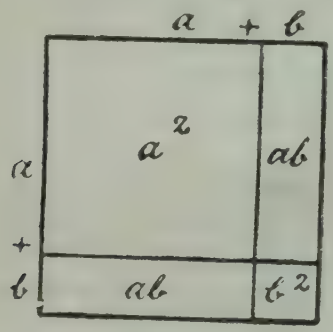

Fig. 3.

$\therefore$ $85 \times 85 = 80 \times 80 + 2 \times 80 \times 5 + 5 \times 5.$

Short method of squaring a No., ending in 5 :—

e.g., $85^2 = 80 \times 80 + 80 \times 10 + 5^2$
$= 80 \times 90 + 25$
$= 7200 + 25 = 7225.$

Multiply 8 by 9, the next higher No., and affix 25 to the right of the product.

5. Multiplying by factors :—

$655 \times 72 = \underline{655 \times 8} \times 9 = 5240 \times 9.$

6. Multiplying in 2 lines, etc. :—

5024
568
40192 $= 5024 \times 8$
281344 $= (5024 \times 8) \times 7$ tens or $\underline{40192 \times 7}$ tens.
2853632

7. In 2005×652, choose 2005 as multiplier, as it contains only *two effective digits* 2 and 5. (The multiplicand and multiplier may be interchanged.)

The result of multiplying more than two Nos. is known as their *continued product*.

e.g., $4 \times 18 \times 20 \times 5 = 7200.$

4. DIVISION.

'$155 \div 5$' means 'find that by which 5 should be multiplied to give 155.'

Division by factors :—

Divide 14498 by 35.

$35 = 5 \times 7.$

5 | 14498
7 | 2899 groups of 5 + 3 units over.
414 groups of 35 + 1 group of 5.

The *remainder* = 1 group of 5 + 3 units
= 8.

14498 is the *dividend*, 414 is the *quotient* and 35 is the *divisor*.

Long division :—

Divide 63428 by 138.

Rough estimate of the quotient :—

$$\frac{63428}{138} = \frac{60000}{150} \text{ or } 400 \text{ roughly.}$$

Ordinary method.

```
      459
138)63428
    552
    822
    690
   1328
   1242
     86
```

Italian method.

```
138 | 459
    | 63428
    | 822
    | 1328
    | 86
```

In the Italian method, the processes $634 - 138 \times 4$, $822 - 138 \times 5$ and $1328 - 138 \times 9$ are done in one step, *e.g.*, in $634 - 138 \times 4$, $4 \times 8 = 32$, $+ 2 = 34$. Put *2;* carry 3; $4 \times 3 = 12$, $+ 3 = 15$, $+ 8 = 23$. Put *8;* carry 2; $4 \times 1 = 4$, $+ 2 = 6$, $6 + 0 = 6$.

Note :—*Dividend = Divisor* $\times$ *Quotient* $+$ *Remainder. The Quotient and the divisor are interchangeable.*

Artifices :—

1. Division by 25, 50, 500, 125, etc.

$$\frac{x}{25} = \frac{x \times 4}{100}; \quad \frac{x}{125} = \frac{x \times 8}{1000}; \quad \frac{x}{500} = \frac{x \times 2}{1000}.$$

$\therefore$ Division by 25 is the same as multiplication by 4 and dividing the product by 100 and so on.

2. Divide 1476 by 99

$$\begin{aligned} 1476 &= 14 \times 100 \quad + 76 \\ &= 14 \times 99 + 14 + 76 \\ &= 14 \times 99 + 90 \end{aligned}$$

$\therefore$ $1476 \div 99$ gives 14 as quotient and 90 as remainder.,

5. The four rules applied to money, weights, etc. may be done by the methods adopted for ordinary Nos.

For, our systems of money, weights, etc, are also written in particular scales of notation, *e.g.*, just as $725 = 5$ units + 2 tens of units + 7 tens of tens, Rs. 7 2 as. 5 p. = 5 pies + 2 twelves of pies + 7 sixteens of twelves of pies.

EXERCISES.

(1) What are the place-values of 6 and 3 in 3264? How many times is the local value of 6 contained in that of 3, and that of 4 in that of 2? If a cipher be added between 2 and 6, how will the place-values of digits be affected?

(2) Find the greatest and the least No. which could be formed with 9, 2, 0, 3, and 4.

(3) Find the value of 'x' in the following :—

(*a*) (Rs. 18-2-9) + (Rs. x) = Rs. 30.

(*b*) $244 \times 11 + x = 4106$.

(*c*) $8850 \times x = 7450 \times 177$.

(*d*) Rs. 468-5-4 $- x =$ Rs. 87-7-0.

(4) Find the value of the following using any short methods you know :—

(*a*) 6084×25; 608×125; 608×625; $5375 \div 125$.

(*b*) 1092×2452.

(*c*) 35367×36124; 868×199; 7265×4812; 23478×398; $78256 \div 199$.

(*d*) $25 \times 8 \times 41 \times 125$; $(23 \times 51 \times 57) \div (19 \times 17)$ $83 \times 219 - 139 \times 75 + 219 \times 27 - 35 \times 139$.

(*e*) $9827 \times 2310 + 9827 \times 4235 + 9827 \times 3454$.

(5) $34034 \div x$ gives the quotient 97 and the remainder 278 Find x. Also find the quotient and remainder when 34034 is divided by 97.

(6) Given that $238 \times 357 = 102816$, deduce the values of 287×357, 288×51, 72×357 and 289×356.

(7) If $144750 \div 2000$ gives a quotient 72 with remainder 750, find the quotient and remainder when 144750 is divided by 1999.

(8) Find the digits marked '*' in the following Italian Division :—

```
        2 * *
  387 | * * * * *
      | 1 0 2 4
      |   2 5 0 3
      |     1 8 1
```

(9) Fill up the vacant spaces in the following division by factors :—

```
4 |_____
  8 |     —3
    236   —2.
```

Find the total remainder.

(10) State how you would find out approximately the number of digits in the products 123×724; 525×7894 and 22456×786.

(11) What must be added to Rs. 72-15-2 in order that the sum may be exactly divisible by 31 ?

(12) Make out a bill for the following :—18 yds. of silk at Rs. 2-4 a yd.; 2 maunds of coffee seed at Re. 1-12-6 a viss; 10 pieces of mull at Rs 10-6 per piece; 30 packets of 6 banyans each at 10 as. a banyan and 15 collars at 6 as. each.

(13) I went to Wahab's gardens, where he gave me 7 cucumbers. I put some into my pocket and cut each of the remaining into 8 pieces. I ate 15 pieces, gave Wahab 10 and the remaining 7 were given to the gardener. How many cucumbers did I put into my pocket ?

(14) A man went to market to sell his cattle and took Rs. 10-15-6 with him. He sold 20 of his buffaloes and bought 4 bulls at Rs. 180 a pair. He then had Rs. 222-3-6 in his pocket. Find the average price he got for a buffalo.

SECTION II.

FACTORS AND MULTIPLES.

6. The following tests of *divisibility* could be easily verified by numerical examples :—

A number is exactly divisible :

(1) by 2, if it contains an exact number of tens or if, its *last* digit (the units' digit) is even

(2) by 4, if it contains an exact number of hundreds, or if its last two digits form a number exactly divisible by 4 ($25 \times 4 = 100$) ;

(3) by 5, if its last digit is '0' or '5' ;

(4) by 8, if it contains an exact number of thousands, or if its last 3 digits form a number exactly divisible by 8 ($1000 = 8 \times 125$).

(5) by 9 or 3, if the sum of its digits is exactly divisible by 9 or 3 ;

For example,—

$$
\begin{aligned}
36594 &= 4 &&= 4 \\
&\quad + 9 \times 10 && + 9 \times 9 + 9 && \because\ 10 = 9 + 1 \\
&\quad + 5 \times 100 && + 5 \times 99 + 5 && \because\ 100 = 99 + 1 \\
&\quad + 6 \times 1000 && + 6 \times 999 + 6 && \because\ 1000 = 999 + 1. \\
&\quad + 3 \times 10000 && + 3 \times 9999 + 3 && \because\ 10000 = 9999 + 1.
\end{aligned}
$$

$$
= \begin{Bmatrix} \text{a multiple} \\ \text{of 9} \end{Bmatrix} + \begin{Bmatrix} \text{the sum of the} \\ \text{digits in the No.} \end{Bmatrix}
$$

(6) by 6, if the tests (1) and (5) above hold good ;

(7) by 11, if the difference between the sum of the digits in the even places and that of the digits in the odd places = 0, or a multiple of 11 ;

For example :—

$$\begin{aligned} 5324 &= 4 && = && 4 \\ &+ 2 \times 10 && + 2 \times 11 - 2 && \because 10 = 11 - 1 \\ &+ 3 \times 100 && + 3 \times 99 + 3 && \because 100 = 99 + 1 \\ &+ 5 \times 1000 && + 5 \times 1001 - 5 && \because 1000 = 1001 - 1 \\ & && = \{ \text{a multiple of 11} \} + 4 - 2 + 3 - 5. \end{aligned}$$

(8) by 12, if the tests (2) and (5) hold good ;

(9) by 99, if the tests (5) and (7) hold good ; and

(10) by 25 if it ends in 2 ciphers, 50 or 75.

Note 1. From test (5) above it follows that the remainder in division by 9 could be found by casting out the nines, *e.g.*, in 875903. $8 + 7 = 15, -9 = 6, +5 = 11, -9 = 2, +9 - 9 = 2, +0 + 3 = 5$. $\therefore$ 875903 ÷ 9 should yield 5 as remainder.

Note 2. Also, *the difference between 2 Nos. formed by using the same digits in different order is exactly divisible by* 9, *e.g.*, $5834 - 4835 = 999$.

Note 3. Testing a product by casting out the nines :—

Let $283 = 9x + 4$ $\quad \because 2 + 8 + 3 = 9 + 4.$

and $584 = 9y + 8.$ $\quad \because 5 + 8 + 4 = 9 + 8.$

$$\begin{aligned} \text{Since } (9x + 4) \times (9y + 8) &= 81x \times y + 36y + 72x + 32 \\ &= \text{multiple of } 9 + 32 \\ &= \text{another multiple of } 9, + 5 \ (= 3 + 2). \end{aligned}$$

$\therefore$ in 283×584 divided by 9, the remainder must be 5.

7. G. C. F. BY MEANS OF FACTORS.

5 multiplied by 6 gives 30. 5 and 6 are *factors* of 30. Also, 30 is a *multiple* of 5 or 6.

Nos. like 5, 7, 47, etc., have no factors other than themselves and unity. They are called *prime Nos.* Nos. which have other factors are called *composite Nos.*

The factors of 30 are 2, 3, 5, 6, 10, 15 and those of 42 are 2, 3, 6, 7, 14.

The factors common to 30 and 42 are 2, 3 and 6. Of these, 6 is the greatest and is called their G. C. F. (= Greatest Common Factor.)

Ex. Fnd the G. C. F. of 144 and 192.

If the same prime No. repeats itself as a factor in the numbers given, it will occur in their G.C.F. the least No. of times that it occurs as a factor in the Nos. given.

The above method of finding the G. C. F. would also reveal whether the Nos. have any common factor. Two Nos., *e.g.*, 35 and 44, have no common factors other than unity and are said to be *prime to each other.*

8. G. C. F. BY LONG DIVISION METHOD.

The following is the principle involved in this method :—

If 'A' divided by 'B' gives 'R' as remainder, the G.C.F of A and B is also the G.C.F. of R and A, *e.g.* 42 ÷ 36 gives 6 as remainder. The G.C.F. of 42 and 36 = that of 36 and 6. By successive application of the above principle the G.C.F. of 3478 and 12956 is found out below :—

A) B ... ...
R

1	3478	12596	3
	2162	10434	
1	1316	2162	1
	846	1316	
1	470	846	1
	376	470	
	94	376	4
		376	

= 94 *is the G. C. F. required.*

[In 3478 ÷ 2162, 2162 ÷ 1316, 1316 ÷ 846, the method of Italian division may be advantageously used.

In the above working, when it is known that the G. C. F. required should be that of 470 and 846, the factor 5 of 470 may be removed as it is not common to both numbers. The G. C. F. could then be read to be 94 by a single division.]

Similarly, the G. C. F. of 3 Nos. has to be found, *e.g.* 3478, 12596 and 44300, first find the G. C. F of 3478 and 12596, to be 94 and then find the G C. F. of 94 and 44300.

EXERCISES.

(1) Find which of the following are prime Nos. :—47, 57, 67, 77, 87, 97, 121, 111, 231 and 441.

(2) Write down the 1st 150 Natural Nos., beginning from 2, strike off every 2nd No. ; beginning from 3, strike off every 3rd No. ; and so on.

Explain why the numbers struck off are composite and the rest are prime.

(3) Find by short tests whether 2, 3, 4, 5, 6, 7, 8, 9, 11, 12 and 99 are the factors of 118764, 90245 and 4837000.

(4) Obtain the remainders in dividing the following by 9 :—

(*a*) $642 + 8556 + 8066$.

(*b*) $54321 - 2345$

(*c*) 84972×45698.

(5) Find x and y if $9y\ 594$ and $1x\ 45$ be exactly divisible by 11. Also, find their values, when the numbers are exactly divisible by 9.

(6) Given that $23a\ 24b$ is exactly divisible by 44, find all the values of 'a' and' 'b'.

(7)

Scale 1 cm. =20'

Find the greatest common length that would measure exactly the lines drawn on the left.

(8) Find the length of the largest square slab, an exact number of which would fully cover a floor $21' \times 13'\ 6''$.

(9) Find by factorising the G. C. F. of :—
81, 120; 192, 240; 56, 63.

(10) Find by Long Division, the G C. F. of 329, 282; 345, 506; and 1316, 4512, 2820.

(11) Find the digits marked '*' below :—

$$\begin{array}{r} * \ * \ * \ * \\ \quad\ * \ * \\ \hline 9\ 9\ 7\ 5 \\ \quad * \ * \ * \ * \\ \hline 1\ 0\ 8\ 3\ 0\ 0 \end{array}$$

$$\begin{array}{r|l} & \quad * \ * \\ * \ * \ ) \ 1\ 4\ 4\ 7 & \\ \quad\ * \ * \ * & \\ \hline \quad\ 4\ * & \\ \quad\ * \ * & \\ \hline \quad 1\ 2 & \end{array}$$

(12) Two rolls of carpet of the same width are of lengths $102'$ and $90'$. They are cut into pieces as long as their greatest common measure. How many pieces could be obtained?

(13) Show with reference to particular Nos, that the G. C. F. of A and B is also a factor of A + B, A — B, mA + nB and of mA — nB.

(14) In finding the G. C. F. of 2 Nos. by Long Division the successive quotients from the beginning are 1, 9 and 4 and the G. C. F. is 23. Find the Nos.

(15) Three baskets contain 252, 260 and 162 apples. These are arranged in heaps so that each heap contains the same No. of apples and the contents of each basket are divided into an integral No. of heaps. Find the largest No. of apples that could be so arranged in a heap and how many such heaps could be formed.

(16) 1431 and 2105, when divided by 'x' yield 3 and 5 as remainders. Find the Nos, that x may represent. Also, find the greatest among them.

(17) From 3 rocks have been split 283, 805 and 661 survey stones respectively. These have to be carried to fields in bandies. If each bandy carries the same No. and if this be the greatest that it could so carry, there yet has to be left 1 stone unremoved near each rock. Find the number of stones carried in each bandy.

(18) I have 2328 marakals of rice and 1284 marakals of black gram. I wish to distribute them in bags so that each bag contains an equal quantity of rice or of black gram, What is the maximum quantity which each bag would then contain ?

9. L. C. M.

The multiples of 30 are *60*, 90, *120*, 150, *180*, 210, etc. ; and those of 20 are 40, *60*, 80, 100, *120*, 140, 160, *180*, 200, 220, etc.

Of these 60, 120 180, etc., are their common multiples and 60 is their Least Common Multiple (L C. M.)

$$30 = 3 \times 10.$$
$$20 = 2 \times 10.$$

And the L. C. M. of 30 and 20.

$$= 3 \times 2 \times 10.$$

The following method of finding the L C. M. is deduced from the above : —

Find the G. C. M. of the 2 Nos. Divide each No. by their G. C. F. Multiply the quotients and multiply the result by their G. C. F.

The final result is their L. C. M. The same method holds good in finding the L. C. M. of more than two Nos.

Ex. Find the L. C. M. of 720, 640 and 935.

$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$

$935 = 5 \times 11 \times 17.$

Multiply together all the prime factors that occur in the given numbers taking care that every factor is taken the largest number of times it occurs in any number. Thus, in this case 2 should be taken 7 times ; 3 twice and 5, 11, 17 once each ; and the L. C. M.

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 11 \times 17$

$= 1,077,120.$

The following method may be adopted where the factors may be read off at sight :—Find the L. C. M. of 18, 30, 48, 54 and 60.

6	*18*, 30, 48, 54, 60	Strike off 18 and 30, as they are contained in 54 and 60.
2	8, 9, 10	9 does not contain 2 and hence must occur as a separate factor in the L. C. M.
	4, 9, 5	

∴ the L. C. M. $= 6 \times 2 \times 4 \times 9 \times 5 = 2160$

EXERCISES.

(19) The number of rupee-coins in a box is such that if it be counted in fifteens or twenties or twenty-fives there would be no balance left. If the money in the box ranges from Rs 1000 to Rs. 1400, find the exact amount kept in it.

(20) A, B and C start from the same straight line and walk perpendicular to it in the same direction with their paces 1'6", 2' and 2' 6" apart Find how often their foot-toes will be in the same straight line in walking a furlong.

(21) An exact No. of tiles would cover the floor of a room if the tiles, 1' × 10", be arranged lengthwise or breadthwise along the length or breadth of a hall. If the dimensions of the hall are between 34' and 42', find them.

(22) In a Drill ground the pupils are arranged in rows of 4, 6 or 8. It is then found that only 3, 5 or 7 remain respectively for the last row If the number of pupils in the drill ground lies between 100 and 130, find it.

(23) A could run once round a circle in 20 secs., B in 25 secs., and C in 30 secs. If all start from the same point at the same time and in the same direction and run round the circle, how often in 24 min. will they all be together at the starting point?

(24) A Tahsildar camps at a place every 14th day, an Inspector every 28th day and an Engineer every 35th day. If they were all in the same camp on 31-8-17, when will they next meet together in that camp?

(25) Show that the L. C. M. of 2 Nos. $= \frac{\text{their product}}{\text{their G. C. F.}}$. The G. C. F. of 2 Nos. = 47. Their L. C. M. = 564. Find the Nos. if both of them have 3 digits.

(26) Find the L. C. M. of 462, 273, 90.

SECTION III.

VULGAR FRACTIONS.

10. The meaning of a Fraction :—

The straight line AB contains 4 units and CD contains 3 units.

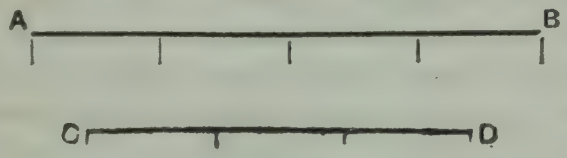

Fig. 4.

∴ CD is *three-fourths* ($\frac{3}{4}$) of AB.

The fraction $\frac{3}{4}$ *means*

(1) $3 \div 4$ or '$\frac{1}{4}$ of 3';

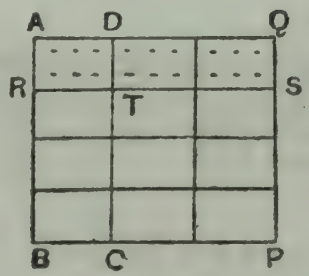

Fig. 5.

(the rectangle ABCD represents 1 unit, ABPQ 3 units, and ARSQ $\frac{1}{4}$ of 3 units)

or

(2) one unit is divided into 4 equal parts and 3 such parts are taken, *e.g.*, RBCT is $\frac{3}{4}$ of ABCD.

The fraction $\frac{a}{b}$ means $a \div b$ or $\frac{1}{b}$ of 'a' and $\frac{a+3}{4} = \frac{1}{4}$ of $(a + 3)$

11. In a fraction $\frac{3}{4}$, '3' and '4' are its *terms*. 3 is its *numerator* and 4 is its *denominator*. The denominator '4' tells that the fraction belongs to the denomination of fourths ($\frac{1}{4}$ths) and the numerator (= numberer) tells how many such fourths form the fraction.

The word 'fraction' means 'part' and hence the numerator is generally less than the denominator. In $\frac{15}{4}$ (= $\frac{1}{4}$ of 15 or 15 times $\frac{1}{4}$) the numerator is greater than the denominator and the fraction is hence called an *improper fraction* as distinguished from a *proper fraction like* $\frac{3}{4}$.

An improper fraction could be expressed as a *mixed number*. For 15 fourths of a Re we could get *three* whole rupees and *three* quarters of a Re. $\therefore \frac{15}{4} = 3 + \frac{3}{4}$ which is written as $3\frac{3}{4}$ and read as '3 and $\frac{3}{4}$'. Since $3\frac{3}{4}$ is a whole number and a fraction mixed together, it is called a mixed number.

12. EQUIVALENT FRACTIONS

The following important principle is useful in comparing the values of fractions as well as in adding and subtracting them:—

For six *eighths* of a Re (or 2-anna pieces) we could get three *quarters* of a Re. (or four-anna pieces).

$$\therefore \frac{3}{4} = \frac{6}{8} \left(= \frac{3 \times 2}{4 \times 2} \right)$$

$$\text{and } \frac{6}{8} = \frac{3}{4} \left(= \frac{6 \div 2}{8 \div 2} \right).$$

$\therefore$ *the value of a fraction is unchanged if both its terms be multiplied or divided by the same quantity.*

$$\therefore \frac{a}{b} = \frac{a \times m}{b \times m}.$$

Such fractions are called *equivalent fractions.*

Ex. Compare the values of $\frac{3}{8}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{12}$ by expressing them as fractions with a common denominator, choosing for it the L. C. M. or any other common multiple of the denominators.

13. ADDITION AND SUBTRACTION.

Quantities must be of the same kind in order that they may be added or subtracted. Similarly, before finding the values of $\frac{3}{4} + \frac{2}{3}$ or $\frac{3}{4} - \frac{2}{3}$, the fractions must be expressed in a common denomination. Each of the above fractions may be expressed in twelfths.

$$\frac{3}{4} = \frac{9}{12} \qquad \frac{2}{3} = \frac{8}{12}$$

$$\therefore \frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{9+8}{12} = \frac{17}{12} = 1\frac{5}{12}$$

$$\text{and } \frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{9-8}{12} = \frac{1}{12}.$$

Ex. A farmer wishes to sow $\frac{2}{3}$ of his land with maize and $\frac{1}{4}$ with black gram. What fraction of the land is left for other purposes?

[Any whole number could be represented as a fraction with its terms equal to each other.

$$1 = \frac{x}{x} = \frac{2 \times (x+3)}{2 \times (x+3)}\Big]$$

14. MULTIPLICATION OF FRACTIONS.

A man left $\frac{4}{5}$ of his property to his children and $\frac{5}{8}$ of this to his eldest son. What portion of the whole property did the eldest son get ?

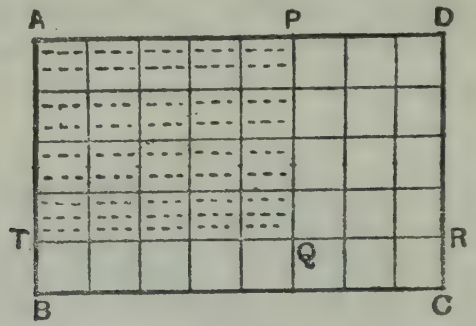

Fig. 6

Make a free-hand sketch of a rect ABCD to represent the whole property. Divide it into fifths and eighths as in the figure. ABCD represents the whole property, ATRD $\frac{4}{5}$ of it, and ATQP $\frac{5}{8}$ of ATRD. $\therefore$ ATQP represents $\frac{5}{8}$ of $\frac{4}{5}$ of the whole property and is the eldest son's share. Since ABCD contains 40 equal divisions and ATQP contains 20 of them, the eldest son's share is $\frac{20}{40}$ of the whole property.

$$\therefore \frac{5}{8} \text{ of } \frac{4}{5} = \frac{20}{40}.$$

NOTE :— $\frac{5}{8}$ of $\frac{4}{5} = \frac{5 \times 4}{8 \times 5}$.

$\frac{a}{b} \times \frac{c}{d}$ means $\frac{c}{d}$ of $\frac{a}{b} = \frac{c \times a}{d \times b}$.

$\frac{5}{8}$ of $\frac{4}{5} = \frac{5 \times 4}{8 \times 5} = \frac{1}{2}$ (by removing the common factors from the terms.)

15. DIVISION OF FRACTIONS.

My father gave me $\frac{5}{12}$ of his money and he gave $\frac{2}{5}$ of it to my sister. Express my sister's share as a fraction of my share.

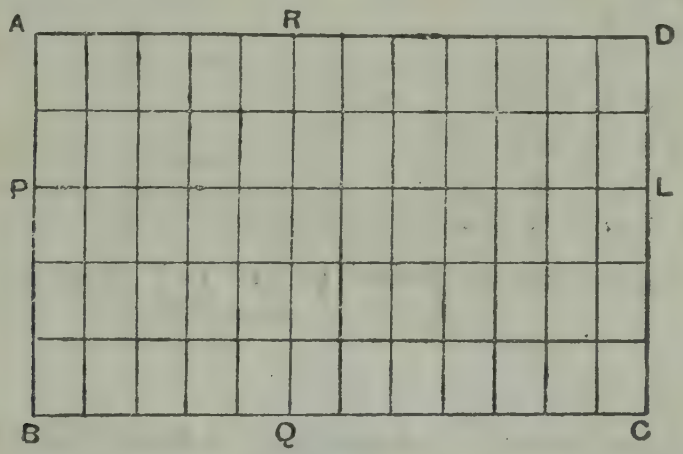

Fig. 7.

Let ABCD represent the whole money, divide it into fifths and twelfths as in the figure, let APLD represent my sister's share ($\frac{2}{5}$) and ABQR my share ($\frac{5}{12}$). Since APLD contains 24 equal divisions and ABQR contains 25 such divisions, my sister's share is $\frac{24}{25}$ of my share.

$\therefore \frac{2}{5}$ is $\frac{24}{25}$ of $\frac{5}{12}$.

The above is expressed otherwise as

$\frac{2}{5} \div \frac{5}{12} = \frac{24}{25}$.

$\therefore$ *the division* $\frac{a}{b} \div \frac{c}{d}$ means '*what fraction of* $\frac{c}{d}$ is $\frac{a}{b}$? or how much of $\frac{c}{d}$ is $\frac{a}{b}$?'

Another meaning may be given to the division $\frac{2}{5} \div \frac{5}{12}$ which holds good in all cases, *viz.*, 'by what should $\frac{5}{12}$ be multiplied to get $\frac{2}{5}$?'

Let $\frac{2}{5} \div \frac{5}{12} = x$.

Then $\frac{5}{12} \times x = \frac{2}{5}$.

$\therefore 12 \times \frac{5}{12} \times x = \frac{2}{5} \times 12$.

$\therefore 5x = \frac{2}{5} \times 12$.

$\therefore \frac{5x}{5} = \frac{2}{5} \times \frac{12}{5}$.

$\therefore x = \frac{2}{5} \times \frac{12}{5}$.

$\therefore \frac{2}{5} \div \frac{5}{12} = \frac{2}{5} \times \frac{12}{5}$.

$\therefore \frac{p}{q} \div \frac{l}{m} = \frac{p}{q} \times \frac{m}{l}$.

[NOTE:—The *reciprocal* of 'a' is $\frac{1}{a}$ and that of $\frac{l}{m}$ is $\frac{m}{l}$.

$\therefore$ dividing by $\frac{l}{m}$ is equivalent to multiplying by its reciprocal $\frac{m}{l}$.]

EXERCISES.

(1) Show graphically that

$\frac{2}{3} = \frac{8}{12}, = \frac{44}{48} = \frac{11}{12}, \frac{57}{12} = 4\frac{3}{4}, 1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$.

$\frac{1}{4} + \frac{1}{3} + \frac{1}{2} = 1\frac{1}{12}, \frac{5}{6} - \frac{1}{2} - \frac{1}{12} = \frac{1}{4}, \frac{5}{6} \times \frac{2}{3} = \frac{5}{9}$ and

$1 \div \frac{1}{4} = 4$.

(2) Supply the missing terms below:—

$\frac{(\ )}{4} = \frac{27}{36}, \frac{75}{(\ )} = \frac{100}{96} = \frac{(\ )}{48}$, and $\frac{16}{28} = \frac{4}{(\ )}$.

(3) Show graphically or otherwise that

(*a*) $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{2+4+6}{(5+10+15)}$.

(*b*) $\frac{2}{5} + 1 = \frac{2+5}{5}$

(*c*) $\frac{7}{6} - 1 = \frac{7-6}{6}$.

(*d*) the product of 2 proper fractions is less than either fraction.

(*e*) $\frac{7}{12}$ is less than $\frac{7+5}{12+5}$ and greater than $\frac{7-5}{12-5}$.

(4) Work out orally:—

$$3\frac{1}{8} + 4\frac{1}{4} + 5\frac{3}{8} + 9\frac{9}{10} + 7\frac{7}{32}; \quad 16\frac{1}{2} + 9\frac{3}{8}.$$

(5) Show, without unnecessary work, that the following cannot be correct:—

$$\frac{5}{8} + \frac{13}{16} + \frac{26}{100} = 1\frac{79}{400}; \quad 206 \times \frac{3}{2} + \frac{88}{39} = 147\frac{20}{39}.$$

(6) Arrange the following in their descending order of magnitude:—

$$\frac{16}{30}, \frac{3}{14}, \frac{30}{47} \text{ and } \frac{9}{10}.$$

How can you tell orally that their sum is greater than 2?

(7) The length of a metre = $39\frac{3}{8}$ inches. Find how many miles there are in 75 kilometres. Verify that a kilometre (= 1000 metres) is nearly equal to $\frac{5}{8}$ of a mile.

(8) A person sold $\frac{3}{5}$ of his estate to A and $\frac{3}{4}$ of the remainder to B. He still has 75 acres of land. Find the area of the whole estate.

(9) A man goes to market with a certain sum of money, buys plantains for $\frac{1}{4}$ of his money and mangoes for $\frac{1}{6}$ of the remainder. He finds he has Rs. 2 8 as. left. How much had he at first?

(10) In a college staff $\frac{2}{3}$ of the number of Professors are Hindus, $\frac{1}{2}$ of the rest Europeans and the remaining 4 are Indian Christians. Find the strength of the staff.

(11) A vertical pole is partly above ground and partly below ground. The part below the ground = $\frac{2}{5}$ of the pole + 2" and that above the ground = $\frac{2}{5}$ of the pole + 3". Find the length of the pole. What fraction of the portion above the ground is that below the ground?

(12) From the corners of a square 3" long, four squares are cut each 1" long. What portion of the original square is the piece left?

(13) A train had a certain number of passengers when it left Egmore. When it came to Chetput, $\frac{1}{10}$ of the number got down

and 10 got in. When it next came to Kodambakam $\frac{1}{10}$ of the number of passengers it then had got down and 2 got in. If 254 persons travelled in the train between Kodambakam and Mambalam, find how many there were in the train when it left Egmore. (Assume that the stations mentioned are in the order in which the train stopped).

(14) P, Q, S, T are 4 villages along a road. S is midway between P and T. Q is nearer P than T by 6 miles. Find the distance between S and Q. If the distance from P to Q $= \frac{2}{3}$ of that from Q to T, find how far S is from T.

(15) Show that the following statement is incorrect:—

$$\frac{\text{Re. 1}}{\text{Rs. 2 8 as.}} = \frac{\text{2 measures 6 ollocks}}{\text{31 measures}}$$

(16) Cut from a sheet of peper fractions of it equal to $\frac{2}{3}$ $\frac{5}{8}$ $\frac{4}{5} \div \frac{5}{6}$ and $\frac{2}{3}$ of $\frac{3}{8}$.

(17) Express the following in simplest form:—

(*a*) $17\frac{1}{13} \times 13\frac{1}{17} - 5\frac{1}{4} \times 4\frac{1}{5}$.

(*b*) $\dfrac{\frac{5}{24} \div \frac{1}{2} \div \frac{5}{8} \times \frac{2}{20}}{\frac{1}{2} \text{ of } \frac{3}{4} + 16\frac{5}{8} \times \frac{1}{11}\frac{6}{12}}$.

(*c*) $\dfrac{1 \div \frac{1}{2} \div \frac{1}{4} \div \frac{1}{8}}{1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8}} \div \frac{3}{35} \times 3$.

(*d*) $1\frac{1}{2} \times 2\frac{1}{8} + 4\frac{1}{7} \div 12\frac{3}{7} + \frac{1}{6}$.

(*e*) $3\frac{1}{2} \times 1\frac{1}{2}$ of $8\frac{1}{3} \div 2\frac{1}{2} \times 3\frac{3}{4}$.

(*f*) $16\frac{1}{13}$ of $38\frac{1}{16} - \frac{5}{9}$.

In simplification involving all the signs +, −, ×, ÷, and 'of' note the following conventions:- Multiplications and divisions must be performed before addition and subtraction; 'of' has the force of a bracket and otherwise, multiplication and division must be performed from left to right.]

16. ALIQUOT PARTS.

When one quantity is expressed as a fraction of another and the fraction has '1' as its numerator, the fraction is called an *aliquot part*. An aliquot part of a quantity is hence contained in it an exact number of times. ('Aliquot' means 'how many').

The following aliquot parts are worth remembering:—

8 as.	= Re. $\frac{1}{2}$	10*s.*	= £$\frac{1}{2}$
4 as.	= ,, $\frac{1}{4}$	5*s.*	= £$\frac{1}{4}$
1 a.	= ,, $\frac{1}{16}$	4*s*	= £$\frac{1}{5}$
5 as. 4 ps.	= ,, $\frac{1}{3}$	6*s.* 8*d.*	= £$\frac{1}{3}$
1 a. 4 ps.	= ,, $\frac{1}{12}$	1*s.* 8*d.*	= £$\frac{1}{12}$
4 ps.	= ,, $\frac{1}{48}$	19*s.*	= $\frac{1}{20}$ of £19

Multiplication of money, weights, etc., may be attempted by means of aliquot parts in questions like the following :—

Find the cost of 139 books at Rs. 2-6-6 per copy.

Rs. 2-6 6 = Rs. 2 + 4 as. + 2 as. 6 ps.

= Rs. 2 + Re. $\frac{1}{4}$ + $\frac{1}{2}$ of 4 as. + $\frac{1}{4}$ of 2 as.

	Rs.	As.	Ps.	
	139	0	0	= Cost at Re. 1 per copy.
			2	
	278	0	0	= Cost at Rs. 2 per copy.
4 as. = Re. $\frac{1}{4}$.	34	12	0	= ,, 4 as. ,,
2 as. = $\frac{1}{2}$ of 4 as.	17	6	0	= ,, 2 as. ,,
6 ps. = $\frac{1}{4}$ of 2 as.	4	5	6	= ,, 6 ps. ,,
Total Rs.	334	7	6	= Cost @ Rs. 2-6-6 per copy.

Find the cost of 2 tons 15 cwt. 70 lbs. of fuel at Rs. 13 8 as. per ton.

	Rs	As.	Ps.	
	13	8	0	= Cost of 1 ton.
			2	
	27	0	0	= Cost of 2 tons.
10 cwt. = $\frac{1}{2}$ a ton	6	12	0	= ,, 10 cwt.
5 cwt. = $\frac{1}{2}$ of 10 cwt.	3	6	0	= ,, 5 cwt.
70 lbs. = $\frac{1}{5}$ of 5 cwt.	0	6	9	= ,, 70 lbs.
Total Rs	37	8	9	= Cost of the fuel reqd.

(The above method is called *practice*.)

EXERCISES.

(18) Find the cost of the following:—

(*a*) 144 pencils at 6 as. 6 ps. per doz.

(*b*) 119 reams of paper at Rs. 2-13-4 per ream.

(*c*) 99 tons of fuel at Rs. 12-15-6 per ton.

(*d*) 284 articles at £11-18-4 each.

(*e*) 1000 articles at Rs. 11-11-11 each.

(*f*) 157 mds. of coffee seeds at Rs. 15-15-8 per md.

(*g*) 229 copies of a book at £2-12-6 per copy.

Show that in (*b*), (*c*), (*d*), (*f*), and (*g*) one line of multiplication and one of division will do.

(19) A bankrupt's *assets* would allow him to pay only 14*s*. 8*d*. per £ of his debts. If his debts amount to £17,000, find the amount of his assets.

(20) Find the cost of making $99\frac{4}{4}\frac{3}{8}$ miles of road at Rs. 215 8 as. per mile. [It is better to begin thus:—Rs. 99-14-4 is the cost of making $99\frac{4}{4}\frac{3}{8}$ miles at 1 Re. per mile.]

(21) If Re. 1 = 1*s*. 4*d*., find in English money the cost of constructing a railway 329 miles long at Rs. 77,386-13-4 a mile.

SECTION IV.

DECIMALS.

17. THE DECIMAL FRACTION IS AN EXTENSION OF THE ORDINARY SCALE.

In Section I it is stated that the decimal scale of notation is generally used in writing numbers. The scale may be extended to fractions less than 1. If each unit be divided into 10 equal parts (look at a foot-rule showing $\frac{1}{10}$"), each such part is a *tenth* ($\frac{1}{10}$) and is written ·1. Thus 6·6 = 6 units + 6 tenths and is read as '6 point 6.' Each tenth may be further divided into 10 equal parts and each such part is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ (*a hundredth*) and is written ·01. Similarly, $\frac{1}{10}$ of a hundredth is a *thousandth* (·001); $\frac{1}{10}$ of a thousandth is a *ten-thousandth* (·0001) and so on. Thus in 66·6666 (read as 'sixty-six point six six six six') the place-value of each 6 is $\frac{1}{10}$ of that to the left of it. Also, any decimal represents a common fraction in another form, *e.g.*, $6{\cdot}526 = 6 + \frac{5}{10} + \frac{2}{100} + \frac{6}{1000}$

$= \frac{6526}{1000}$ or 6526 thousandths.

Note 1.—The decimal point (·) separates the integral from the fractional part.

Note 2.—Any number of zeroes may be added to the right of a decimal without altering its value while each zero added immediately to the right of the decimal point reduces the value of the decimal part by one-tenth, *e.g.*, ·440 = ·44 but ·044 = $\frac{1}{10}$ of ·44.

18. THE METRIC SYSTEM.

In our system of weights and measures, no uniformity is observed in deriving higher and lower units from the main unit. In France the systems of lengths, weights, capacity, etc., are in the decimal scale. One *metre* (m.) is chosen as the unit for measuring lengths.

For smaller measurements, a *deci*-metre (1 dm. = $\frac{1}{10}$ metre), a *centi*-metre (1 cm. = $\frac{1}{100}$ m.), a *milli*-metre (1 mm. = $\frac{1}{1000}$ m.) are used (see the graduations in the metre-ruler) and for longer measurements a *Deka* metre (1 Dm. = 10 m.), a *Hecto*-metre (1 Hm. = 100 m.), a *Kilo*-metre (1 Km. = 1000 m), and a *Myria*-metre (1 Mm. = 10000 m.) are used. The prefixes (Myria-), (Kilo-), (Hecto-), (Deka-) (deci-), (centi-) and (milli-) are used in the same meaning with reference to 1 *litre*, the unit for measuring capacity (= *the capacity of a cubic decimetre*) and to 1 *gram*, the unit of weight (= the weight of 1 cubic centimetre of water at 4°C.) The advantage of such a decimal scale of tables is that quantities can be expressed at sight from one denomination to another and the four rules applied to them as in ordinary numbers. Hence a decimal scale of tables is often used in scientific calculations

Ex. 1. Express 5 Km. 8 Dm. 5 m. 4 dm. 6 cm. in metres and decimals of a metre.

	Km.	Hm.	Dm.	m.	dm	cm.
The length given =	5	0	8	5	4	6

= 5085·46 m.

Similarly 506·04 grams = 5 Hg. 6 g. 4 cg.

Ex. 2. Find the sum of 5 Km. 6 Dm. 7 m. 4 cm. and 7 Km. 6 Hm. 5 m. 4 dm.

Km.	Hm.	Dm.	m.	dm.	cm.
5	0	6	7	0	4
7	6	0	5	4	0
12	6	7	2	4	4

The sum = 12672·44 m.

19. ADDITION AND SUBTRACTION OF DECIMALS.

Since the scale of notation is the same, the processes are similar to those for integers.

Ex. 1. Find the sum of 7·56, 8·54 and 90·464.

Hundreds	Tens	Units		Tenths	Hundredths	Thousandths
		7	·	5	6	
		8	·	5	4	
	9	0	·	4	6	4
1	0	6	·	5	6	4

The addition may be done by keeping the decimals in the same row and adding up the digits of *like* place-value or, more conveniently, by writing them one below the other as shown in the margin.

Ex. 2. What must be added to 48·95 to get 121·2?

$$\begin{array}{r} 121\cdot20 \\ 48\cdot95 \\ \hline 72\cdot25 \end{array}$$

(5 + 5 = 10; 1 + 9 = 10, + 2 = 12; 1 + 8 = 9, + 2 = 11; 1 + 4 = 5, + 7 = 12.)

20. APPROXIMATIONS.

Experience would show that our measurements are necessarily inexact. In measuring the length of a page, suppose we find that it lies between 18·6 cm. and 18·7 cm. Our ordinary measuring instruments would not help us to find out how many hundredths of a centimetre there are in the length in addition to 18·6 cm. Further there is no use of our expressing the above result in such minute details. Hence in practice we have to decide whether it should be taken as 18·6 cm. or as 18·7 cm. In either case there will be an error. Yet, if the length be nearer to 18·6 cm. than to 18·7 cm., we take it as 18·6 cm. as such an approximation involves a less error. In making such an approximation, lengths equal to about $\frac{1}{10}$ of a cm. are considered negligible and are hence neglected. Similarly

in measuring the length of a foot-ball field, we express the result to the nearest whole number of yards and in measuring the length of a road we express it to the nearest whole number of miles or furlongs, certain negligible lengths in excess or in defect of that number being overlooked. In order to have some uniformity in making such approximations the following convention is adopted by mathematicians:—

"*With reference to a chosen unit, count as* '1' *fractions of it equal to, or greater than* $\frac{1}{2}$ *and count as* '0' *fractions less than* '$\frac{1}{2}$'."

Thus 88 yds. 1 ft. will be approximately taken as 88 yds.; 5 miles 7 furlongs as 6 miles; and 6 furlongs 5 chains as 7 furlongs.

Ex. 1. Find the value of 254·648 to the nearest hundredth.

254·648 lies between 254·64 and 254·65. The error in taking 253·648 as 254·64 is ·008 *too little* and that in taking it as 254·65 is ·002 *too much.*

∴ 254·648 = 254·65 to the nearest hundredth as the error is less by so taking it.

[The value to the nearest hundredth is also said to be the value *correct to two places of decimals*]

Since 254·648 = 254·6 + $\frac{48}{100}$ of a tenth, its approximate value = 254·6 to the nearest tenth, by the convention above stated. Similarly,

254·649 = 255 to the nearest unit.

and = 25 tens, to the nearest ten. (Why?)

Ex. 2. Find the values of Rs. 6 12 as. $2\frac{1}{8}$ ps. and of Rs. 6 2 as. $2\frac{3}{16}$ ps. *to the nearest pie.*

Ex. 3. Find to the nearest pie the sum a man should receive for working 17 days @ Rs. 11 per month.

21. DECIMAL MULTIPLICATION.

Multiplication and Division by 10, 100, 1000, etc. :—

In multiplying a decimal by 10, 100, 1000, etc., the place-value of each digit in the multiplicand would increase, 10, 100, 1000, etc. times and hence the decimal point would be shifted 1, 2, 3, etc. places to the right, *e.g.*, $25{\cdot}556 \times 100 = 2555{\cdot}6$.

Similarly in dividing by 10, 100, 1000, etc., each digit in the dividend would be reduced to $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., of its value and hence the decimal point would be shifted 1, 2, 3, etc places to the left, *e.g.*, $\frac{25{\cdot}56}{100} = {\cdot}2556$.

Multiplication of one decimal by another :—

Multiply 20·65 by 56·9.

$20{\cdot}65 \times 56{\cdot}9 = 206{\cdot}5 \times 5{\cdot}69$.

∴ R. E. (= rough estimate of product = 200×6 or 1200.

And the product required = that obtained by multiplying 206·5 by 5·69.

Note that the multiplier is so altered as to have only the units' digit in its integral part, that is to say, is expressed as a decimal in *its standard form* and the multiplicand is correspondingly changed so as not to affect the entire product. The advantages of expressing the multiplier in its standard form are seen in the rapid formation at sight of R. E. of the product as well as in the working shown below :—

206·5
5·69

1032·5[a]	*a.* 5 tenths × 5 units = 25 tenths = ·5 + carry 2 units.
123·90[b]	*b.* ·5 × ·6 = *o* hundredth + *3* tenths.
18 585[c]	*c.* ·5 × ·09 = *5* thousandths + *4* hundredths.
1174·985	

[In the above working, the multiplier in its standard form is so written that its units' digit is below the digit of the least place-value in the multiplicand. Multiplication is begun with the digit of the highest place-value in the multiplier. The digit of least place-value in the partial products obtained by multiplying by 5 units, 6 tenths and 9 hundredths are written below the digits 5, 6 and 9 as well as below the proper digits in the multiplicand in point of place-value].

Note.—Since $206{\cdot}5 \times 5{\cdot}69 = 1174{\cdot}985$

$\therefore\ 2{\cdot}065 \times 5{\cdot}69 = 11{\cdot}74985$ (how ?)

and $\cdot 2065 \times 569 = 117{\cdot}4985$ (how ?)

Thus a decimal fraction with 'm' places of decimals multiplied by another with 'n' decimal places would give a product having '$(m + n)$' decimal places *in general* : in general, because the last digit in the product may at times be a '0' and hence be omitted

22. DIVISION OF ONE DECIMAL BY ANOTHER.

General Method :—

Divide $4144{\cdot}656$ by $75{\cdot}84$.

Express the divisor in its *standard* form and make the necessary alteration in the dividend so that the quotient may be unaltered. Thus in this question the result is the same as if $414{\cdot}4656$ is divided by $7{\cdot}584$. The quotient may then be found to be roughly equal to $400 \div 8$ or 50. The digit of the highest place-value in the quotient should hence be in the tens' place and the division may be worked as in the ordinary method.

In the above division the quotient could be found out accurately to be $54{\cdot}65$. But in the divisions like $2{\cdot}2 \div 7$ any number of steps in division will always leave some remainder and the quotient is a decimal which does not terminate at any stage. Such a decimal is called a

non-terminating decimal and if the same digits occur in it again and again in the same order, *e.g.*, 3·142857142857.....it is called a *recurring* (*repeating* or *circulating*) decimal. In writing such a decimal, two dots are marked above the extreme digits of the recurring portion to indicate its nature. Thus $22 \div 7 = 3\cdot\dot{1}42857\dot{}$.

23. CONVERTING DECIMALS INTO COMMON FRACTIONS AND *vice versa*.

Ex. i. $\cdot3125 = \frac{3}{10} + \frac{1}{100} + \frac{2}{1000} + \frac{5}{10000}$

$= \frac{3125}{10000} = \frac{5}{16}$ by reduction to lowest terms.

Any common fraction could be expressed as a decimal by a process of decimal division :—

Ex. ii. $\frac{11}{32} = \frac{110}{32}$ tenths $= \cdot3 + \frac{140}{32}$ hundredths

$= \cdot34 + \frac{120}{32}$ thousandths

$= \cdot343 + \frac{240}{32}$ ten-thousandths

$= \cdot3437 + \frac{160}{32}$ hundred-thousandths

$= \cdot34375$.

With fractions like $\frac{1}{3}, \frac{5}{6}, \frac{1}{9}, \frac{4}{11}$, etc., the equivalent decimal is recurring.

NOTE.—*The denominator of a vulgar fraction must have only 2 or 5 as its prime factors if the equivalent decimal should terminate.*

Converting a recurring decimal into a common fraction :—

(1) Let $S = \cdot\dot{1}\dot{8} = \cdot1818\ldots\ldots$ (a pure recurring decimal).

Then $100\,S = 18\cdot1818\ldots\ldots$

$\therefore 99\,S = 18$ and $S = \frac{18}{99}$.

(2) Let $S = \cdot4\dot{2}\dot{5} = \cdot4252525\ldots\ldots$ (a mixed recurring decimal)

Then $1000\,S = 425{\cdot}2525\ldots\ldots$

and $10\,S = 4{\cdot}2525\ldots\ldots$

$\therefore\quad 990\,S = (425 - 4)$

$\therefore\quad S = \dfrac{(425 - 4)}{990}$.

From (1) it follows that if the decimal has no non-recurring portion the equivalent fraction is formed by keeping the recurring portion as the numerator and by writing down as many nines as there are digits in the recurring portion for the denominator. From (2) it follows that, if there be any non-recurring period, the numerator of the equivalent fraction = a number formed of the non-recurring and recurring digits in the order in which they are found in the decimal, *less* the number formed by the non-recurring period and its denominator is formed by writing down as many nines as there are digits in the recurring period followed by as many ciphers as there are digits in the non-recurring portion.

24. EXPRESSING ONE QUANTITY AS A DECIMAL OF ANOTHER.

Ex. i. Express 6 annas as a decimal of Rs. 2.

$$\frac{6 \text{ as.}}{\text{Rs. } 2} = \frac{6 \text{ as.}}{32 \text{ as.}} = \tfrac{3}{16} = {\cdot}1875.$$

$\therefore$ 6 as. is ·1875 of Rs. 2.

Ex. ii. What decimal of a mile is 4 fur. 4 chs.?

Ex. iii. Express £7 19*s.* 4*d.* as the decimal of a £.

EXERCISES.

(1) (*a*) Draw a straight line AB, 12·8 cm. long. Mark a point C in it. Measure AC and CB. Verify if AC + CB = AB.

(*b*) Mark three points A, B, C so as to be roughly in a straight line. Measure the distances from A to C, from B to C and from A to C. Verify whether AB + BC = AC.

(Note that three points A, B, C are collinear if AB + BC=AC).

Work (*b*) with 4 points A, B, C, D and verify whether AB + BC + CD = AD.

In (*a*) mark a point D in AB produced and verify whether AD—DB = AB.

(2) Express 5 litres 6 decilitres 4 centilitres in litres and decimals of a litre. Express 506·827 kilograms in grams, centigrams and milligrams respectively.

(3) In 2354·86 find how many times the local value of 6 is contained in that of 3 and that of 8 in that of 2.

(4) Find the sum of the digits marked * below :—

```
 *  **   ****   **    *
546·782 + 48·092 + 6·464 + 5·21.
```

(5) Supply the missing portions below :—

(*a*) 416·456 + 78·64 + 78·562 + ( ) = 1200·6866.

(*b*)
```
    4 1 4 4 · 4 4
+     7 7 6 · 8 8 8
+ * * * * · * * *
+   4 0 4 4 · 4 4 4
  ---------------
= 1 8 5 2 1 · 5 4 4
```

(6) Show by forming rough estimates that the answers given below are wrong :—

(*a*) The British equivalent of 1825 francs at 25·185 francs per £ = £652·4.

(*b*) 77·8 × 76·5 = 60457·3.

(*c*) $\dfrac{212{\cdot}45 \times 398{\cdot}15 - 97{\cdot}6 \times 97{\cdot}54}{146{\cdot}87.} = 14070.$

(7) Find the products of digits marked with like letters in the following :—

g f e d c b a *a b c d e f g*
1 4 6 · 7 4 3 2 × 5 6 5 1 · 6 7 6

(8) Find the values of the following, correct to 3 places of decimals :—

444·85 × 77·25 ; 888·99 × ·107 ;
564·2 × 7·577 ; ·027 × 407·07 ;
75·56 × 8·887 ; ·04 × 50 × ·07877.

(9) Given that $x \times 8{\cdot}897 = 755{\cdot}5 \times 889{\cdot}7$, find x.

Find the missing number in

$$1644{\cdot}48 + {\cdot}0645 = (\qquad) + 6\,45.$$

(10) During each month in a year a company earns Rs 1,225·75, spends ·548 of it and saves the remainder. Find its annual savings.

(11) Find the quotients in the following divisions :—

$$\frac{83{\cdot}4898}{25{\cdot}4} ; \frac{6399\,99}{{\cdot}0799} ; \frac{4518{\cdot}39}{14{\cdot}25} ; \frac{139{\cdot}73652}{1\,386}.$$

Deduce, from the results obtained, the values of :—

$$\frac{8348\,98}{2{\cdot}54} ; \frac{6{\cdot}39999}{79{\cdot}9} \quad \frac{4518{\cdot}39}{{\cdot}01425} ; \text{ and } \frac{13973652}{1386}.$$

(12) The product of two numbers = 1441795·88 and one of them = 3800·2. Find the other.

(13) My book is 1·75" thick. Each cover is ·05" thick. If there are 125 leaves in it, find the thickness of a leaf.

(14) The greatest weight which Prof. Ramamurthi is able to bear is 2824·825 lbs. If the ordinary weight of a fully developed man be 148·675 lbs., how many such men's weight is he able to bear ?

(15) An iron rod 2' 8" long is heated, when it becomes 1·0003 times as long. Immediately it is cut into pieces ·32" long in order to make nails How many pieces would there be and what length of the original rod would be left ?

(16) If P = 40·875 and Q = 37·5.

find (P + Q), (P—Q), P × Q, $\frac{P}{Q}$ and $\frac{P+Q}{P-Q}$.

(17) A six-inch ruler, supposed to be divided into inches, has its so-called first, second, &c., inches really ·98, ·99, 1·01, 1·02, 985 and 1·05 inches long. If each inch in it is further subdivided into tenths, find the real length of a line which, according to measurement with that ruler, is 5·7".

(18) Convert $\frac{1}{11}$, $\frac{4}{5}$, $\frac{42}{64}$, $\frac{73}{75}$, $\frac{1}{7}$, $\frac{3}{7}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{8}$, and $\frac{7}{8}$ into decimals. Show how you would test at sight whether the equivalent decimals would terminate or not.

(19) A train moves at the rate of 12·8 miles an hour on the average. But it takes 51·75 hours to travel 556·8 miles, as it had to stop at several stations. How long on the whole did the train stop at the stations?

(20) A man sold ·27 of his estate for Rs. 54,000 and cleared his debts. He next sold at the same rate ·34 of the remaining portion for his daughters' marriages. What decimal of the estate still remains with him? Find its worth at the rate mentioned.

(21) Show that (*a*) £7 19*s*. 4*d*. = £7+9 florins + 64 farthings = £7 + 9 tenths of a £ + $64 \times 1\frac{1}{24}$ thousandths of a £ = £7·967 to the nearest thousandths of a £ and (*b*) £ 2·785 = £ 2475 hundredths of a £ + 35 thousandths of a £ = £2 + 15 shillings + $(35 - 35 \times \frac{1}{25})$ farthings = £2 15*s*. 8*d*. to the nearest penny.

Hence or otherwise express (*a*) £ 2 14*s*. 7*d*; £ 5 14*s*. 6*d*, £ 3 10*s*. 10*d*.; £5 5*s*. 6*d*. in £ and decimal of a £ and (*b*) £8·881; £ 7·856, £ 12·0564 in £ *s*. *d*.

CHAPTER II.

SYMBOLIC ARITHMETIC.

SECTION I.

SYMBOLIC REPRESENTATION.

25. Consider the following question :—' For constructing a school-building a zemindar gives Rs. 25,000 and 20 other gentlemen contribute Rs. 700 each. What is the total contribution obtained ?'

The total contribution = Rs. $20 \times 700 + 25{,}000$
= Rs. 39,000.

The same method holds good if any other numbers be employed instead of those given. Thus, if the zemindar gives Rs. 'x,' 'n' other gentlemen contribute Rs. 'g' each and if the total contribution represented by Rs. 'C,' the equality

$$C = n \times g + x$$

indicates the process of working in general *form* and from it the amount of contribution in particular cases could be calculated by substitution of the numerical values of 'n,' 'g' and 'x.' Such an equality is called a *formula*. In a similar manner, if a number 'N,' divided by another number 'D,' leaves a quotient 'Q' and a remainder 'R,' the Arithmetical Law governing them may be represented symbolically by the formula

$$D \times Q + R = N, \text{ or } R = N - D \times Q.$$

Many of the processes in Arithmetic may thus be expressed in a concise, general form by using the

symbols 'a,' 'b,' 'c'...'p,' 'q,' 'r'...'x,' 'y,' 'z' in addition to the ordinary numbers and the signs +,—, × and÷.

Ex. Convert Rs. p q as. r ps. into pies.

$p \times 192 + q \times 12 + r$ is an algebraic expression for the number of pies in Rs. p q as. r ps.

It is generally written as $192p + 12q + r$.

The total number of pies in Rs. 2 4 as. 8 ps. is $192 \times 2 + 4 \times 12 + 8$ which can be simplified into 440. But in Rs. p q as. r ps. we can only state in a general way that there are $192p + 12q + r$ pies and cannot express the total number of pies by a numerical quantity unless we know the numerical values of p, q and r. Such a general way of stating is characteristic of the *algebraic expression.*

Note :—$q \times 12$ is written as $12q$ and $p \times q$ as $p.q$ or as pq [6×8 should not be written as 68]. In an algebraic expression like $a + 12b - \frac{c}{4}$ the parts a, $+ 12b$ and $- \frac{c}{4}$ are known as *its* terms.

EXERCISES.

(1) Express in symbolic language the following Arithmetical Laws :—(*a*) The value of a product is unchanged if its multiplicand and multiplier are interchanged. (*b*) The value of a fraction is unaltered if both its numerator and denominator are multiplied by the same quantity.

(2) Convert Rs. p q as. r ps. into rupees and fractions of a rupee.

(3) Write down formulæ to find

(*a*) the daily collections (C) in Rupees of a Tramcar Company which sells every day blue tickets (*b*) at 3 pies each, green tickets (*g*) at 6 ps. each, pink tickets (*p*) at 9 pies each and yellow tickets (*y*) at one anna each :

(*b*) the length in inches (P) of the piece remaining after '*n*' pieces, each '*c*' inches long are cut from a wire '*d*' yds. long ;

(*c*) the monthly salary in Rs. (S) of a clerk in the *n*th year of his service if he is started on Rs. '*r*' per mensem and given an annual increment of Rs. '*i*' ;

(*d*) the velocity in feet per second (*v*) of a body after '*t*' seconds which moves with an initial speed of '*u*' ft. per second and whose speed decreases at '*f*' feet per second during each successive second ;

and (*e*) the total monthly savings (S) in Rupees of a family in which '*m*' members earn Rs. '*d*' each per month and '*n*' members spend Rs. '*r*' each per month.

(4) If there are '*x*' divisions in a class, each division contains '*y*' boys and each boy pays '*r*' rupees per instalment, calculate the fee-income got from the class per instalment.

(5) The cost of an inland telegram is stated to be 12 as. for any number of words up to 12 and one anna extra for each additional word. State the formula, giving the cost *in annas* (*c*) of an inland telegram consisting of any no. of words (*w*) not less than 12.

(6) Convert £ X Y *s*. Z *d*, (1) into pence, (2) into pounds.

(7) A train runs at the rate of '*y*' miles per hour. If the speed corresponds to '*x*' feet per second, find the formula connecting '*y*' and '*x*,' Also express in miles per hour a speed of '*m*' yds. in '*t*' minutes

(8) Write down the value of a number having '*x*' as its units' digit, '*y*' as the tens' digit and '*z*' as the hundreds' digit.

(9) A man bought '*l*' viss of brinjals at '*p*' annas a viss, '*m*' viss of ladies' fingers at '*q*' annas a viss, '*t*' drumsticks at '*r*' annas a dozen and 400 leaves at '*x*' annas a hundred. What sum had he to pay? [Express in Rupees.]

SECTION II.

26. POSITIVE AND NEGATIVE QUANTITIES.

Let the straight line A'PA represent a straight ascent uphill. Suppose a person starts from P, goes 5 steps upwards in the direction PA and then 3 steps down in the

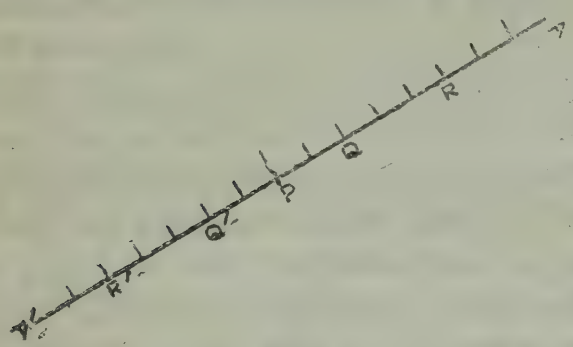

Fig. 8.

direction AP. The number of steps he would then be from P=5—3 or 2. Here '5' indicates the number of steps moved upwards and '3' indicates the number of steps downwards. Thus the '—' sign before '3' indicates not only subtraction but also a *reversal* of direction, *i.e.*, from a movement uphill to one downhill. If after going 5 steps from P upwards along PA, the person comes down 10 steps where would he then be? Representing as before, the number of steps he would then be from P may be expressed as 5—10. Here the 'minus' sign before 10 could not be taken to indicate subtraction as 10 could not be subtracted from a smaller number 5. Yet the 'minus' sign may be taken to indicate reversal of direction. After moving 5 steps upwards from P, the person would be at R and then by going 10 steps downwards he would be at R' which is 5 steps below P. The combination of the two movements can be represented by 5—10 and its result by —5, which means that the final position is 5 steps below the starting point. In the above, the 'minus' sign before a number as in —10 or —5 has been taken to indicate

steps measured downwards or steps measured below the starting point. If a person starts from P, first moves 2 steps upwards along the ascent and then 3 steps upwards, he would finally be 2+3 or 5 steps above P. The combination of the movements may be represented by +2+3 and its result by +5, which means that the final position is 5 steps above the starting point. Thus the 'plus' sign may be put before a number not only to indicate addition but also to indicate steps measured upwards along A'A or steps measured above the starting point. Numbers beginning with the ' + ' sign in the above sense are called ' positive ' (written as '+*ve*') numbers and those beginning with the '—' sign are called 'negative' (written as '—*ve*') numbers.

If a person moves 'x' steps upwards from P along the ascent and then moves 'x' steps downwards along the path, he would be where he started from. Hence if $+x$ is any quantity called $+ve$ when it has a certain meaning, a quantity which has the exactly opposite meaning is $-ve$ and is represented by $-x$. Or, the effect of the '—' sign is the reversal of the meaning when the '+' sign is used.

The same idea of $+ve$ and $-ve$ quantities may be extended. Thus,

(1) If a measurement of 5 steps made to the right of a fixed point O along a horizontal line X'OX be represented by +5, a measurement of 5 steps to the left would be indicated by —5.

(2) If a measurement of 5 steps taken upwards of a fixed point O along a vertical line YOY' be indicated by +5, a measurement of 5 steps taken downwards would be indicated by —5.

(3) If + 8 means 8 degrees above the freezing point in a thermometer, '—8' would mean 8 degrees below the freezing point.

(4) If + 1000 means 1000 ft. above the sea-level, — 1000 would mean 1000 ft. below the sea-level.

(5) If + 400 means 400 years after the birth of Christ, — 400 would mean 400 years before the birth of Christ.

(6) If + 5 means 5 miles walked towards the north, — 5 would mean 5 miles walked towards the south.

(Note that the meanings above given to numbers beginning with the + and — signs are purely conventional).

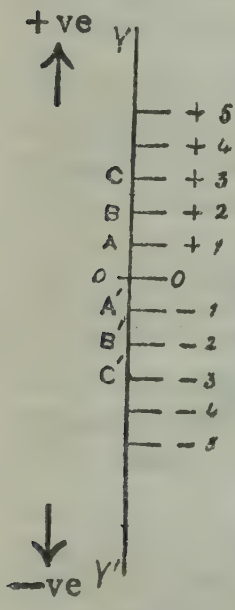

Fig. 9.

27. The interpretation above given to numbers beginning with the '+' and '—' signs leads to the series indicated in the marginal diagram, where measurements taken upwards along a verticalline Y'OY are considered + *ve* and those taken downwards — *ve*. The arrangement shows that (1) for every + *ve* number, there is a corresponding — *ve* number.

(2) While hitherto zero was considered to be the lowest in the scale of numbers, the series can be extended indefinitely on either side of zero.

(3) Some definite order is followed in assigning a rank to each number in the above scale. In the above fig. it may be seen that + 3 is as much higher than 'O' as —3 is lower than it, — 2 is lower in the scale than —1

by + 1 and also that the value of each number in the scale ... + 5, + 4, + 3, + 2, + 1, 0, — 1, — 2, — 3, — 4, — 5.....is 1 lower than its immediate predecessor and 1 higher than its immediate successor.

Ex. Suppose 5 boys A,B O,C.D go to a sweetmeat-shop, each with 4 as of pocket money. A orders sweetmeat for 2 as.. B for 3 as.. O for 4 as., C for 5 as. and D for 6 as. Assign them a rank one after another in respect of the pocket-money they have after the purchase,

28. ADDITION OF + VE AND — VE QUANTITIES.

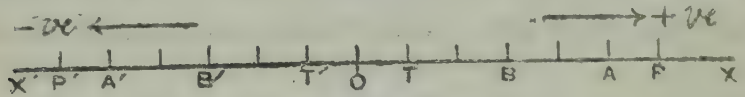

Fig. 10.

Let the number of steps measured to the right along a horizontal line X'OX be considered + *ve* and hence the number of steps measured to the left —*ve*. Then OA may be taken to represent + 5, AB — 2, A'B' + 2, OB' —3, B'P' — 3, &c.

If a person starts from O, goes 3 steps to the right and again goes 2 steps to the right, the result of his movements would be to keep him 5 steps to the right of O. The combination of the above movements and its result may be expressed by the statement OB + BA = OA.

$\therefore \quad +3+2=+5 \quad \ldots \quad \ldots \quad \ldots \quad (1)$

Similarly, since OB' + B'A' = OA', $-3-2=-5$ (2)

Since OA + AB = OB, $+5-2=+3 \quad \ldots \quad (3)$

Since OA' + A'B' = OB', $-5+2=-3 \quad \ldots \quad (4)$

Since OB + BO = 'o', $+3-3=0 \quad \ldots \quad (5)$

Since OA + AB + BB' = OP' + P'T' + T'B'

= OB', $+5-2-6=-6+5-2=-3 \ldots \quad (6)$

In the above six cases + *ve* and — *ve* quantities are added. $-6+5-2$ is spoken of as the *Algebraic sum* of — 6, + 5 and — 2. From (6) it follows that *the algebraic sum of quantities would be unchanged in whatever order they be taken.*

This law would facilitate the work of finding the value of an algebraic sum. Thus,

$$\begin{aligned} &+3x-2x-5x+7x-12x+8x \\ &=+3x+7x+8x-2x-5x-12x \\ &=+18x-19x \\ &=-x. \end{aligned}$$

[NOTE 1. If in the last fig. a movement from O to T be represented by + x, that from O to T' would be represented by — x, OB would represent + $3x$ and OB' — $3x$.

NOTE 2. The — *ve* numbers — 1, — 2, — 3, ... etc., give the values of n for which $n+2=1$, $n+3=1$, $n+4=1$, ... etc.]

EXERCISES.

(10) Find the values of the following:

$-5+6$; $-2-3-5$; $+5\ 2\ 7$; $-5+4+4+-2$; $-1\cdot8+4\cdot2-5\cdot6+6\cdot8+1\cdot7-19\cdot8$

(11) The Salivahana era began in 78 A D and Buddha was born about 500 B.C. Find the No of years between the two events.

(12) The Ramayana was written 25 centuries before the birth of Christ How many centuries have elapsed since then till now?

(13) Find the values of x, y and z for which $x+10=5$, $y+3=2$ and $z+100=80$.

(14) Find the algebraic sum in the following: —

(*a*) $+2x+3x-4x+5x-6x-7x+2x$

(*b*) $+4y+5y-2y+13y-8y$.

(*c*) $-1+1-2-2+3-4+5-18+990$.

(*d*) $-2-3-4-5-6-7-8$.

(*e*) $+1+2+3-4+5-6-7$.

(15) A boy running up a stair-case ascends 14 stairs, when he finds that he has dropped his kerchief 8 stairs below, next descends and picking it up, again ascends 14 stairs, when he drops his pencil which rolls down the stair-case to the point from which he began to ascend. Represent the number of steps he has to descend by a $-ve$ number. By what number would you represent the stairs he ascended or descended each time?

(16) Give a reasonable interpretation for the following:—

(a) The temperature in a certain place was $-12°$C. on a certain night.

(b) In a certain race A is given a start of $+$ 20 yds. and B $-$20 yds. from a fixed point.

(c) The height of a hill is $+$ 1000 ft. and the depth of a cave $-$100 yds. with reference to the sea-level.

(d) A stone thrown from a place reaches a height of $-$120 ft. in 3 seconds.

(17) A liquid at 15°C. is cooled through 20°C. Find the temperature after cooling.

(18) A stranger asked a man in the street for the way to A's house. He directed the person to go 15 houses up the street. He wrongly went 50 houses up. He was then directed to go about 25 houses back to ascertain the house. He counted wrongly and went 30 houses back. How far and in what direction is he finally from A's house?

29. SUBTRACTION OF +VE AND −VE QUANTITIES.

Find the value of $(-5)-(-4)$, or subtract (-4) from (-5).

This means that the quantity should be found which when added to (-4) would give (-5).

If a body starts from O, (refer to Fig. 9) and goes 4 steps in the direction OY', it must still go *one more* step in the same direction so that it may finally be 5 steps below the starting point.

$\therefore$ (— 1) should be added to (— 4) to give (—5).

$\therefore$ (— 5) — (— 4) = — 1.

Similarly to subtract (—4) from (+ 5) we find we have to add (+ 9) to (— 4) to get 5.

$\therefore$ (+ 5) — (— 4) = + 9, where the '—' sign separating the bracketed numbers indicates subtraction.

[NOTE :—Subtracting (— 4) from any number, *e.g.*, + 5 is equivalent to adding + 4 to it.]

EXERCISES.

(19) Subtract—$2x$ from $+3x$, $5a$ from $10a$, + 6 from —12, — 6 from — 1, — 12 from — 15, — 18 from — 6, —12 from + 12, — 13 from 0 and + 18 from — 1;

(20) A liquid at + 15°C is cooled to — 20°C. Through how many degrees is it cooled?

(21) A liquid boils at x°C and freezes at $-2x$°C. Through how many degrees must it be heated from the freezing state before it boils?

(22) A merchant gains Rs. 'z' in one transaction, loses Rs. x in another transaction, again gains Rs. $5z$ and loses Rs. $2x$. If he has gained on the whole, find a simple expression for his gain. Interpret the result when z = 1,000 and x = 5,000.

(23) Starting from the school-gate, I walk 'x' yds. north, turn back and walk '$2y$' yds. south. How many yards north am I now from the school-gate? Interpret your result when x = 130, y = 120.

(24) After leaving a station (S) a train runs $2x$ miles north, returns $8x$ miles south and again goes $5x$ miles north along the same line. How far and in what direction is it then from the station (S)?

(25) A man was born in 1887 A. D., which corresponds to the year '+ 1062' in Quilon era. Express the date of beginning of the Christian era in terms of the Quilon era.

(26) It is stated that Alexander the Great died in the year 323 B. C. If that year be represented by + 33 with reference to

the year of his birth, by what number in respect of the same will you represent the year of his invasion of India which took place in 330 B. C. ?

(27) A Boy-Scout who is in search of a bunch of keys hidden beneath the soil in a foot-path walks 'm' yds. west, then 'n' yds. east, again 'x' yds. west, and then 'y' yds. east, when he finds the keys beneath his foot. How far west is he finally from the starting point ? Interpret your result by drawing a plan of his walk when $m = 100$, $n = 35$, $x = 27$, and $y = 113$, and marking in it the points where he stopped, turned and started

(28) The road from A to C passes though T. From A to T it rises 'x' ft., falls 'y' ft. and again rises 'b' feet. From T to C. it rises 'x' ft., falls '$2b$' ft. and again rises 'b' ft. How high is C above A ? How far is T below C ? Interpret your results when $x = 36$, $b = 40$, and $y = 72$.

30. THE EFFECT OF THE '+' AND '−' SIGNS BEFORE BRACKETS.

In Art. 29, the '−' sign has been used between the bracketed numbers (+ 5) and (−4) to indicate subtraction and the numbers + 5 and −4 have been enclosed within brackets to indicate that the '+' and '−' signs in them should be distinguished from the '−' sign indicating subtraction. In a similar manner the numbers may be enclosed within brackets when they have to be added as in (+ 5) + (− 4), where the '+' sign between the bracketed numbers indicates addition while the '+' and '−' signs before 5 and 4 indicate the nature of the quantities.

In expressions like 18−(14−4), 3 (14−4), (14−4) ÷ 2, the brackets are used to show that (14−4) should be treated as one whole, that is to say, the result (14−4) should first be calculated and then it may be deducted from 18, or multiplied by 3, or divided by 2. To serve the same purpose a line may be drawn above or below

the part to be so treated as in $18-\overline{14-4}$ and $\frac{14-4}{x}$.

This line is called a *vinculum* (= holding together).

It may be easily shown that $(-6)+(-18) = -6-18 = -24$; $(-6)+(+18) = -6+18 = +12$; $(-6)-(-18) = -6+18 = +12$; and $(-6)-(+18) = -6-18 = -24$, where the + and — signs between the bracketed numbers indicate addition and subtraction. Hence it follows that before proceeding to the actual calculation the '+' sign may be removed as well as the brackets without in any way affecting the final result. But in removing the '—' sign, the sign of the bracketed number following it should be reversed if the result should be unaltered. The same rule may be extended. Thus

$$\begin{aligned} &-9+(-5-6+7-8) \\ &= -9+(-5-6-8+7) \\ &= -9+(-19+7) \\ &= -9+(-12) \\ &= -9-12. \\ &= -21. \end{aligned}$$

Also $-9-5-6+7-8$
$= -9-5-6-8+7$
$= -28+7$
$= -21.$

$\therefore -9+(-5-6+7-8) = -9-5-6+7-8.$

In a like manner it may be verified that

$$-9-(-5-6+7-8) = -9+5+6-7+8.$$

The following illustrations would suggest the general rules:—

(1) $\underline{9+(5+3)} = \underline{9+5+3}.$

Suppose you go to your grand-mother's house with Rs. 9 in your pocket. There your uncle gives you Rs. 5 and your grand-mother Rs. 3. You then get Rs. (5 + 3) extra and thus have Rs. 9 + (5 + 3). This may be expressed in another way:—After getting Rs. 5 from your uncle you have Rs. 9 + 5

and after getting Rs. 3 more from your grand-mother you must have Rs. $9 + 5 + 3$.

$\therefore \quad 9 + (5 + 3) = 9 + 5 + 3.$

Similarly $a + (b + c) = a + b + c$ ... ... (1)

where a, b, c stand for numbers like 9, 5, 3.

(2) $\quad 9 + (5 - 3) = 9 + 5 - 3.$

Suppose in the above example your grand-mother does not give you any money but you give her Rs. 3. Then the extra money you get is only Rs. $(5-3)$ and altogether, you have Rs. $9 + (5-3)$. This may be otherwise expressed as follows :—After getting Rs. 5 from your uncle you have Rs. $9 + 5$ and after giving away Rs. 3 to your grand-mother you have Rs. $9 + 5 - 3$.

$\therefore \quad 9 + (5-3) = 9 + 5 - 3.$

Similarly $a + (b - c) = a + b - c$ ... ... (2)

where a, b, c stand for numbers like 9, 5 and 3.

(3) $\quad 9 - (5 + 3) = 9 - 5 - 3.$

Suppose you start with Rs. 9, buy cloth for Rs. 5 and fruits for Rs. 3. You then spend Rs. $(5 + 3)$ on the whole and have Rs. $9-(5 + 3)$ on the whole. This balance may otherwise be expressed as follows:—After buying cloth you have Rs. $9-5$ and after buying fruits you have Rs. $9 - 5 - 3$.

$\therefore \quad 9-(5 + 3) = 9 - 5 - 3.$

Similarly, $a - (b + c) = a - b - c$ ... ... (3)

where a, b and c stand for numbers like 9, 5 and 3.

(4) $\quad 9-(5-3) = 9-5+3.$

Suppose you go to a shop with Rs. 9, buy cloth and pay a five-rupee note. After deducting the price of the cloth, the shop-keeper returns you Rs. 3. Then

the price of the cloth = Rs. $(5-3)$. After paying for the cloth you therefore have Rs. $9-(5-3)$. The result may be otherwise expressed thus :—When you pay a five-rupee note you have Rs. 9—5 and when the shop-keeper returns you Rs. 3, you have Rs. 9—5 + 3.

$\therefore\ 9-(5-3)=9-5+3.$

Similarly $a-(b-c)=a-b+c$ ... ... (4)

where a, b, c stand for numbers like 9, 5, 3.

The above illustration shows that the equalities

$a+(b+c)=a+b+c$ ... ... (1)
$a+(b-c)=a+b-c$ ... ... (2)
$a-(b+c)=a-b-c$ ... ... (3)
$a-(b-c)=a-b+c$ ... ... (4)

are true for certain valhes of a, b and c. The equalities may be verified to be equally true if any + *ve* or —*ve* numbers be substituted for a, b and c.

The significance of the above equalities is that if any expression or arithmetical number could be expressed in the *form* found on one side of the sign of equality it could also be expressed in the *form* found on the other side of the sign of equality.

Hence the laws for the removal and insertion of brackets :—

1. *When a '+' sign precedes the brackets, they may be removed without changing the signs of any term within, and vice versa.*

2. *When a '—' sign precedes the brackets, they may be removed provided the sign of each term within is changed, and vice versa.*

From the above laws, it follows that the terms of an expression may be grouped *in any manner*, *e.g.*,

$$+ a - b + c - d + e = (+ a - b) + (+ c - d) + e$$
$$\text{or} = + a - (+ b - c) - (+ d - e)$$
$$\text{or} = + a - (+ b - e + d - e).$$

The importance of such grouping may be seen in adding and subtracting expressions.

Ex. 1. Add $a + b$, $a - b + c$ and $c - b$.

The sum $= (a + b) + (a - b + c) + (c - b)$
$= a + b + a - b + c + c - b$
$= a + a + b - b - b + c + c = 2a - b + 2c$

or thus:—

$$\begin{array}{r} a + b \\ + a - b + c \\ \quad - b + c \\ \hline = \quad 2a - b + 2c. \end{array}$$

Ex. 2. Simplify:—

$a - (2a + b + c) + (2b - 1) - (b - a + 1)$.

The expression $= a$

$$\begin{array}{rrrr} & - 2a & - b & - c \\ & & + 2b & - 1 \\ + & a & - b & - 1 \\ \hline = & & & - c - 2 \end{array}$$

$= - (c + 2)$.

(The terms in a and b cancel).

Ex. 3. Subtract $2x + y - z$ from $x - 2y + z$.

The result required $= (x - 2y + z) - (2x + y - z)$.

$$\begin{array}{rrrr} = & x & - 2y & + z \\ & -2x & - y & + z \\ \hline = & - x & - 3y & + 2z \end{array}$$

(Subtracting $2x + y - z$ is the same as adding $(- 2x - y + z)$.

EXERCISES.

(29) Add :—

(*a*) $2x + 3y - 5z$; $3x + 7y - 4z$; $2y - 4x + 12z$.
(*b*) $2x - 4y - 8z$ and $4z + 4y - 4x$.
(*c*) $p + q + r$: $2p - 3q + r$: $4p + q + 1$.
(*d*) $l + m + n$; $l - 2m - 2n$: $2l + m - n$; $2l - m - n$.
(*e*) $-x - y - z$; $-2x - 3y - 4z$: $2x + 2y + z$.

(30) Subtract $(m + n)$ from $(2m - n)$; $(a - b)$ from $(a + b)$; $(r + l)$ from 0 ; $5x - 2y + 3z$ from $4x + 4y - 4z$.

(31) Illustrate by measurements taken along a straight line that

$$+4 + (+2 + 3) = +4 + 2 + 3;$$
$$+4 + (+2 - 3) = +4 + 2 - 3;$$
$$+4 - (+2 + 3) = +4 - 2 - 3;$$
$$+4 - (+2 - 3) = +4 - 2 + 3.$$

(32) What expressions must be inside the brackets below?—

$p - q - r = p - (\quad)$; $x - c + p = x - (\quad)$;
$+5 + 4 - 3 = +5 + (\quad)$; $-5 - 3 + 4 = -5 - (\quad)$.

31. MULTIPLICATION AND DIVISION OF +VE AND −VE NUMBERS.

Suppose that the values of 'S' have to be found from the formula $S = (t + 5)(u - 5)$, when $t = -30$, $u = -5$, $t = +10$ and $u = -1$.

Then the product of two *−ve* numbers has to be found. An interpretation will be found in the illustration given below for products involving *−ve* as well as *+ ve* multipliers.

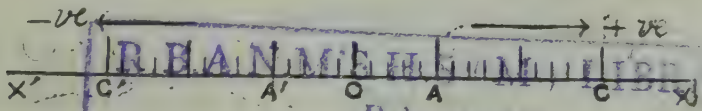

Fig. 11.

[Scale 1 unit = 1 mile.]

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Suppose X′OX in the above figure represents a road. Let distances traversed in the direction X′X be represented by + *ve* numbers and those traversed in the direction XX′ by —*ve* numbers. Then if a man on horseback travels at the rate of 5 miles an hour from X′ to X his speed in miles per hour may be represented by + 5. Also let time in hours after he passes a place O on the road be represented by a + *ve* number and the time before by a — *ve* number. Suppose also that a jutka moves at the rate of 5 miles an hour from X to X′ and passes him at O. Then its speed in miles per hour may be represented by — 5.

Since the distance of '*d*' miles traversed in time '*t*' hours by any moving body with the speed of '*s*' miles per hour is given by the formula

$$d = (s) \times (t)$$

the following considerations would lead to products obtained by + *ve* and — *ve* multipliers.

(1) Three hours after they meet, the man must be $(+5) \times (+3)$ miles from O. But by calculation he must be at C, which is 15 miles from O in the direction OX.

$$\therefore (+5) \times (+3) = +15 \quad \ldots \quad \ldots \quad (1)$$

(2) Three hours before they meet, the man must be $(+5) \times (-3)$ miles from O. But by calculation he must be at a place C′ which is 15 miles from O in the direction OX′.

$$\therefore (+5) \times (-3) = -15 \quad \ldots \quad \ldots \quad (2)$$

(3) Three hours after they meet the jutka must be $(-5) \times (+3)$ miles from O. But by calculation it must be at C′ which is 15 miles from O in the direction OX′.

$$\therefore (-5) \times (+3) = -15 \quad \ldots \quad \ldots \quad (3)$$

(4) Three hours before they meet, the jutka must be $(-5)\times(-3)$ miles from O. But by calculation it must be at C which is 15 miles from O in the direction OX.

$$\therefore\ (-5)\times(-3) = +15 \quad \ldots \quad \ldots \quad (4)$$

From (1), (2), (3) and (4) above, it follows that

$$\left.\begin{aligned}(+p)\times(+q) &= +pq\\ (-p)\times(-q) &= +pq\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \text{I}$$

$$\left.\begin{aligned}(-p)\times(+q) &= -pq\\ (+p)\times(-q) &= -pq\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \text{II}$$

From I, the product of two factors is + *ve* if both have the same signs, and from II, the product is − *ve* if they have opposite signs.

It is also seen that

$$\frac{+pq}{+q} = +p;\ \frac{-pq}{-q} = +p \ldots\ldots\ldots \text{III}$$

$$\frac{-pq}{+q} = -p;\ \frac{+pq}{-q} = -p \ldots\ldots\ldots \text{IV}$$

[The division in these cases means 'by what quantity should $+q$ or $-q$ be multiplied to give the result $+pq$ or $-pq$?]

From III and IV it is seen that the quotient is + *ve* if the dividend and the divisor have the same signs and it is −*ve* if they have opposite signs.

Ex. From the formula $t = \frac{u+5}{u-5}$, find the values of t when $u = +15, -16, -3$ and -1.

32. THE LAWS OF INDICES.

$a\times a\times a\times a$ is shortly written as a^4. When the factors of a product are thus equal, the product is called a *power* of the factor. a^4 is read as 'a to the fourth power, or as the fourth power of a'. 'a' is called the root of the power and '4' is the *index* or *exponent*. p^2 is read as 'p squared' and b^3 as 'b cubed.'

x^4 means 1 multiplied by x *four* times and $= 1 \times x \times x \times x \times x$. The index shows how many times '1' should be multiplied by the root to give the power required.

$\therefore \quad a^m \quad =$ 1 multiplied by 'a' m times,

and $a^n \quad =$ 1 ,, 'a' n times.

$\therefore \quad a^m \times a^n =$ 1 ,, 'a' $m + n$ times.

$$= a^{m+n} \quad \ldots \quad \ldots \quad \ldots \quad \text{V}$$

Similarly $a^m \div a^n = a^{m-n} \quad \ldots \quad \ldots \quad \ldots \quad \text{VI}$

NOTE :—$a^2 \div a^2 = a^{2-2} = a^0$ by VI and $a^2 \div a^2 = 1$.

$\therefore \quad a^0 = 1$. Also 'a^0' means '1' multiplied by 'a' zero times and hence is equal to 1.

$$a^{-4} = a^2 \div a^6 = \frac{1 \times a \times a}{1 \times a \times a \times a \times a \times a \times a}$$

$$= \frac{1}{1 \times a \times a \times a \times a} \text{ or } \frac{1}{a^4}.$$

$$(a^4)^3 = 1 \times a^4 \times a^4 \times a^4$$
$$= 1 \times a^{4+4+4}$$
$$= 1 \times a^{12}$$

$$\therefore \quad (a^m)^n = a^{m \times n} \quad \ldots \quad \ldots \quad \ldots \quad \text{VII}$$

Similarly $x^3y^2 \times x^2y^3 \div x^2y^2$

$$= \frac{x^3 \times x^2}{x^2} \times \frac{y^2 \times y^3}{y^2} = x^{3+2-2} \times y^{2+3-2}$$

$$= x^3y^3.$$

The equalites V, VI and VII are true for all values of 'm' and 'n'.

EXERCISES.

(33) Find the values of x^2, $-7x+7$, $2x-2x^2$, x^2-3x+3, x^4+1, $3x-2$, $\frac{3x-2}{5}$ and $x^3 \div 3x^2 + 20$, when $x = -3$, -2, -1, 0, 1, $1\frac{1}{2}$, 2 and 20.

(34) Show that when $x = -1$, $x^3 = -1$ and when $x = +2$ or -2, $x^2 = +4$. Find the values of x, when $x^2 = 16$ and $x^2 = \frac{1}{16}$.

(35) Find the product in each case below :—

$a \times b \times (-a) \times (-b) \times (-c)$; $x^4 \times x^3 \times (-x)^2$; $(-p)^2 \times (-p)^2 \times (-p)^6$; $(2-3)(2-3)$; $(4-4)^6$, $(-3)^2$; $(2-4)^2$; $(3-4)^2$; and $(1-2)(1-3)(1-4)$.

(36) Find the quotients in :—

$(-a)^5 \div (+a)^2$; $-7\frac{1}{4}a^2x^4 + (-2)(-a)^2 \times (-x)^3$.

(37) The product of 'A' and 'B' $= 4q^6p^4$ and A $\div$ B $= p^2q^2$. Express 'A' and 'B' in terms of 'p' and 'q.'

(38) Express $\frac{a}{b} \times \frac{b^2}{c^2} \div \frac{c^2}{a^3}$ in its simplest form and find its value when $b = 2$, $c = 2$ and $a = 1$.

(39) Find by what power of 10 each of the following should be multiplied in order to express it as a decimal in its *standard* form :—

15000000000000; 1560000; ·00346; 156798756 and ·000012578.

(40) Find the values of :—

$$\frac{(\cdot5)^4 \times (2\cdot1)^2 \times (6\cdot4)^5}{(1\cdot5)^2 \times (5\cdot6)^4 \times (1\cdot6)^2}; \quad \frac{17^2 \times 52^4 \times 57^5}{38^6 \times 34^3 \times 39^4}$$

$$\frac{14 \times 10^{-4} - 11 \times 10^{-5} \times 8}{4^{-2} - 2^{-5}}$$

SECTION III.

33. SOME IDENTITIES AND THEIR APPLICATIONS.

It has been shown in Chapter I, Art. 3 that

(1) $(p + q) \times c = pc + qc.$

$$\therefore \frac{pc + qc}{c} = p + q \text{ and } \frac{pc + qc}{p + q} = c.$$

(2) $(a + b + c) \times n = an + bn + cn.$

$$\therefore \frac{an + bn + cn}{n} = a + b + c \text{ and } \frac{an + bn + cn}{a + b + c} = n$$

and (3) $(a + b)^2 = a^2 + 2ab + b^2.$

The following diagram shows that

(4) $(a - b) \times c = ac - bc.$

$$\therefore \frac{ac - bc}{c} = a - b.$$

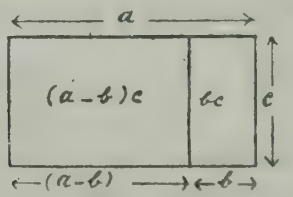

Fig. 12.

$$\left[\frac{ac - bc}{c} = \frac{ac}{c} + \frac{-bc}{c} = \frac{ac}{c} - \frac{bc}{c}.\right.$$

$$\left.\therefore \frac{x - y}{k} = \frac{x}{k} - \frac{y}{k}.\right]$$

The following diagrams show that

(5) $(a - b)^2 = a^2 + b^2 - 2ab.$

(6) $(a^2 - b^2) = (a + b)(a - b)$.

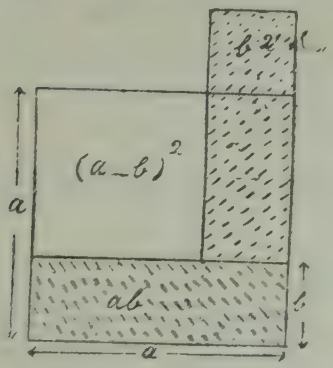

Fig. 13.

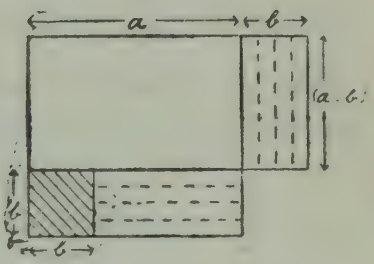

Fig. 14.

It may easily be verified that the equalities above stated are true for all values of the quantities involved. Such equalities are called *identities* and *have important applications in rendering some numerical calculations easier* as may be seen from the following examples :—

Ex. 1. $\dfrac{1755 + 635}{5} = 351 + 127$;

$\dfrac{1755 + 635}{351 + 127} = 5$; $\dfrac{1755 - 635}{5} = 351 - 127$;

$\dfrac{1755 - 635}{351 - 127} = 5$.

Ex. 2. Find the value of $\dfrac{1\frac{5}{6} - \frac{9}{4}}{\frac{5}{6} - \frac{3}{4}}$, using any artifices.

$$\frac{1\frac{5}{6} - \frac{9}{4}}{\frac{5}{6} - \frac{3}{4}} = \frac{3\left(\frac{5}{6} - \frac{3}{4}\right)}{\left(\frac{5}{6} - \frac{3}{4}\right)} = 3.$$

Ex. 3. Find the value of 995^2.

$$\begin{aligned} 995^2 &= (1000 - 5)^2 \\ &= 1000^2 - 2 \times 1000 \times 5 + 5^2 \\ &= 1{,}000{,}000 - 10000 + 25 \\ &= 990025. \end{aligned}$$

Ex. 4. Find the value of $199^2 - 195^2$.

The value required

$= (199 + 195)(199 - 195) = 394 \times 4 = 1576.$

[NOTE: $\frac{199^2 - 195^2}{199 + 195} = 199 - 195$].

Ex. 5. Find the product of 397 and 403 without directly multiplying them.

$397 \times 403 = (400 - 3)(400 + 3) = 400^2 - 3^2$

$= 160000 - 9 = 159991.$

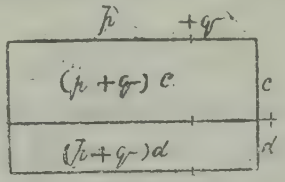

Fig. 15.

Fig. 15 shows that $(p + q)(c + d)$

$= (p + q)c + (p + q)d$

$= pc + qc + pd + qd.$

Hence the following algebraic method of multiplication :—

$$\begin{array}{rl} & (a + b + c)(d + e) \\ = & a + b + c \\ & d + e \\ \hline = & ad + bd + cd = (a + b + c)d. \\ + & ae + be + ce = (a + b + c)e. \\ \hline = & ad + bd + cd + ae + be + ce. \end{array}$$

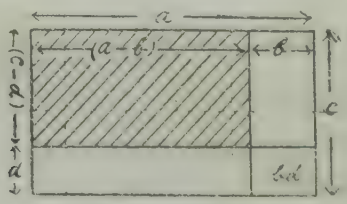

Fig. 16.

Fig. 16 shows that

$$(a - b)(c - d) = ac - bc - ad + bd.$$

Hence the method :—

$$\begin{array}{r} a - b \\ c - d \\ \hline ac - bc = c(a - b) \\ -ad + bd = -d(a - b). \end{array}$$

The significance of the above equalities is that if an algebraic expression or an arithmetical number could be expressed in the form $(a - b)(c - d)$ it could be replaced by another of the form $ac - bc - ad + bd$.

NOTE :—$(20-5)\times(+3)=(20)\times(+3)+(-5)\times(+3)$.

$(20-5)\times(-3)=(20)\times(-3)+(-5)\times(-3)$.

$-a(c + d - e) = -ac - ad + ae$.

$$1 + \frac{x-1}{2} = 1 + \tfrac{1}{2}(x-1) = 1 + \tfrac{1}{2}x - \tfrac{1}{2};$$

$$1 - \frac{x-1}{2} = 1 - \tfrac{1}{2}(x-1) = 1 - \tfrac{1}{2}x + \tfrac{1}{2}.$$

EXERCISES.

(41) If $(a - b)(c - d) = ac - bc - ad + bd$ for all values of a, b, c and d, deduce that $(-2) \times (-3) = (+6)$.

(42) Given that $(a + b)(c + d) = ac + ad + bc + db$, deduce that $(x+p)(x+q) = x^2 + (p + q)x + pq$. Hence or otherwise find out the expanded products in $(x - 2)(x + 3)$, $(x+2)(x+3)$, $(x + 2)(x - 3)$ and $(x - 2)(x - 3)$.

(43) Simplify, using any artifices you know :—

(i) $995 \times 253 + 747 \times 995$;

(ii) $(3100 - 395) \div (1033\frac{1}{3} - 131\frac{2}{3})$;

(iii) $\frac{5}{11}$ of Rs. 1,265 — $\frac{5}{12}$ of Rs. 1,205 ;

(iv) $299^2 - 1$; $98^2 - 4$; and $997^2 - 9$.

(44) Simplify :—

$[4 - \{2 - (4 - \overline{2-4})\}]$; $\frac{2a^3}{4a^2}$; $\frac{2a}{b} \div a$; $\frac{2a - 4b}{2}$;

$\frac{a}{b} + \frac{c}{d} \times \frac{d}{a}$ and $p(p+q) - q(p-q)$.

(45) Find the distributed products in the following :—

$(x+1)(x+2)$; $(x-1)(x+2)$; $(x-1)(x-2)$; $(x+2)(x-1)$; $(2a+b)(2a-b)$; $(3a+2b)(3a-2b)$; $(a^2+b^2)^2$; $(a^2-b^2)^2$; $(x^2+y^2)(x^2-y^2)$; $(x+2)(x+3)$; $(x-5)(x-6)$; $(x+5) \times (x-5)$; $(a+b)^2$; $(x-1)(x-2)(x-3)$; $(a-b)(b-c) \times (c-a)$; $(x+1)(x+3)(x+4)$; $(x+2)(x+5)$.

(46) Factorize :— $2a + 2b$; $a^2 + ab$; $a^2 - ac$; $ap + bp + cp$; $25x^2 - 64y^2$; $(a+b)^2 - a^2$; $\left(1 + \frac{r}{100}\right)^2 - \left(1 + \frac{r}{100}\right)$; $a + ax$; $1 + 2x + x^2$; $1 - 2x + x^2$.

(47) Find the values of $a - b + c$; $ad - bd + cd$; $xb + xa + xc$; $a^3(b-c) + b^3(c-a) + c^3(a-b)$; $x^2 + \frac{x^2}{12} + \frac{x}{16} + \frac{1}{16}$, when $x = -2$, $a = -3$; $b = -5$, $c = 1$ and $d = 0$.

(48) The sum of the first 'n' natural numbers $= \frac{n(n+1)}{2}$.

Find therefrom the sum of the first 10, 15 and $(n - 10)$ natural numbers.

(49) If $P = x + y$ and $Q = x - y$, express in terms of x and y the values of $P + Q$, $P - Q$, $P^2 + Q^2$, $P^2 - Q^2$ and $(P + 1) \times (Q + 2)$.

(50) A man bought $(a + b)$ maunds of sugar at Rupees $(a+b)$ per maund and sold them at $(4a - 2b)$ annas per viss. Find his gain in the transaction.

(51) Find the values of $\frac{x^2 + 3x + 2}{2x^2 + 3x + 2} \div \frac{2x + 3}{2x - 3}$.

$\frac{(x+1)(x+2)}{(x+3)(x+4)}$ when $x = 0, -1, -2, -5, -6$.

R.B.A.N.M.S. HIGHER SECONDARY SCHOOL BANGALORE-1.

CHAPTER III.

THE SIMPLE EQUATION AND ITS APPLICATIONS.

34. In Art. 33 it has been stated that an equality like $(a + b)^2 = a^2 + 2ab + b^2$ could be verified to be true for all values of 'a' and 'b' and is called an identity. But an equality like $2x + 5 = 17$ is true only for a particular value of the unknown quantity 'x' involved in it. Such an equality is called an *equation*. Many problems can be solved by expressing the conditions given by means of equations as in the following:—

Ex. 1. In one scale-pan of a balance a weight of 30 palams is placed and in the other 3 palams and a stone. What is the weight of the stone?

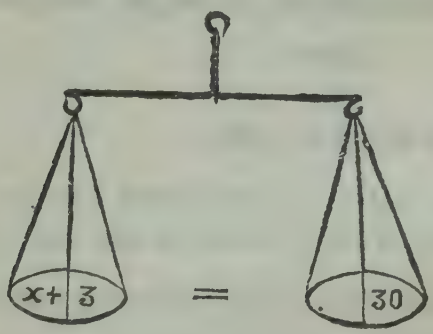

Fig. 17.

Let 'x' palams be the weight of the stone.

Then $x + 3 = 30$.

Taking away 3 palams from each scale-pan, we get

$x = 30 - 3$ or 27.

Also, the addition of equal weights to both the scale-pans would leave the balance undisturbed.

Hence $x + 3 + 6 = 30 + 6$
or $x + 9 = 36$.

We therefore get that an equation is unaltered if the same quantity be added to or subtracted from, each side of the equation. Thus,

if $2x - 3 = 17$,
$2x - 3 + 3 = 17 + 3$,
or $2x = 20$;
again, if $2x + 25 = 43$,
$2x + 25 - 25 = 43 - 25$,
or $2x = 18$.

Ex. 2. If 4 equal weights placed in one scale-pan are equal to 1 lb., find the weight of each.

If 'x lb.' represents each weight, we get the equation

$4x = 1$.
$\therefore \quad x = \frac{1}{4}$.
$\therefore$ each weight $= \frac{1}{4}$ lb.

Hence an equation is unaltered by the division, or multiplication, of both sides by the same quantity.

We therefore derive the following *axioms* in solving equations :—

(1) If $x + a = b$, $x = b - a$.
(2) If $x - a = b$, $x = b + a$.
(3) If $x \div a = b$, $x = b \times a$.
(4) If $x \times a = b$, $x = \frac{b}{a}$.

Hence '*any term on one side of the sign of equality may be transposed to the other side provided its sign is reversed*,' the principle underlying the process being one of subtraction or addition.

Ex. 3. I have two packets, each containing an equal number of two-anna pieces. I spend 20 pieces and find 50 still with me. How many pieces had each packet originally?

Let 'x' be the no. of two-anna pieces in each packet.

Then the no. in the two packets $= 2x$.

After spending 20, there would be $(2x - 20)$ pieces left.

But in the question it is given that there are 50 pieces left.

$\therefore$ the equation for this question is

$$\begin{aligned} 2x - 20 &= 50 \\ +\ 20 &\quad +\ 20 \\ \therefore\ 2x &= 70 \\ \therefore\ \frac{2x}{2} &= \frac{70}{2} \\ \therefore\ x &= 35. \end{aligned}$$

$\therefore$ the no. of two-anna pieces in each packet $= 35$.

[VERIFICATION. $x \times 2 = 35 \times 2$ or 70; $70 - 20 = 50$. $\therefore$ the result 35 must be correct.]

When we find that the value 35 for x makes $2x-20$ equal to 50, this value of x is said to satisfy the equation $2x - 20 = 50$ and is called the root or solution of the equation. When from a given equation its root is found we are said to solve the equation.

EXERCISES.

1. **Solve the equations:—**

$5x = 10$; $7x = 12$; $\frac{x}{2} = 8$; $\frac{5}{9}x = 15$; $x \times \frac{3}{4} = \frac{3}{6}$; $x + 7 = 21$; $x - 7 = 6$; $x + 4 = 20$ and $2x - 5 = 10$.

2. **Express the condition given in the following question in the form of an equation and find the result required by solving the equation:—**

Think of a number, double it, add 20 to the result. The final result is 24. Find the original number thought of.

Ex. 4. A bought some yards of silk at Rs. 2 per yd. and sold them all for Rs. 20. B bought as many yards of a superior silk at Rs. 5 per yd. and sold the whole for Rs. 38. If A's gain was equal to B's, how many yards of silk did each buy?

Suppose A as well as B bought 'x' yds.

A's gain in rupees $= 20 - 2x$ (why?)

B's „ „ $= 38 - 5x$ (why?)

$\therefore$ the equation for this question is

$$20 - 2x = 38 - 5x$$

Adding $+ 5x \qquad + 5x$

$$20 + 3x = 38$$

Subtracting $20 \qquad = 20$

$$3x = 18.$$

$$\therefore \quad x = 6.$$

$\therefore$ A as well as B bought 6 yds. of silk.

[VERIFICATION: A's gain = Rs. $20 - 6 \times 2$ = Rs. 8 and B's gain = Rs. $38 - 6 \times 5$ = Rs. 8. $\therefore$ the result obtained should be correct.]

NOTE:—When terms involving 'x' are found on both sides of the sign of equality, it is desirable to collect all terms in 'x' to one side and terms without 'x' to the other side of the sign of equality.

EXERCISES.

3. Solve :—

$3(x+3)+2(x-2) = x \div 4$; $5(x-2) = 7(x-4)$; $4x+4 = 5x$; $3x - 9 = x - 21$; $x - 5 = 5 - x$ and $x = 13(x - 15)$.

Ex. 5. A man lived $\frac{1}{4}$ of his life as a boy, $\frac{1}{5}$ as a youth, $\frac{1}{3}$ as a man and spent the remaining 13 years of his life in old age. How many years did he live ?

Suppose he lived for x years.

Then the equation for this question is $\frac{x}{4}+\frac{x}{5}+\frac{x}{3}+13 = x$. This equation involves fractions and may be solved thus :—

Clear the equation of fractions by multiplying either side of the sign of eqality by the least common multiple of the denominators.

Then, multiplying either side by 60.

$$60 \times \frac{x}{4} + 60 \times \frac{x}{5} + 60 \times \frac{x}{3} + 60 \times 13 = 60 \times x$$

$\therefore 15x + 12x + 20x + 60 \times 13 = 60x$

$\therefore 47x + 60 \times 13 = 60x$

Subtracting $47x$ $= 47x$

$60 \times 13 = 13x$

$\therefore 13x = 60 \times 13$

$\therefore x = \frac{60 \times 13}{13}$

$= 60.$

EXERCISES.

(4) Divide Rs. 2225 among A, B and C so that A may have half as much as B and B Rs. 150 less than C.

(5) Think of a number ; double it ; subtract 800 and divide the result by 10. If the quotient be 5, find the original No. thought of.

Devise a simple method by which the original No. could be easily deduced from the result obtained.

(6) An engineer calculated that the cost of constructing a dam, x furlongs long, across a river was $\frac{7}{8}(35-4x)$ lacs of rupees and a second engineer calculated that the cost would be $\frac{7}{2}(9x-35)$ lacs of rupees. If the estimated cost was the same in both cases, find the length of the dam and the estimated cost.

(7) A gentleman left $\frac{1}{6}$ of his property to a school, $\frac{1}{2}$ of it to his sons, $\frac{1}{4}$ of it to his wife and distributed the remaining Rs. 10,000 among his daughters. Find the worth of the whole property.

(8) A man's assets would allow him to pay 12 as. only for every Re of his debts. His debtors on the whole allowed him non-payment of Rs. 10,000 of his debts. After fully clearing the remaining debts he found that he had Rs. 4,500 left with him. Find the amount of his original assets and of his debts.

(9) A boy went to the bazaar with Rs. x in his pocket, spent $\frac{1}{3}$ of it in oranges, $\frac{1}{4}$ of the remainder in biscuits and had 3 as. left. Find the value of x.

(10) The sum of three consecutive numbers is 51. Find them.

(11) The sum of three consecutive even numbers=54. Find them.

(12) At 4 A.M. a thief stole a cycle and biked at 8 miles an hour along a certain road. The theft was discovered at 6 A.M. and the owner pursued him immediately in a motor-bike at 12 miles an hour. At what time will he overtake the thief?

(13) A train moves at the rate of 15 miles per hour. I travel in it and pass a railway bridge in 10 seconds. Calculate the length of the bridge.

A porter observes that the above train passes a semaphore in 8 seconds. Fnd the length of the train.

(14) Find the value of x for which $\frac{1}{3}(x+1)+\frac{1}{5}(x+4)$ is equal to $(x-1)-\frac{1}{9}(2x+5)$. How will you test whether your result is correct or not?

(15) Solve the equations :—

$$\frac{1}{2}x + \frac{1}{4}x + 560 = x;$$

$$\frac{x-3}{2} = \frac{x-2}{3};$$

$$x + \frac{99}{100}\left(\frac{1}{2}x\right) = \frac{9}{10}\left(56900 - \frac{3x}{2}\right);$$

$$x - \left(20 + \frac{x}{12}\right) = 60;$$

$$10x + (10 - x) = 18 + 10(10 - x) + x;$$

$$4\left(x - \frac{1}{4}\right) - \frac{1}{2}(1 - x) = 5\left(\frac{3x-1}{3}\right);$$

$$5 - 3x = 2(2x - 1);$$

$$\frac{5}{11}x - 11 = \frac{7}{12}x - 18;$$

$$\frac{x}{6} + \frac{x}{3} = x + \frac{3}{4}x - 1;$$

$$\frac{3}{4}(2x - 3) + \frac{1}{2}(7x - 8) = \frac{1}{6}(18 - x);$$

$$(x + 2)(x + 3) \times (x - 4)^2 = (2x - 5)(x - 8).$$

Verify the correctness of your solution in each case.

(16) I bought some mangoes at 6 as. a dozen and thrice the number at 8 as. a dozen. I sold them all at 7 as. a dozen and found that I was a loser by Rs. 7. How many mangoes of each sort did I buy?

(17) On a certain day 200 single and 400 return tickets were taken out from one railway station to Rsmesvaram in the S.I.R., the total receipts from these amounting to Rs. 1,800. If the return ticket cost Rs. $1\frac{1}{2}$ more than the single ticket, find the cost of a single ticket.

(18) To finish a piece of work within a limited number of days, a man was engagad on condition that he would be paid Rs. 5 for every such day and that if he should fail to finish the work within the time allotted he would be paid nothing for the extra days but fined Rs. 3 for each such day delayed.

The man finished the work in 36 days aud was paid Rs. 132. By how many days did he delay the work beyond the time allotted?

(19) In a battle between the French and the Germans, the French army which consisted of 200,000 fought with the German

army consisting of 350,000. If the number of survivors in the German army was $1\frac{1}{2}$ times the corresponding number in the French army, find their respective losses given that the losses in the German army were twice those in the French army.

(20) The length of a rectangular field is thrice its breadth. Each side was increased by 10 yds. and the area then increased by 900 sq. yds. Find the dimensions of the field before extension.

(21) The length of a grass-plot was 2 yards more than its breadth. The length was cut down by 2 ft. and the breadth increased by 1 ft. Then its area diminished by 20 sq. ft. Find the length of each side of the new plot.

(22) Two years ago I weighed $\frac{1}{6}$th heavier than my friend. On weighing ourselves to-day, I find I have lost since then $\frac{1}{4}$th of the weight my friend has gained. If my weight is 130 lbs. now and that of my friend 140 lbs., find how much we weighed two years ago.

(23) A's age was twice B's 10 years ago. Now B's age $= \frac{5}{8}$ of A's. Find their present ages.

(24) The sum of the interior angles of a convex polygon of 'n' sides $= (2n - 4)$ right angles. In a particular polygon, each interior angle $= 108°$. How many sides has the polygon?

(25) AB is a st. line 15 cm. long. C is a point in it such that $AC = \frac{2}{3} \times CB$. Find the length of AC.

(26) A boat goes up-stream at three miles an hour and down-stream at 4 miles an hour. I travelled in the boat from P to Q up-stream, halted at Q for 4 hours and then returned to Q. I took 18 hours in all. Find the distance between P and Q.

(27) From A to B a man travelled along a short-cut at the rate of 3 miles per hour. He returned by a route 3 miles longer at $2\frac{1}{2}$ miles per hour and the return journey took him two hours onger. Find the distance between A and B by each route.

(28) A bag contains rupees, half-rupees, quarter-rupees and quarter-anna pieces—923 coins in all. If the sum of money represented by the different coins be the same, find the number of each kind of coin.

(29) In a whole number of 2 digits, one digit is twice the other. The digits are written in the reverse order and the new number formed is 36 less than the original number. Find the original number.

(30) S has given p acres and q acres of land to his sons A and C respectively. At his death he leaves 'r' acres to be distributed between his sons so that they may have equal property after the division. Show how the distribution may be done. How many acres of land will A and C newly receive ?

(31) A bag consists of an equal No. of quarter-anna pieces, one-anna pieces and two anna pieces, their total value being Rs 39. Find the No, of each kind of coin in the bag.

(32) If 'x' minutes after 3 o'clock the minute and hour-hands of the school clock are found coincident, show that $x = 15 + \frac{x}{12}$.

Find at what time between 3 and 4 o'clock the two hands of a clock will be (1) oppositely directed in the same straight line, and (2) perpendicular to each other. [Note that the speed of the moving end of the minute-hand is 60 minute-spaces per hour and that of the hour-hand 5 minute-spaces per hour].

(33) Between one Census and the next the population of a district increased by $\frac{1}{6}$th. The No. of Europeans doubled and that of Indians rose from 999,950 to 1,199,920 How many Europeans were there in the district at the time of the first Census ?

(34) In an action between two battle-ships P and Q, P fired twice as many shells as Q. P's hits were 60 and Q's 40. If P's misses were 4 times Q's, find how many shells each of them fired.

(35) If $\frac{am + b}{b} - c = d$, express 'm' in terms of 'a', 'b,' 'c', and 'd.'

(36) In each of the formulæ $\frac{an + b}{bn - a} = c$; $\frac{l - cn}{b + an} = d$; $S = -\frac{1}{5} - \frac{1}{4}n$, express '$n$' in terms of the other quantities involved.

(37) If $G = \frac{Wh}{d\,(W+P)}$, express 'W' in terms of the other quantities and thence find its value when $G = 224$, $d = 4{\cdot}5$, $h = 350$ and $P = 10$.

(38) If $S_1 = \frac{1}{2}m + 60$ and $S_2 = \frac{15}{n+2}$, express 'm' and 'n' in terms of the other quantities involved and hence find the value of 'm' and 'n' when $S_1 = 120$ and $S_2 = 2$.

(39) Find the numbers represented by letters in the following equalities: —

$$2(5x - 7) = 14{\cdot}4; \quad \frac{15}{13\frac{1}{2} - 6m} = 2;$$

$$\frac{4\frac{1}{2}x + 10}{3} = 15; \quad 2\frac{1}{2}(3\frac{1}{2}m - 4); - 5m = 6;$$

$$\frac{33}{4n - 4} \div 5 = 16; \quad \frac{6n + 1}{2} - 5 = 10; \quad \frac{2}{n} + \frac{1}{8} = \frac{5}{n}.$$

and $\frac{1}{2}\{2 \times 50 + (n - 1) \times (-2)\} = \frac{1}{2}\{2 \times 55 + (n - 3)(-4)\}$.

(40) Two brothers A and B have equal amounts of money. A spends Rs. 45 per month out of his money while B adds Rs. 25 per month to the money he has. If after 12 months B has twice as much as A, find the amount each had at first.

(41) Two officers A and B are started in service on Rs 200 and Rs. 400 respectively on the same date. A being given a biennial increment of Rs. 100 and B one of Rs. 50. How many years hence will they draw equal salaries?

(42) The formula $C = \frac{3}{64}b \div \frac{1}{32}p + \frac{1}{8}w + \frac{1}{4}g$ gives the collections in Rs. C at a flower-show where 'b' blue tickets, 'p' pink tickets, 'w' white tickets and 'g' green tickets were sold. From the formula, find 'b' when $C = 260$, $p = 1600$, $w = g = 400$.

CHAPTER IV.

CIRCLES, ANGLES AND PARALLELS.

SECTION I.

35. THE CIRCLE.

Let a boy stand at A. Let some more boys stand so that each is 6 ft. from A. Their positions may be indicated by points marked as in fig. 18.

Fig. 18.

If these points be jointed by a smooth curve the space enclosed is called a *circle*. And the curved line enclosing it is called the *circumference of the circle*. The point A is the centre of the circle. Hereafter the symbol '⊙' will be used for 'circle' and 'O^{ce}' for 'circumference.')

All points on the O^{ce} *of a* ⊙ *are equidistant from its centre*. A straight line like AR drawn from the centre to the O^{ce} to represent such distance is called a *radius* of the ⊙.

If one end of a string be held fixed at a point A on a flat surface and, holding the string straight, the other end be moved about, the space traversed by the string is a circle.

NOTE 1. *All radii of a* ⊙ *are equal.*

NOTE 2. *If a point moves so as to be at the same distance from a fixed point it must move along the* O^{ce} *of*

a ⊙. This is the principle involved in the construction of a ⊙ with a pair of compasses. (Suggest other methods of drawing ⊙s on the floor and on paper).

NOTE 3 *A point lies outside, or inside, a* ⊙ *according as its distance from the centre is greater, or less, than the radius.*

Cut out a paper-circle with radius 4 cm. Fold it along its centre. It will be divided into two halves exactly coinciding with each other. Each half is called a *semi-circle*. Its straight line boundary is a *diameter* of the circle and its curved-line boundary is the *semi-circumference*. ('Semi' means 'half.')

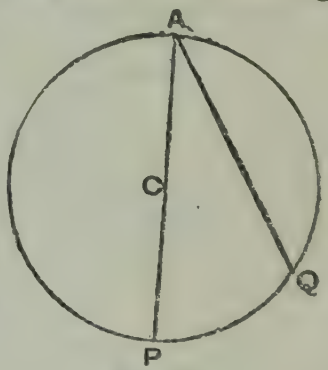

Fig 19.

Note that a diameter AP = twice the radius CP. If instead of folding along AP the circle be folded along AQ it will be divided into two parts (called *segments* of the circle) which are unequal. The straight line AQ is called *a chord* of the circle and the two parts into which the O^ce is divided at A and Q are called *arcs* of the circle. The arc greater than the semi O^ce is called the *major arc* and the other the *minor arc*.

Ex. 1. Show with the help of the dividers that the diameters of a ⊙ are equal.

Ex. 2. Draw a ⊙ with rad. 2·5″. Mark point B in it 2″ from its centre C. Draw a diameter through B. Draw some more st. lines from B to the O^ce. Find by measurement and comparison which is the greatest of such st. lines and which is the least.

36. *Comparison of chords of a ⊙ drawn from a point* A *on the* ⊙ce :—

Take a piece of string. Fix one of its ends at A. Holding the string straight, rotate it as indicated in fig. 20 so that it passes through several points on the ⊙ce. Then portions of the string will represent chords of the ⊙. It would be seen that the chords increase in length as they approach the centre and decrease in length as they recede from it and also that the diameter through A is the greatest of such chords.

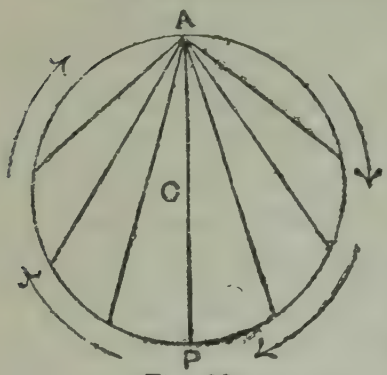

Fig. 20.

37. *To place a chord of given length in a ⊙ through a given point* A *on the* ⊙ce :—

Let P be the other end of the chord. Since AP should be equal to the given length, P must lie on the ⊙ce of a ⊙ whose centre is A and whose radius = the given length. Hence, with centre A and radius = the given length, draw two arcs cutting the ⊙ce of the given ⊙ at P and Q. Join AP, AQ. AP or AQ is the chord required. (Why?)

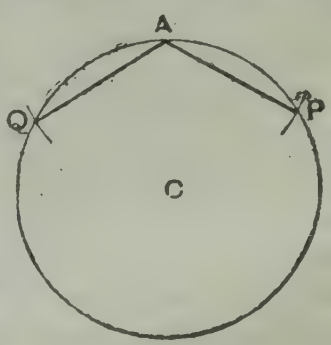

Fig. 21.

NOTE:—*Two, and only two, equal chords could generally be drawn from a point on the* Oce. (Why?)

When is one chord alone possible? When is the construction impossible?

Ex. 1. Draw a ⊙ with rad. 2 cm. Starting from a point A on the ⊙ce place six successive chords AP, PQ, QR, RS, ST, TU, each equal to the radius. Where does U fall? Shade the fig. formed by the chords.

Ex. 2. Two boys A and B can each throw stones to a distance of 30 yds. at the greatest. Standing at a distance of 20 yds. from each other along an east-to-west line, they are both just able to throw a stone to hit a monument M to the north of the line. Fix the position of M in a diagram drawn to scale.

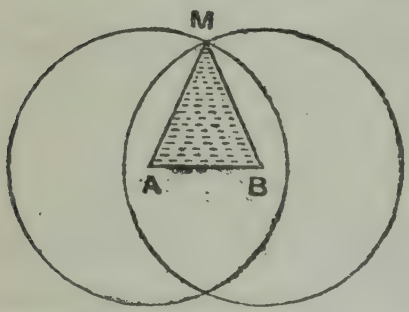

Fig. 22.

Let 1 cm. represent 10 yards. Along an east-to-west line, mark two points A, B, 2 cm. apart to represent the positions of A and B. Since the monument is 30 yds. from A it must be on the ⊙ce of a ⊙ whose centre is A and rad. = 30 yds. Similarly the monument must be on the ⊙ce of a ⊙ with centre B and rad. = 30 yds. Hence, draw two circles with centres A and B and with rad. = 3 cm. and let their ⊙ces cut at M and P. If M be to the north of the line AB, M would represent the position of the monument.

The Oce of each ⊙ need not be drawn in full. It is enough if the intersecting arcs be shown to represent the pt. M.

[In the above figure, the shaded portion has M, A, B, as its three corners. It is called the triangle MAB and its sides MA, AB, BM are 3 cm., 2 cm. and 3 cm. long. Hence triangles could be constructed if the lengths of their sides be given.

EXERCISES.

(1) P and Q represent two fixed stones in the rear-yard of a house. They lie in a north-to-south line and are 20' apart. The house-owner has secretly hidden his treasure below a spot in the rear-yard which is at the same distance of 15' from P and Q but lies to the east of PQ. Fix the position of the spot. Draw a diagram to a scale of 1 cm. to 5'.

(2) Work the above exercise when the treasure spot is 15' from P and 10' from Q.

(3) Construct a triangle with sides 4", 3", 3"; each 4; 4", 3", 2"; 2", 1", 1·5". Show that *no triangle could be constructed unless any two of the sides given are together greater than the third side.* Verify by choosing lengths for the sides of a triangle.

(4) A village B is three furlongs east of another village A. A school is to be built which shall not be more than 2 furlongs from either village. Mark in a plan drawn to scale the area in which the school may be built.

(5) A and B are two forts 5 miles apart and contain guns whose ranges are 4 miles and 3 miles respectively. Draw, to any convenient scale, a diagram showing the ranges of the guns and shade the portion covered by the guns of both the forts.

(6) Two fortresses P and Q are at a distance of 5 miles from each other. Where must a gun be mounted whose greatest range is 4 miles if it should be able to fire shots which would just hit both the fortresses?

(7) The minute and hour-hands of a watch are respectively 3 cm. and 2 cm. in length. Draw a diagram showing the paths of their moving ends.

[NOTE:—*Any number of circles may be drawn so as to have the same centre.* Such circles are called *Concentric* circles.]

(8) Construct 2 ⊙s with centres A and B 4 cm. apart and with rad., 2 cm., 2 cm.; 1·5 cm., 2·5 cm.; 1·5 cm., 1·5 cm.; 2 cm., 1·5 cm.; 2 cm., 3 cm.; 3 cm., 4 cm.

[Note that *two ⊙s touch each other externally if the distance between their centres = the sum of their radii.* When will

they cut? When will one ⊙ lie entirely outside the other? Note also that, *if two ⊙s touch each other, the point where they touch is called the point of contact and it lies on the straight line joining the centres.*]

(9) Construct two ⊙s with centres, P, Q 2 cm. apart and with radii 3 cm., 1 cm.; 4 cm., 2 cm.; 5 cm.; 3 cm.; 3 cm., 5·5 cm.; 4 cm., 5 cm.; 3 cm., 4 cm.

[Note that *two ⊙s touch each other internally if the distance between their centres = the difference between their radii* and also that *the point of contact is in a line with the centres.*]

(10) Construct a circle with rad. 4 cm. Draw two more ⊙s to touch it (*a*) internally at a point A on the ⊙ce, (*b*) externally at the same point and (*c*) one internally and the other externally at the same point A on the ⊙ce.

How many such ⊙s could be drawn? Where will their centres lie? What other condition should be given to draw any particular circle you may have in view?

(11) Construct two circles with radii 6 cm and 4 cm. to touch each other (*a*) externally, and (*b*) internally, at a given point A.

(12) Mark three points P, Q, R so that PQ = QR = PR = 4·6 cm. Construct equal circles with centres P, Q and R so that each ⊙ touches the other two.

(13) Find four points P, Q, R S such that PQ = QR = 3 cm., PR = 4 cm., PS = 3·5 cm., and RS = 4 cm. Find the distance between Q and S for the different positions they may assume.

(14) Describe a triangle with the sides 6 cm., 8 cm., 10 cm., long. Draw circles with the sides as the diameters.

(15) Describe a circle with radius 2.5". Which is the longest chord that can be drawn in it? Draw several chords and compare their lengths by using your dividers or by measurement.

(16) Construct a triangle ABC in your note-book so that the point A is 5" from the top-right corner and 4" from the top-left-corner. B is 6" from the top-left corner and 5" from the bottom-left-corner and C is 5" from each of the bottom-corners. Discuss the different cases when the construction would be impossible.

(17) Draw two ⊙s with centres 3·5 cm. apart touching each other (*a*) externally and (*b*) internally, it being given that the radius of one of them = 2·5 cm.

(18) Construct triangles after choosing lengths for its three sides. Verify by using the dividers the truth of the following statement :—The difference between any two sides of a triangle is always less than the third side.

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SECTION II.

38. ANGLES.

If a man starts from P, walks in the direction PQ, turns at A to the direction AR, he may be said to have turned through the *angle* QAR.

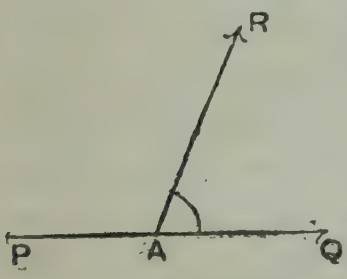


Fig. 23.

Thus, to represent an angle two st. lines which branch out from the same point are necessary, *e.g.*, AQ, AR in fig. 23. These two lines are called the *arms* of the angle and their point of intersecton is the *vertex* of the angle.

The "angle QAR" may be written "∠QAR" or "$\hat{QAR}$." In naming an angle the name of its *vertex* occurs in the middle.

Comparison of angles :—

(i) Place your dividers with the fixed end at A and both the legs coincident on AQ. Then keeping one leg on AQ rotate the other leg until it assumes in turn the positions AR, AS. The amount of turning is greater with the ∠QAS and the ∠QAS is greater than the ∠QAR.

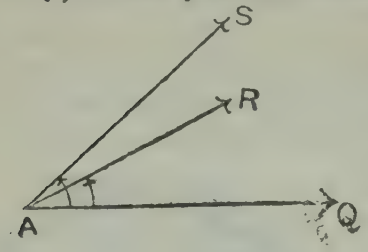


Fig. 24.

[Note that the *size of an angle is independent of its arms*].

(ii) Trace the ∠PST on tracing paper, or cut out on paper the ∠PST, and place it over the ∠ACB so that S falls on C, SP along CA. If ST falls along CB, the ∠s PST, ACB are equal.

If ST falls between CB, CA, the ∠PST is less than the ∠ACB. (When is the ∠PST greater than the ∠ACB?)

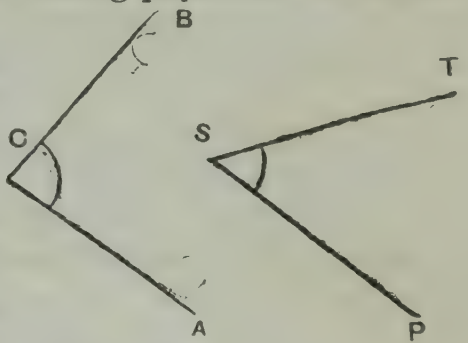


Fig. 25.

(Note that there are 2 ways of turning from the direction SP to ST as shown by arrow marks in fig. 26. Of the 2 ∠s thus formed, the ∠PST or ∠TSP refers only to the ∠ which requires the less amount of turning and the other is called the *reflex angle* PST or TSP.)

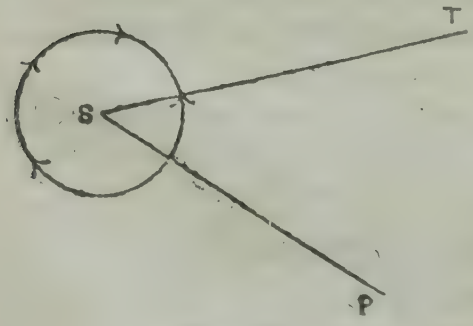


Fig. 26.

Ex. A man walks round a field ABCD, going from A to B, B to C, C to D and D to A. Mark the ∠s he

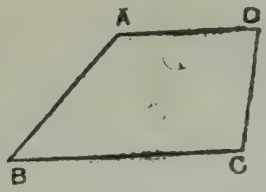


Fig. 27.

turns through.

39. THE STANDARD ANGLE.

Take a sheet of paper having a straight edge AB.

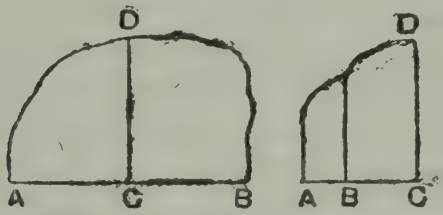


Fig. 28.

Mark a point C in it. Fold the sheet so that CB falls along CA. The line of crease CD is straight. The angles DCA, DCB are equal. Each of them is called *a right angle* (rt. ∠). The rt. ∠ is the simplest angle to make and is hence chosen as the standard ∠ of measurement.

CD is said to be *at right angles to*, or *perpendicular* (⊥) *to*, AB.

Each of the angles P, Q, is less than a right ∠ (Why?)

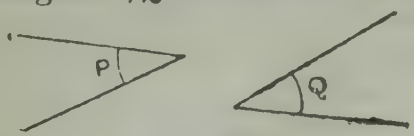


Fig. 29.

and is called an *acute* angle.

The angles m, b, are each greater than a right angle (Why?) and are hence called *obtuse* angles.

Fig. 30.

Suppose a stick is placed in the direction OP. It is revolved ronnd O so as to face the direction OS.

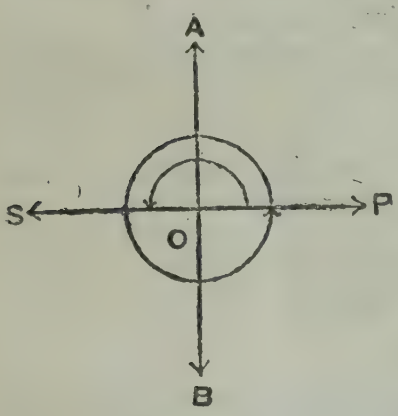


Fig. 31.

From OP to OA it must turn through one right angle and from OA to OS thro' another rt. ∠. The ∠POS = 2 rt. ∠s and is called a *straight* ∠. Also, one complete revolution round O means turning through 4 rt. ∠s. Hence *the ∠ turned thro' in one quarter of a complete revolution is a right ∠.*

40. THE PROTRACTOR OR THE ANGLE-GAUGE.

Cut out a paper circle with radius 4 cm. Fold it along a diameter. Again fold it along a radius ⊥ to the diametrical line of fold. The circle will then be divided into 4 equal parts called *quadrants* having their ∠s at the centre right ∠s. Cut out a quadrant. Fold it twice so as to divide the angle at the centre into 3 equal parts. Similarly divide each such part into 3 smaller equal parts.

It will be found that each small ∠ at the centre $= \frac{1}{9}$ of a right angle. It will also be found that, if in a circle arcs be equal, ∠s opposite to them at the centre are also equal.

Applying this principle, the rt. ∠ at the centre of a quadrant may be divided into 90 equal parts. One such part ($= \frac{1}{90}$ of a rt. ∠) is called a degree (written 1°) and is used as the *unit* of angle measurement.

(Show how you would practically divide the semi-⊙ce of a ⊙ into 180 equal parts).

A semi-circular protractor may be thus constructed and, by means of it, angles could be measured in degrees.

(A rectangular, triangular or a quadrant protractor could easily be cut from it. How?)

To measure an ∠ PQS with the protractor, place the centre of the semi-circular protractor on Q and its diameter along QP as in fig. 32. Read the graduation against QS. If it be 40, $\angle PQS = 40^\circ$. (Since ∠PQS is acute, the other reading 140 should not be given).

How to draw an ∠ of given size, *e.g.*, of 40° :—

(i) Draw a st. line QP. Place the protractor so that its

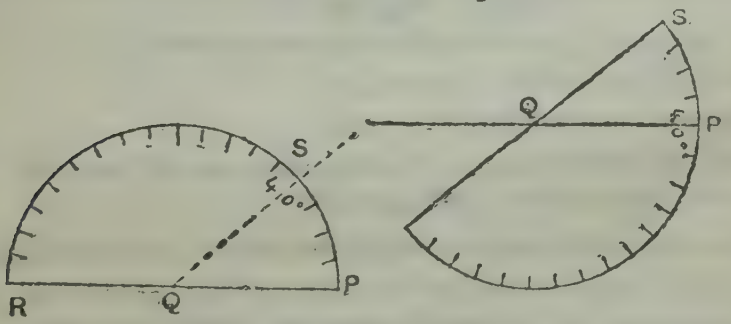


Fig. 32.

centre falls on Q and its diameter along RP. Mark a pt.

S where the st. line proceeding from Q to the 40° graduation ends. Join QS. Then the $\angle PQS = 40°$.

(ii) Place the protractor so that its centre falls on Q and the straight line proceeding from Q to the 40° graduation falls along QP. Draw a st. line along the straight edge of the protractor. Then the $\angle SQP = 40°$.

EXERCISES.

Fig. 33.

(19) Measure the $\angle$s in the above figure and find their sum.

(20) In (*a*), (*b*) and (*c*) below, two angles are drawn with the

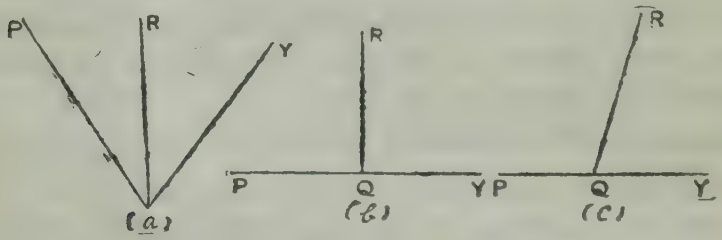


Fig. 34.

same vertex and on either side of a common arm.

(Such $\angle$s are called *adjacent* $\angle$s).

Measure them and find their sum.

(21) Draw 2 adjacent $\angle$s whose sizes are 40° and 40° ; 40° and 60° ; 40° and 140° ; 50° and 130°.

From the above exercises verify the truth of the following :—

If the two outer arms of two adjacent $\angle$s are in a st. line, their sum = 2 rt. $\angle$s ; and conversely. *If two adjacent $\angle$s are together equal to 180°, their outer arms will be in the same st. line.*

When two ∠s are together = 2 rt. ∠s they are said to be supplementary, or one of them is said to be the supplement of the other.

There are 2 nos. against each division of the protractor. Why is the sum of each pair 180? Show how you would choose only one of them in measuring a given angle.

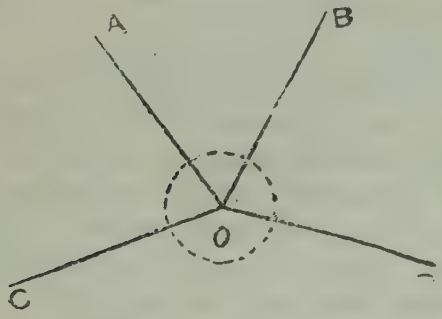


Fig. 35.

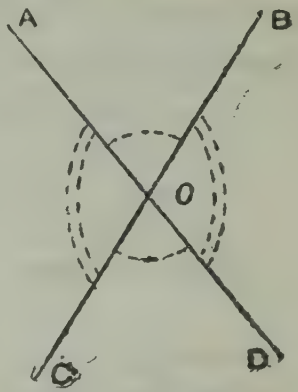


Fig. 36.

(22) Compare the ∠s AOB, COD and also the ∠s AOC, BOD in figs. 35 and 36. Note that *when two st. lines intersect the vertically opposite ∠s are equal*.

(23) Draw any no. of st. lines meeting at a point O. Measure the ∠s around O in order and find their sum. Also, find the sum of the angles around O in figs. 35 and 36. Hence show that *the sum of the ∠s around a point = 360°*.

Measure the ∠s in the figs. below. Also, measure the ∠s

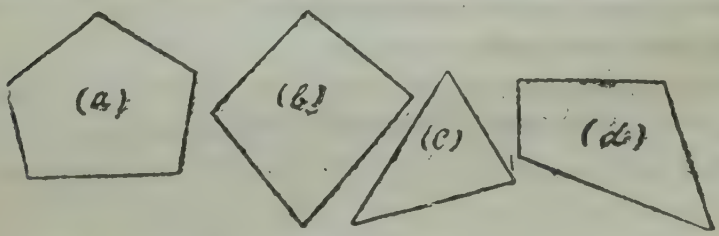


Fig. 37.

a man may have to turn through in walking round them. [Fig. (*a*) is a triangle. It has three sides and three angles. Fig. (*b*) and

(*d*) have four sides. They are called *quadrilaterals*. A figure with five sides is a *pentagon*, with six sides a *hexagon*, with eight sides an *octagon*, with many sides a *polygon*.]

(24) Draw six different triangles, measure their angles with the protractor and find the sum of the three angles in each triangle. Verify if the three angles of a triangle are together equal to two rt. ∠s.

[Note the following methods:—

(*a*) Cut out a piece of triangle ABC. Fold it through A so that the line of crease AD is ⊥ to BC. (This could be done by making DC fall along DB). Find, by folding, E, F the middle pts. of DB, CD. Fold through E and F so that the crease lines are perpendicular to BC and B and C fall on D. Fold along OP so that A falls on D. Show that the three ∠s of a △ are thus equal to 2 rt. ∠s.

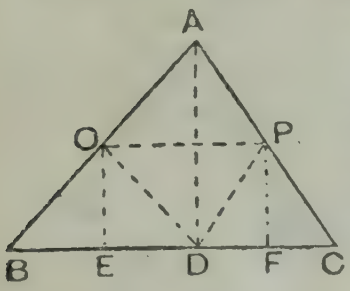


Fig. 38.

(*b*) Cut out the 3 ∠s of a △ and see if they together form a straight-angle.

(*c*) A man stands at C facing the direction CB (Fig. 39), turns thro' the ∠C, goes to A in the direction AD, turns at A thro' the ∠ DAE, goes backwards to B and turns there towards C. Show that in so doing he must turn thro' the ∠s of the triangle or through 2 rt. ∠s.

Hence or otherwise, show that the four ∠s of a quadl. are together equal to four rt. ∠s.]

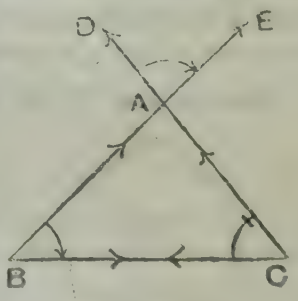


Fig. 39.

(25) A man walks 3 miles due North, turns thro' 45° to his left, walks 3miles, turns thro' 45° to his right and goes 1mile. How far is he finally from his starting point? (Find from a fig. drawn to scale).

(26). Construct a pair of △s ABC, DEF such that

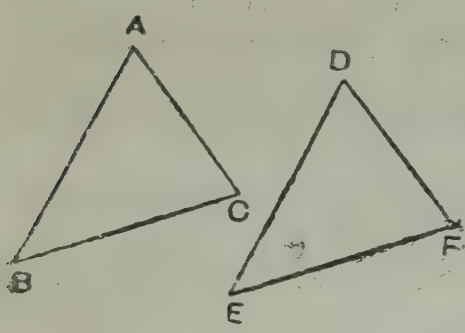


Fig. 40.

(*a*) AB = 3" = DE
BC = 4" = EF
AC = 4·5" = DF;

(*b*) AB = 3" = DE
AC = 4" = DF
∠A = 38° = ∠D

(*c*) BC = EF = 4"
∠B = ∠E = 30°
∠C = ∠F = 50°

(*d*) AB = ED = 3 4"
∠B = ∠E = 60°
∠C = ∠F = 88°;

(*e*) ∠B = LE = 90°
AC = DF = 5"
BC = EF = 4°; and

(*f*) ∠B = ∠E = 90°, AB = DE = 3", BC = EF = 4".

Show, by cutting and superposition:

(i) from (*a*) that two △s will exactly coincide with each other and hence will be *congruent* or *identically equal* if three sides of the one be respectively equal to three sides of the other.

(ii) from (*b*) that two △s are congruent if two sides and the included ∠ in the one are respectively equal to two sides and the included ∠ in the other,

(iii) from (*c*) and (*d*) that two △s are congruent if two angles and one side of the one are respectively equal to two angles and one side of the other.

(iv) from (*e*) and (*f*) that two right-angled triangles are congruent if two sides of the one are respectively equal to two sides of the other.

(27) Construct a triangle ABC

having (*a*) AB = 3", ∠B = 30°, AC = 2";
(*b*) AB = 3", ∠B = 100°, AC = 5";
(*c*) AB = 3", ∠B = 30°, AC = 1·5".

Show with reasons that the construction gives two triangles of different sizes and shapes in (*a*), only one in (*b*) and (*c*) and that if AB = 3", ∠B = 30°, AC = 1·2" the construction is impossible.

(28) Draw three △s which are equiangular to one another. Show that they resemble one another in shape but need not be identically equal. (The 3 △s are said to be *similar*).

(29) Show that the following method of construction would give an ∠ exactly equal to a given ∠BAC:—

Draw a straight line PQ. With centres A and P and with the

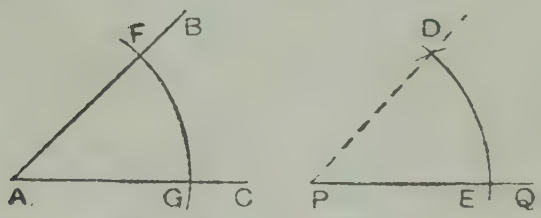


Fig. 41.

same rad. draw (1) the arc FG cutting AB, AC at F and G and (2) the arc DE. With centre E mark off ED = the distance FG. Join DP. Then the ∠DPQ is the one required.

(30) Construct a quadrilateral PQRS, given the following:—

(*a*) PQ = QR = 4 cm.
PS = RS = 3 cm.
QS = 5·5 cm.;

(*b*) ∠PQR = 100°, PQ = 4 cm., QR = 2 cm, PS = 4 cm. SR = 5 cm.;

(*c*) ∠PQR = 76°, ∠QRS = 83°, ∠RSP = 99°, PS = 4 cm., SR = 2·5 cm.;

(*d*) ∠PQR = 96°, ∠QRS = 76°, PQ = 4 cm., QR = 3·5 cm., SR = 5·6 cm.;

(*e*) PS = 3 cm. SQ = 6 cm., PR = 5cm., PQ = 4 cm. RS = 4 cm.;

(*f*) PQ = 4·7 cm., ∠PQR = 76°, ∠QPS = 106°, PR = 8 cm., QS = 7·5 cm;

(*g*) QR = 4 cm., RP = 6 cm., ∠RQP = 76°, QS = 5·9 cm., ∠QRS = 100°,

(*h*) QR = 2 cm, $\angle QRP = 30°$, $\angle SQR = 45°$, $\angle QRS = 75°$, $\angle PQR = 95°$.

Measure the other sides and angles in the quadrilaterals thus constructed,

Show that five independent conditions are necessary for constructing a quadrilateral.

(31) Copy out the figs. given in Exercise 23 as accurately as you can. [Only a set of compasses and ruler are allowed].

(32) Draw a diagram of a four-sided field ABCD from the following data :—

A man walking along its boundary goes from A to B 250 yds., turns 74° to the right, goes from B to C 270 yds., turns 82° to the right and goes from C to D 310 yds. What is the length of the fourth side of the field ? (Represent 100 yds. by 1 inch)

(33) Show how the construction in Exercise 30 (*h*) would help us to *find the distance between two inaccessible objects* P and S.

(34) Construct a $\triangle$ with each side 3 cm Measure its angles. Repeat the above exercises with 5 more $\triangle$s having its three sides equal.

[A triangle whose three sides are equal is called an *equilateral* triangle. Its three $\angle$s are equal and are hence each 60°. 'Equilateral' means 'equal sided.']

(35) Construct a $\triangle$ with two sides each = 3 cm. Measure the third side and the $\angle$s. Which of its angles are equal ? Repeat the above exercise with 5 more triangles having two sides equal.

(A triangle with two of its sides equal is called an *isosceles* triangle and the angles opposite to the equal sides will be found to be equal. ('Isosceles' means 'having equal legs.")

(36) Draw five triangles with all the sides of each unequal. Measure the angles in each.

(A triangle with all its sides unequal is called a *scalene* triangle. Its three angles will be unequal.

(37) Cut out a triangular sheet of paper ABC. Find by folding the middle points D, E, F of its three sides. Fold along DE, DF, and EF. Show by paper-cutting and superposition that the triangular sheet of paper is divided into four congruent smaller triangles.

(38) Construct an isos $\triangle$ ABC with AB = AC. $\angle BAC = 30°$. Show that an $\angle$ could be divided into 2 equal parts (*bisected*) by the following methods:—

(1) Cut out on paper an angle BAC. Fold it along AD so that AC falls along AB. Then AD is the bisector.

(2) Measure the $\angle BAC$ in degrees. Draw with the protractor an $\angle BAD$ within the angular space BAC so as to contain half the number of degrees in $\angle BAC$.

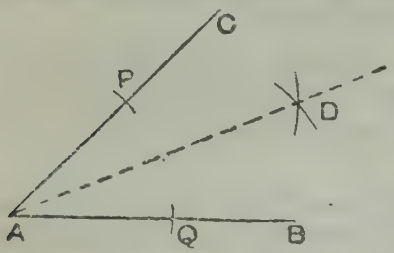


Fig. 42.

(3) With centre A draw an arc cutting AC, AB at P, Q. respectively With P. Q. as centres and with the same rad., draw arcs cutting at D Join AD. Then AD is the bisector of the $\angle BAC$. *Vide* Ex. 26 (1).

[Note that the *construction method will give the exact bisector.*]

(39) Draw with the protractor two straight lines NS, WE to

Fig. 43.

cut at O at 90° as in fig. 43. Divide each ∠ at O into 2, 4, 8, 16 equal parts. Adopt the construction method and thus mark the points of the Mariners' compass. Point out the lines ⊥ to each other in the figure obtained.

What is the difference in degrees between two consecutive points of the compass?

[NOTE.—The direction of a point which is not indicated definitely by the Mariners' compass can be described as 35° N of E for the point P in fig 44, where the ∠EOP = 35°.

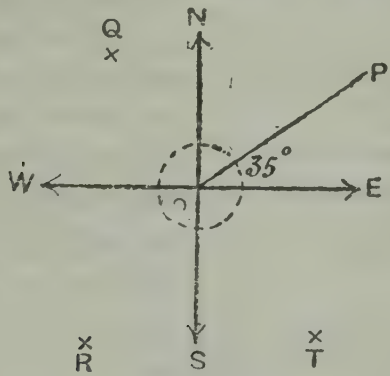


Fig. 44.

The position of P may also be described as 55° E of N, the ∠NOP being 55°. The point P is then said to *bear* 35° N of E or 55°E of N from a point O. Similarly, the point R may be described as *bearing* 55° S of W. Read out the bearings of T and Q.]

(40) A point P bears 62° N of W from a point A; and, from a point B situated 2½ miles due west of A, P bears 42° N of E. Find from a diagram drawn to scale the distance of P from A and B.

(41) A ship moving directly S W. is observed from a light-house and is found to bear 27° N of E at 3 P. M. At 4-30 P.M. it reached a harbour which bears 17° S of E. If the distance between the light-house and the harbour be 20 miles, find the speed of the ship in miles per hour from a diagram drawn to scale.

SECTION III.

41. DRAWING ⊥s WITH SET SQUARES.

The figs. drawn below suggest two methods :—

(*a*) Suppose a ⊥ to AB has to be drawn. Place the set square in position (1) indicated in fig. 45. Place the ruler along the longest edge of the set sq. Holding the ruler firmly, slide the set sq. up to the position (2), *i.e.* to say, till the edge PQ passes thro' the particular point through which the required perpendicular is to be drawn. Draw a straight line P'Q' along the edge.

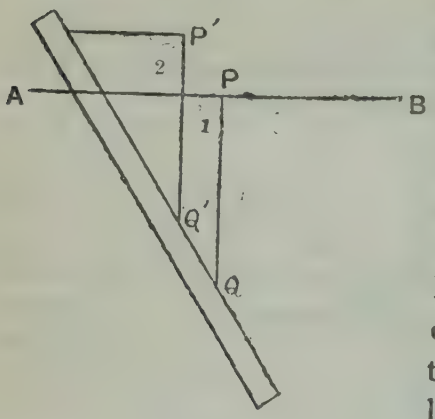


Fig. 45.

(*b*) Place the set sq. as in fig. 46 so that its longest edge P*q* falls along AB. Holding the ruler firmly along the edge P*r*., turn the set square so that P*r* occupies a position (as in fig. 47 above) which is ⊥ to its former

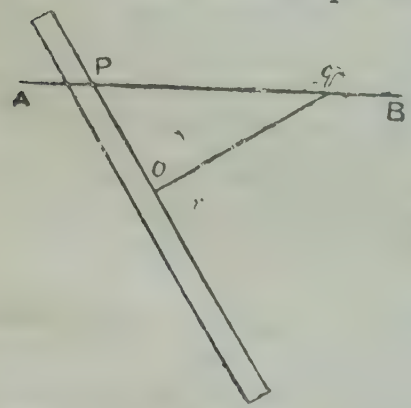


Fig. 46.

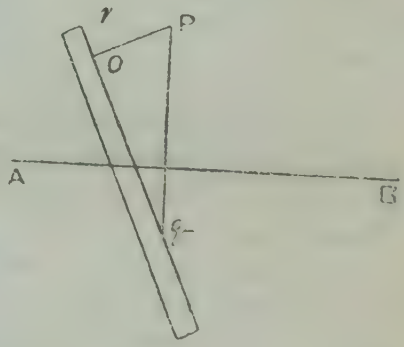


Fig. 47.

position. Slide it up or down the ruler till Pq passes through any particular point from which the $\perp$ is to be drawn. Draw a st. line along Pq.

EXERCISES.

(42) Draw a st. line AB and mark a pt. P outside it. Draw a $\perp$ PQ to AB. How many such $\perp$s could be drawn through P?

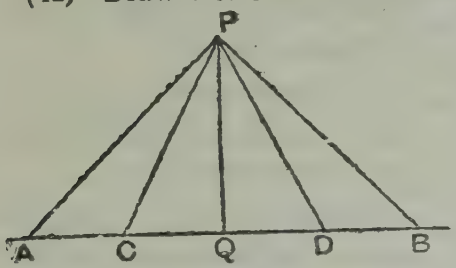


Fig. 48.

Draw some more st. lines PA, PC, PD, PB. Which of the st. lines drawn from P to AB is the shortest?

[Since PQ is the shortest of such st. lines, the *distance of a point* P from a st. line AB *means always the* $\perp$ *distance* PQ. Note that *of all the straight lines that can be drawn to a given straight line from a point, the perpendicular is the shortest*].

(43) Construct a quadrilateral with each $\angle$ a rt. $\angle$ and with each side = 2". (The fig. formed is a *square* which has all its sides equal and all its angles rt. $\angle$s.)

(44) Construct a quadrilateral with each $\angle$ a rt. $\angle$ and with 2 adj. sides 4 cm. and 3 cm. (The fig. is a *rectangle*.)

Divide the rect. into sq. cm.

(45) Find the $\perp$ distance of P from AB. How would you test whether the $\perp$ edges of your set squares are at exact right angles to each other or not?

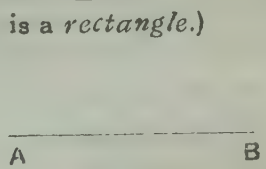


Fig. 49.

(46) Show by measurement that, in one set square, each acute $\angle = 45°$ and the arms of the rt. $\angle$ in it are equal.

Show also that in the other set square the acute $\angle$s are 30°, 60° and that the longest edge is double the shortest.

SECTION IV.

42. PARALLELS.

A straight line PQ cuts two other st. lines AB, CD.

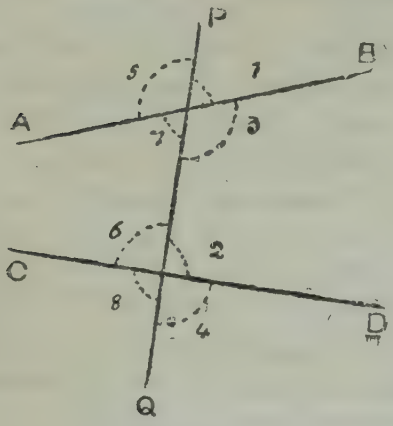


Fig. 50.

(Such a st. line is called a *transversal*). Eight ∠s are formed.

The pairs of ∠s (1, 2), (3, 4), (5, 6), (7, 8), are said to be *corresponding ∠s*.

Angles like (7, 2), (3, 6) are called *alternate ∠s*.

Measure the angles 1, 2 and calculate the size of each of the rest.

Note that in the above neither the corresponding nor the alternate ∠s are equal. AB and CD move away from each other in the direction of B and D and move to meet each other in the direction of A and C.

Thro' 2 pts. P, Q in a st. line draw two st. lines CD, RS so that the coresponding ∠s are equal. If one of the ∠s be 60°, find the size of each ∠ in the fig.

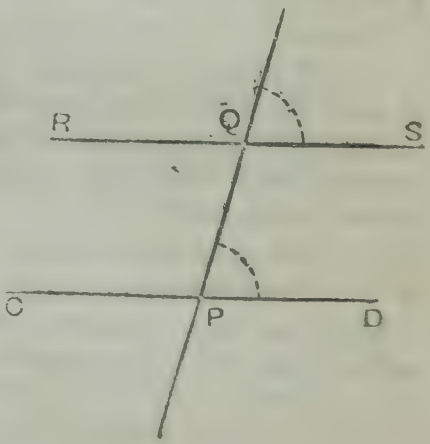


Fig. 51.

Note (1).—*When the corresponding ∠s are equal*, RS, CD attempt to meet neither towards S, D nor towards R, C] and *are said to be parallel* ('||').

Note (2).—*When the st. lines are thus* ||, *the alternate*

angles RQP, DPQ *are equal*, as also the angles QPC, PQS. (Why ?) Also, ∠ SQP + ∠ QPD = 180° (Why ?) and ∠ RQP + ∠ CPQ = 180°. (Why ?)

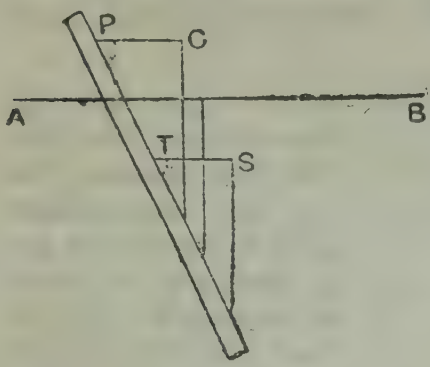


Fig. 52.

The principle stated in note (1) above underlies the method of drawing parallels with the set squares indicated in Fg. 52 where PC and TS are drawn || to AB.

NOTE 1 :—In fig. 52 the || st. lines PC, AB, TS point to the same *direction*.

NOTE 2 :—Thro' the same pt. T only one || could be drawn to a st. line.

EXERCISES.

(47) Construct any triangle ABC. Through A draw PAQ || to BC. Hence show that the 3 ∠s of the △ are together equal to 180°.

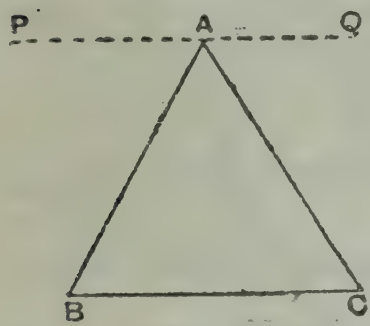


Fig. 53.

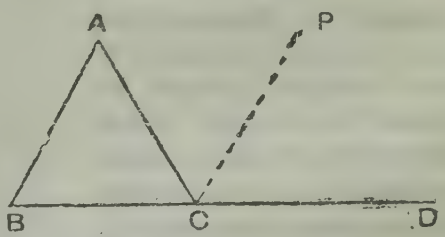


Fig. 54.

(48) Construct a △ ABC. Produce BC to D. Through C draw CP || to BA. Hence show that the *exterior angle* ACD *is equal to the sum of the two interior opposite* ∠s ABC, BAC. Verify your result by measurement.

(49) Draw any $\angle ABC$. Within the angular space mark a point Q and draw QD, QH $\parallel$ to BA, BC respectively. Join BQ and produce it. Show that $\angle DQH = \angle ABC$. Verify your result by measurement.

(50) Draw AB $\parallel$ to CD. Draw PQ, RS, LN $\perp$ to AB, CD.

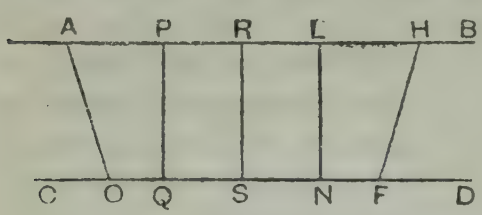


Fig. 55.

Draw other st. lines like OA, FH between the $\parallel$ s. On measurement it will be found that the $\perp$ distance is the shortest between the two $\parallel$ s. Also note that (1) *the distance between two $\parallel$s is the same throughout ; (2) if PQ is drawn $\perp$ to CD, it is also $\perp$ to the $\parallel$ line AB ; and (3) if a pt. moves so as to be always at the same distance NL from CD, its path must lie along one of the two $\parallel$s that could be drawn to CD at the given distance NL from it.*

(51) Draw two $\parallel$ s AB, CD and draw two other $\parallel$s PQ, NO cutting one another at V, R, S, U. The figure VRSU has its opp. sides $\parallel$ and is hence called a parallelogram ($\parallel^{m}$). Measure the sides and angles of the $\parallel^{m}$ and compare. It will be found that *its opp. sides are equal; its opp. $\angle$s are equal; two of its $\angle$s with a common arm are together equal to two rt. $\angle$s.* Join the diagonals UR, VS. Let them cut at T. On measurement, it will be seen that VT = TS, TR = TU. *The diagonals of a parallelogram bisect each other.*

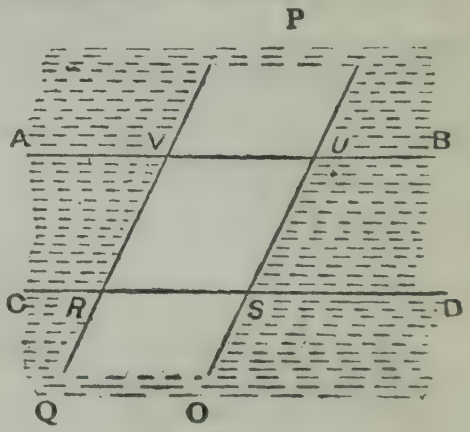


Fig. 56.

(52) Construct a $\parallel^{m}$ ABCD, having

(*a*) AD = 3″, DC = 4″, AC = 6″.

(*b*) AD = 3″, DC = 4″, $\angle ADC = 100°$.

(*c*) AD = 3 cm., ∠DAB = 108°, DB = 5 cm.

(*d*) AC = 4 cm., DB = 8 cm., the angle bet. AC, DB = 41 degrees.

(*e*) ∠DAC = 45°, ∠ADC = 34°, BC = 4 cm.

(53) Show that the following construction method may be used in drawing a || to a given st. line RS through a given point T :—

With cent. T draw any arc RQ. With centre R and with the same radius draw an arc TH. Set off RQ = HT. Join QT. Then QT is || to RS. (Why ?)

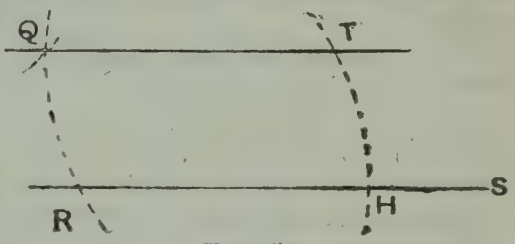


Fig. 57.

(54) Draw a rt. ∠BAC. In its angular space mark a point 2" from AB and 3" from AC.

(55) Draw PQ, XY so as to intersect at an ∠ of 75°. Mark 4 pts. each of which is 2" from PQ and 2·5" from XY. Also, mark the points taking PQ, XY at rt ∠s.

(56) Draw a st. line PQ and mark a pt. X, 3 cm. from it. Mark another point which is 4 cm. from X and 4 cm. from PQ.

(57) Draw an ∠PQR = 68°. Draw any △ like PAB on AP,

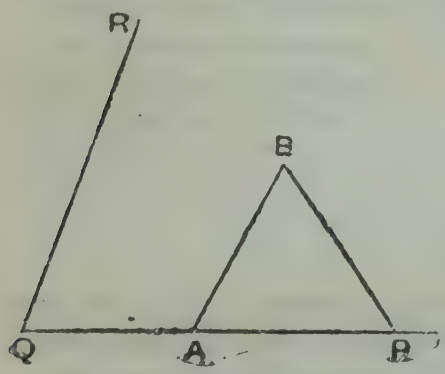


Fig. 58.

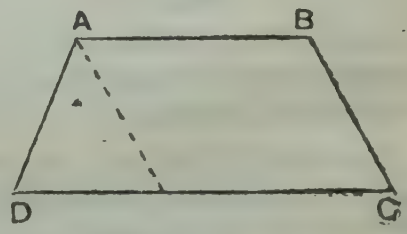


Fig. 59.

a segment of QP. Construct a △ exactly equal to PAB but with its vertices lying on QR, QP, produced if necessary.

(58) Construct a quadl. ABCD with CD, BA parallel, BA = 4 cm., BC = 2 cm., CD = 6·5 cm., AD = 2·9 cm. Write down the steps of your construction. (Refer to Fig. 59).

(59) Construct a ||^{m} ABCD with its sides each = 3·6 cm. and one ∠ = 54°.

Join the diagonals and measure the ∠s at the point of their intersection. (Such a ||^{m} is called a rhombus. It may be seen that *the diagonals of a rhombus bisect each other at right angles.*)

(60) Construct a rhombus with its diagonals 6 cm., 4 cm. Measure its sides and angles.

(61) Draw any straight line AX and step off equal distances AB, BC, CD, DE, etc., along it. Through the points A, B, C, D, E, draw a series of parallels. Draw a few straight lines to cut the parallels, taking some of them to pass through the points A, B, C, D, or E, some parallel to AX and some at random. Measure the intercepts made on each of these straight lines by the set of parallels and compare their lengths? What result do you get? If a set of parallels be drawn so as to make equal intercepts on one straight line, it will also make equal intercepts on any other straight line cutting the parallels.

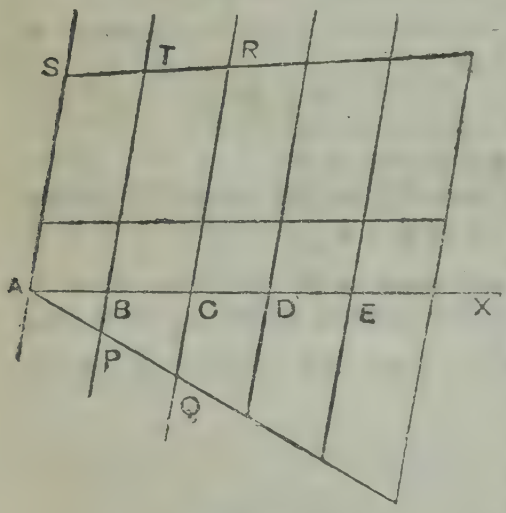


Fig. 60

Note from the above fig. that if in a △ ACQ, B the middle point of one side AC and if BP be drawn || to another side CQ, to meet the third side in P, then P is the middle point of AQ; and also that if, in a trapezium AQRS with AS, QR parallel, PT be drawn || to QR through the middle point P of AQ to meet SR in T, then T is the middle point of SR.

CHAPTER V.

RATIO AND PROPORTION.

SECTION I.

43. RATIO.

Suppose A is worth Rs. 5,000 and B, Rs. 2,000. Their fortunes may be compared by stating (1) that A is worth Rs. 3,000 more than B, or (2) that A is $\frac{5}{2}$ times as rich as B. The former method is by subtraction and the latter by division. Similarly, B's worth may be expressed to be $\frac{2}{5}$ of A's. When the relation between two *like* quantities is expressed by stating what multiple, part or parts one quantity is of another, the *relation* is called a **ratio**.

In Fig. 61, the relation of AR to RB is that of 3 to 2.

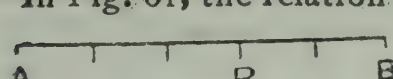


Fig. 61.

This statement is written as AR : RB :: 3 : 2 and read as AR *is to* RB *as* 3 *is to* 2.

It means the same as $\frac{AR}{RB}=\frac{3}{2}$. Thus the ratio of '$a$' to '$b$' is the fraction $\frac{a}{b}$, 'a' and 'b' being called the *terms* of the ratio $a:b$ and the rules applicable to fractions are also applicable to ratios, *e.g.*:—

$\frac{a}{b}=\frac{a\times n}{b\times n}=\frac{a\div n}{b\div n}$; similarly $a:b=(a\times n):(b\times n)$ $=(a\div n):(b\div n)$.

Hence, *the value of a ratio would be unaltered if its terms be multiplied or divided by the same quantity.*

Comparison of Ratios:—Ratios may be compared just as fractions.

Ex. Which is the greater ratio 4 : 5 or 12 : 13? The question reduces to :—'Which is greater, $\frac{4}{5}$ or $\frac{12}{13}$?' Reducing to a common denominator

$$\frac{4}{5} = \frac{52}{65}; \frac{12}{13} = \frac{60}{65},$$

$\therefore$ 12 : 13 is greater than 4 : 5.

Equality of Ratios:—When two ratios are equal, the statement of equality is called *a proportion* and the four terms taken in order are said to be *in proportion.* Also in a proportion, *e.g.* 12 : 18 = 20 : 30, the product of the middle terms 18 and 20, which are called *the means* = that of the other terms 12 and 30, which are called *the extremes.*

Conversely, if $2x = 3y$, $x : y = 3 : 2$.

EXERCISES.

(1) ABC is a triangle and P is a point in AB. PQ is drawn

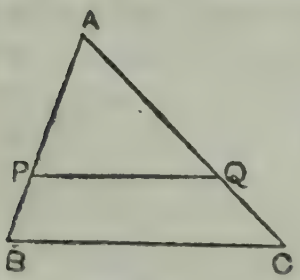


Fig. 62.

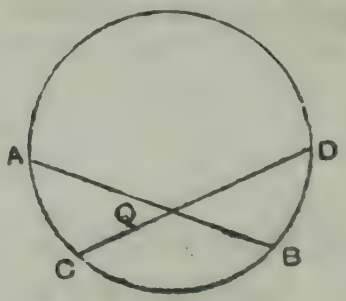


Fig. 63.

parallel to BC. Verify by measurement if AP : PB = AQ : QC. AP : PQ = AB : BC. AQ : QP = AC : CB and AP : AQ = AB : AC.

(2) In a circle two chords AB CD intersect at Q. Show by measurement that AQ : CQ = QD : QB and also that AQ × QB = CQ × QD.

(3) Find the value of x below :—

Rs. 4-8 as. : Rs. 13-8 as. = 5 tons : x.

£8-14*s.* : $8x$ = 16 gallons : 334 gallons.

(4) Supply the missing terms below :—

( ) : 5 mds. 4 viss = 8 lbs. : 16 lbs. 8 oz.

Rs. 46-11 as. : ( ) = £10-7*s* : £5-15*s*.

(5) (*a*) 6 : 5 : (6 + 2) : (5 + 2) : (6 — 2) : (5 — 2).

(*b*) 5 : 6 : (5 + 2) : (6 + 2) : (5 — 2) : (6 — 2).

Arrange the above ratios in the ascending order of magnitude.

(6) The rates of cycling of two persons A and B are as 8 : 6. A completes a journey in 4 hrs. In what time will B complete the same journey? (Assume that A bikes at $8x$ miles per hour).

(7) The time taken by A to complete a journey : that taken by B is as 4 : 3. A travels at 10 miles per hour. What is B's rate of travelling?

SECTION II.

44. PROPORTION, DIRECT AND INVERSE.

Quantities are of two kinds :—(1) *Constants* like 5, 8, etc., which are so called because they have a fixed value and (2) *variables*, *e.g.*, the growth of a tree in inches, the price of ghee in rupees, etc., which are so called because they often change their values or *vary*. At times, quantities may vary simultaneously so as to have some relation between them, *e.g.*, the price to be paid depending on the number of books to be bought or the number of days required to finish a work depending on the number of workmen employed, etc., as in the following examples :—

Ex. 1. If 12 copies of a book cost Rs. 25, find the price of 18 copies at the same rate.

Method (i) Generally, the method of solution by Unitary Analysis is employed as follows :—

12 copies cost Rs. 25.

$\therefore$ 1 copy costs Rs. $\frac{25}{12}$.

$\therefore$ 18 copies cost Rs. $\frac{25}{12} \times 18$ or Rs. $37\frac{1}{2}$.

Method (ii) A practical use of the ratio may be made in working as follows:—

The no. of copies has increased in the ratio $\frac{18}{12}$.

$\therefore$ the price should increase in the ratio $\frac{18}{12}$.

$\therefore$ the price required = Rs. $25 \times \frac{18}{12}$.

Similarly, in the case of 8 copies, the price should decrease in the ratio $\frac{8}{12}$ and is $\therefore$ Rs. $25 \times \frac{8}{12}$. To what extent the number of copies increases or decreases, to the same extent the price in rupees should increase or decrease.

$\therefore$ the price of copies *varies directly* as their number or is said to be *directly proportional* to it.

Ex. 2. If 40 men can dig a trench in 15 days, in what time will 60 men be able to dig it, working at the same rate?

Method (i) by Unitary Analysis:—

40 men will take 15 days.

$\therefore$ 1 man „ 40×15 days.

$\therefore$ 60 men „ $\frac{40 \times 15}{60}$ days.

It may be easily seen that if the number of workmen be doubled the time in days would be reduced to $\frac{1}{2}$, also that if the number of workmen be reduced to $\frac{1}{4}$, the number of days required would become fourfold and that if one of the two variables becomes 'x' times its previous value, the other would be reduced to $\frac{1}{x}$ of its value.

Method (ii) by the use of a multiplying ratio :—

The no. of men has increased in the ratio $\frac{40}{40}$.

$\therefore$ the no. of days should decrease in the ratio $\frac{40}{60}$.

$\therefore$ the no. of days required $= 15 \times \frac{40}{60}$ or 10.

Similarly, if 30 men be employed, the no. of days should increase in the ratio $\frac{40}{30}$ and is $\therefore 15 \times \frac{40}{30}$ or 20.

In the above example the no. of days *varies inversely* as the no. of men and is said to be *inversely proportional* to it.

EXERCISES.

(8) If I mess in a hotel for 16 days and am charged Rs. $6\frac{1}{2}$ for the period, calculate the charge for 65 days at the same rate.

(9) Just when the shadow of a man, 5 ft. 3 in tall, is 8 ft. that of a cocoanut tree = 44 ft., calculate the height of the tree. Also, find the length of shadow then cast by a building 84 ft. high.

(10) A man's monthly salary is Rs. 133-5-4. Find his annual salary.

(11) The height of a boy was measured on his 12th and 16th birth-days and was found to be 4 ft. 4 in. and 5 ft. 3 in. respectively Test whether the height is proportional to the age.

(12) A motor leaves P at 10 a.m., reaches S 6 miles away at 10-15 a.m., waits there for 20 minutes and then leaves for C, 20 miles away at the same rate. When will it reach C ?

(13) A boy saves 6 as. per week out of his pocket-money and deposits it in the Savings Bank. How much will he have saved at the end of a year ?

(14) Walking at my usual rate I could reach school from my house just in time. Walking at $\frac{4}{5}$ of my usual rate, I reach it 15 min. late. What is the usual time I take to reach school from my house ?

(15) By travelling in a train moving at 15 miles per hour, I complete a journey in 10 hrs. What time would the same

journey take if I travel in a train moving at the rate of 20 miles per hour?

(16) From a bar of gold 12 pairs of bangles, each pair weighing 12 sovereigns, could be made. Find the weight of a bangle when 36 bangles are made out of the bar. Also, find how many bangles, each weighing 8 sovereigns, could be made from the same quantity of gold.

(17) From a roll of carpet, 27 pieces, each 5 cubits long, could be cut. How many pieces, 9 cubits long, could be cut from the same roll?

(18) *Show that a speed of 60 miles per hour = that of 88 ft. per second*

(19) I can walk from Saidapet to Chembrambakam in 3 hrs. 30 min. Starting from Saidapet I walked for an hour along the road to Chembrambakam. Then a friend took me in his coach to Chembrambakam. Thus my journey occupied 2 hrs. 30 min. How many hours' drive is Chembrambakam from Saidapet by the above coach?

Ex. 3. 120 men take 15 days to build a fortification wall 20 ft. high. How many days will 80 men take to build a similar wall but twice as long and 24 ft. high?

(Assume that each man works at the same rate daily).

(*a*) Length of the second wall = twice that of the first.

∴ No. of days should be twice or 15 × 2, *i.e.*, 30.

(*b*) The height is $\frac{24}{20}$ times as large.

∴ No. of days = $30 \times \frac{24}{20} = 36$.

(*c*) No. of men is reduced to $\frac{80}{120}$.

∴ No. of days should increase in the

ratio $\frac{120}{80}$ and $= \frac{36 \times 120}{80}$ or 54.

(20) 4 cows give 72 measures of milk in 12 days. If I require 900 measures of milk to be given in 6 days, how many such cows should be had?

(21) If ten men earn Rs. 10 each per month, how much will they have earned at the end of 13 years?

(22) My bulls will plough 25 acres in 40 days, working 6 hrs. per day. In how many days will they plough 55 acres working 8 hrs. a day?

(23) When milk sells at 3 measures a Re, the monthly cost of feeding 4 infants with milk is Rs. 15. Find the annual cost of feeding 6 infants with milk when it sells at 4 measures a Re.

(24) A besieged garrison of 1400 men have provisions for 6 weeks at 12 oz. per man per diem. Three weeks hence, 280 more men come and join. Thenceforward, each man is allowed 10 oz. per diem. How long will the provisions last from the date of their joining?

(25) A vessel when half full of ghee weighs 2 viss 10 palams. When it is three-quarters full of the same kind of ghee, it weighs 2 viss 35 palams. What would be the weight of the vessel when empty? What weight of ghee can the whole vessel hold?

(26) The cost of printing a notice consists at (1) a fixed charge for setting up the type, independent of the no. of copies printed, and (2) a charge for printing and paper proportional to the number of copies printed. It costs Rs. 3 for printing 300 copies and Rs. 4-4 as. for printing 600 copies. Reckon the cost of printing 570 copies.

(27) While carrying luggage by a train, a fixed no. of seers is allowed free of charge and the rest is charged at a certain rate per maund. For a journey of 127 miles, I paid Rs. 3-3 as. for 10 mds. and my friend Rs. 6-6 as. for 19 mds. Find the rate charged per maund of excess luggage.

(28) The illumination from a light 'L' varies inversely as the square of the distance. An object 'O' is at a distance of 6 ft. from the light 'L'. If the illumination at 'O' is to be reduced to $\frac{1}{4}$ of what it is in that position, how far from 'L' should 'O' be placed?

(29) The cost of constructing a roadway 17 miles long, and 16 feet wide = Rs. 54,400. Find the cost of constructing a like roadway 22 miles long and 30 ft. wide when materials are twice as costly and the conditions of labour, etc., are unaltered.

SECTION III.

45. PROPORTIONAL DIVISION.

Fig. 64.

AP is a st. line divided into 8 equal parts. AR contains 5 and RP 3 of the parts. If AR is 5, RP is 3.

$$\therefore \frac{AR}{RP} = \frac{5}{3}.$$

AP is hence divided at R in the ratio 5 : 3.

Similarly AP may be said to be divided at R and S in the ratio 5 : 2 : 1.

When AR : RP = 5 : 3, AR = $\frac{5}{(5+3)}$ of AP (Why?)

When AR : RS : SP = 5 : 2 : 1, AR = $\frac{5}{(5+2+1)}$ of AP. (Why?)

Suppose AP represent Rs. 1,000; what does AR represent, if AR : RP = 5 : 3?

AR represents $\frac{5}{(5+3)}$ of Rs. 1 000 or $\frac{5}{8}$ of Rs. 1,000 of Rs. 625.

Similarly RP represents $\frac{3}{8}$ of Rs. 1,000 and RS $\frac{2}{8}$ of Rs. 1,000.

EXERCISES.

(30) A st. line 9·6" long is divided in the ratio 5 : 7. Find the length of each part.

(31) A st. line AB is divided at C such that AC : CB = 5 : 8. What fraction of AB is BC?

(32) Divide Rs. 3,850 between two brothers in the ratio 4 : 3.

(33) The gold mohur consists of pure gold and alloy in the ratio 22 : 2. Find how many mohurs' weight of pure gold is used in coining a million mohurs.

(34) Two persons start a business concern with capitals of Rs 5,000 and Rs. 6,500 respectively. The agreement is that $\frac{1}{3}$ of the total profits shall be divided equally between them and the balance be divided in proportion to the capitals invested by them. If the total profits amount to Rs. 1,035, find each man's share.

(35) Chalk consists of lime and carbonic acid gas in the ratio 14 : 11. On strongly heating chalk carbonic acid gas goes away and lime alone remains. How much of lime can thus be obtained by heating 6 tons of chalk?

(36) A rectangular field requires 250 yds. of fencing. Its sides are in the ratio 5 : 3. What is the length of each side?

(37) If in fig. 64 AP represents 2888 miles, find the distances represented by AR, RS and SP.

The question is the same as dividing 2888 in the ratio 5 : 2 : 1. AR represents $\frac{5}{(5+2+1)}$ of 2888 miles or 1805 miles. RS represents $\frac{2}{8}$ of 2888 miles or 722 miles and SP $\frac{1}{8}$ of 2888 miles or 361 miles.

(38) A straight line 36 cm. long is divided into four parts in the ratio 2 : 3 : 5 : 8. Find the length of each part.

(39) A bell is composed of copper, zinc and lead in the ratio 9 : 5 : 2 by weight. If the weight of the bell is 35 lbs., find the weight of each metal used.

(40) A maximum of 60 marks is allotted among the five questions of a question paper in the ratio 6 : 2 : 3 : 4 : 5. Find the marks allotted to each question.

(41) A sweetmeat consists of sugar, wheat-flour and ghee in the ratio $\frac{1}{2}$: 1 : $\frac{1}{8}$ by weight. How much of each ingredient is contained in 52 seers of sweetmeat?

[Note:—$\frac{1}{2}$: 1 : $\frac{1}{8}$ = $\frac{1}{2} \times x$; 1 $\times x$: $\frac{1}{8} \times x$, where 'x' is any number, not a cipher, and hence $\frac{1}{2}$: 1 : $\frac{1}{8}$ may be expressed so as to have whole numbers as its terms.]

(42) Fresh-water pipes have to be newly opened in three wards A, B and C of a Municipality. If the population in A, B and C be 1500, 2000 and 2500 respectively, and if an expendi-

ture of Rs. 10,800 has to be met for the purpose by A, B and C in the ratio of their population, find how much should be contributed by each ward.

(43) In a District Board Railway the annual receipts amount to Rs. 2,60,000. The receipts per mile got from passengers = Rs. 654 and those from goods = Rs. 646. Find the total annual receipts from passengers and from goods.

(44) The cost of maintaining a motor car consists of the items: pay to the chauffeur, repairs and petrol. in the ratio 8 : 3 : 2. If the yearly expenditure comes to Rs. 2,860, find how much on the average has to be spent on each item per month.

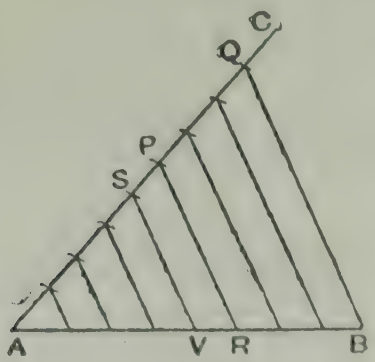


Fig. 65.

(45) Divide a straight line AB in the ratio 5 : 3.

[Draw AC making an angle with AB. Along AC step off 8 equal lengths and let AS contain 4 of them, SP 1 and PQ 3. Join QB. Draw PR and SV parallel to QB.

Then AR : RB = 5 : 3

and AV : VR : RB : = 4 ; 1 : 3.]

(46) Show how you would produce a straight line AB to C such that AC : CB = 5 : 3.

[Note that when a point C taken in a st. line AB divides it into two segments AC, CB in a given ratio. AB is said to be divided *internally* at C *in the given ratio*. When the point C

is taken in AB produced so that AC and CB are in the given ratio, AB is said to be divided *externally* in the given ratio.]

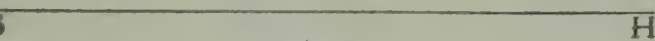


Fig. 66.

(47) SH represents the sum of the three sides of a triangle which are in the ratio 4 : 3 : 3. Construct the triangle. Measure the length of each side and verify your result by calculation.

(48) Divide Rs. 45,000 among three persons A, B and C so that A's share is half as much again as B's and B's is half as much as C's. (Note that A's share : B's = $1\frac{1}{2}$: 1 and B's : C's = $\frac{1}{2}$: 1).

(49) The butter brought by a Kannadian consists of 2 parts of wheat flour to every 5 parts of pure butter. When he measures it he manages to mix 1 part of buttermilk with every 10 parts of the butter. Find the quantity of genuine butter in one measure of butter brought from him.

(50) Butter sold by a woman in the street consists of pure butter and rice flour in the ratio 5 : 1. If one measure of pure butter yields $\frac{7}{8}$ measure of good ghee and rice flour subsides after boiling, find how many measures of good ghee could be got from two measures of butter sold in the street.

(51) Two alloys are composed of copper, tin and zinc in the following ratios by weight : – 10 : 3 : 3 in the first and 9 : 2 : 1 in the second. These alloys are fused together in equal quantities by weight. Find the ratio of copper, tin and zinc in the resulting alloy.

(52) A vessel which could just hold 28 measures is filled with a mixture containing ghee and oil in the ratio 5 : 2. From the vessel 10 measures of the mixture are taken away and the vessel is then filled with oil. Find the ratio of oil to ghee in the vessel then.

(53) B, S and N opened a Bank. B advanced Rs. 10,000 and after 6 months invested another Rs. 10,000. S advanced Rs. 50,000 and after 5 months another Rs. 50,000. N advanced Rs 30,000 and after 3 months withdrew Rs. 20,000. If the annual profits

amount to Rs. 26,200, how much should B, S, and N receive, assuming that the profits are to be divided in the ratio of the capitals invested.

[**Note**:—Investing Rs. 'x' for 'y' months = investing Rs $x \times y$ for 1 month.]

(54) Stracey and David own 280 and 350 acres of tea plantations. They unite their plantation grounds, taking at the same time an additional partner Palman, who pays them Rs. 1,05,000 on the understanding that a third portion of the plantation shall in future belong to each. How much out of Rs. 1,05,000 should each of Stracey and David receive?

(55) Our district contains a population of 2,189,775. The number of literates for every 1000 of illiterates is 455. Also, there are 60 male literates for every 26 female literates. Calculate the number of female literates in the district.

CHAPTER VI.

AVERAGES.

46. Suppose a man's earnings to be as follows :—

January	Rs. 100.	July	Rs. 120.
February	„ 180.	August	„ 100.
March	„ 90.	September	„ 140.
April	„ 20.	October	„ 120.
May	„ 30.	November	„ 100.
June	„ 70.	December	„ 130.

Then a general idea of his monthly earnings may be formed by finding the average of the above, *viz.*, by dividing the total earnings by 12, the number of months.

The man's average earnings may hence be calculated to be Rs. 100 a month.

Rs. 100 is the **arithmetic mean** or the average per month of the earnings above shown. [Note that *this mean lies between the greatest and least of the monthly earnings.*]

The average per month of the earnings for 12 months

$$= \frac{\text{the sum of all the earnings}}{12}.$$

∴ The sum of all the earnings = 12 times the average.

Ex. 1. The average of two numbers = 18. One of them is 12. Find the other.

The sum of the numbers = $2 \times 18 = 36$.

One of them = 12.

∴ the other = $(36 - 12)$

$= 24$.

Ex. 2. The average of 18 numbers = 12 and that of 17 of them = 9. Calculate the 18h number.

The sum of 18 numbers = $18 \times 12 = 216$.

The sum of 17 of them = $17 \times 9 = 153$.

$\therefore$ The eighteenth number = $216 - 153$ or *63*.

Ex. 3. The marks obtained by a pupil in Ele. Mathematics for the six terms of his S. S. L. C. course are 30; 42; 72; 80; 70 and 44 respectively. Find how much he has scored on the average per term.

Instead of adding all the marks and dividing by 6, the following method may be adopted :—

From a glance, presume that 60 may be near the average. Then, taking the marks in order, find how far this '*trial*' average is greater or less than each mark given above and take the algebraic sum of the difference as follows :—

$30 = 60 - 30$;	
$42 = 60 - 18$;	$-30 - 18 = -48$;
$72 = 60 + 12$;	$-48 + 12 = -36$;
$80 = 60 + 20$;	$-36 + 20 = -16$;
$70 = 60 + 10$;	$-16 + 10 = -6$;
$44 = 60 - 16$;	$-6 - 16 = -22$.

Divide this algebraic sum -22 by 6 and add the quotient $-3\frac{2}{3}$ to the 'trial' average 60. The result gives the true average to be $60 - 3\frac{2}{3}$ or $56\frac{1}{3}$.

(Practically, the true average to the nearest unit would suffice).

EXERCISES.

(1) (*a*) The average of 11, 12 and x is 14. Find 'x.'

(*b*) In 12 successive months the rainfall in a village was 2·1″, 2·5″, 1·5″, 1·5″, 1·2″, 3·4″, 2·4″, 1·8″, 1·6″, 2·8″, ·2″ and ·6″. Calculate the average monthly rainfall during the year.

(2) The following table gives the runs of three batsmen of our school team in five cricket matches :—

	Krishnaswami.	Ranganathan.	Subramanian.
Match I	100	88	44
,, II	72	44	142
,, III	18	44	0
,, IV	128	78	100
,, V	10	81	48

Find the average number of runs scored by each per match. If a prize is to be given to that batsman whose average score is the highest, find to whom it should be given.

(3) Six pupils appear from a school for Algebra and Geometry in the S. S. L. C. public examination. Their marks are 80, 10, 25, 47, 88 and 68 respectively. Calculate the school average in that subject in the examination. How many boys are above the school average and how many are below the average ?

(4) The marks got by a boy in English in Forms IV, V and VI are as follows :—

	Long Term.	Short Term.
Form IV	56%	48%
,, V	45%	60%
,, VI	48%	86%

If the final marks awarded to him by the school be the average of the entries made above, find, to the nearest unit, the final marks awarded to him by the school.

(5) In Form VI of our school there are two sections. Section A contains 35 boys and Section B 30 boys. The class average in English in Section A is 40% and that in Section B is 44%. Find the average percentage in English for the whole of Form VI. (Answer to the nearest unit.)

(6) You have to revise your lessons for an examination and have only a fortnight's holiday before you. Devoting 2 hours per day you can revise the A group subjects in 20 days. At 3 hours a day you can finish the B group subjects in 10 days. And the other subjects you can finish in 7 days if you work an hour a day. How many hours on the average should you work a day, if you want to revise all your lessons within the holidays available.

(7) In a school there are 400 boys of whom 105 are in Forms IV, V, VI and pay a school fee of Rs. 5-4 as. each per instalment, 205 are in Forms I, II, III and pay Rs. 2-12 as. each per instalment and the rest are in the lower classes and pay Re. 1-4 as. each per instalment. Find the average fee income got by the school per boy per instalment. If fees be collected only for 8 instalments during the year, find the average fee income got by the school per month.

(8) There are 12 peons in a college. The average monthly pay per peon is Rs. 9, that of the first three is Rs. 10 and that of the next seven is Rs. $9\frac{1}{2}$. The eleventh peon is paid Rs. 6 per month. Find the pay of the 12th peon.

(9) The average daily attendance in a theatre for the first three days of a week is 395 and that for the next 4 days is 500. Find the total attendance for the whole week and also the average daily attendance for the week.

(10) In 1918 the number of deaths due to influenza was as follows in a certain town :—January 29, February 12, March 7, April 0, May 18, June 45, July 37, August 27, September 19, October 36, November 17. The number of deaths in December is forgotten but the average number of deaths per month is remembered to be 25. Find the number of deaths by influenza in December 1918.

(11) A class of 15 boys sat for an examination. The first boy got 57 marks. But, while transferring the marks from the answer-book to the marks-register, his marks were entered as 87 by mistake The class average as shown by the marks-register was 47. Calculate the actual class-average.

(12)

Name of Taluk.	Its area in square miles.	Its population in hundreds.
A	843	4480
B	484	2884
C	546	1240

Find the average population per square mile in each Taluk. (Answer to the nearest unit) and hence find which is the most thickly-populated Taluk.

(13) I went to market, bought 500 cocoanuts at Rs. 3-8 per hundred, 1600 at Rs. 32 per 1000, 120 at 6 as. per dozen and sold them at 1 anna per cocoanut. Find my average gain per cocoanut.

(14) In a year, the average land-tax per acre of cultivable area was as follows in four districts:—

District.	Average land-tax per acre.
A	Rs. 12
B	,, 10
C	,, 8
D	,, 6

while the cultivable areas in them expressed in milllons of square miles were as follows:—

A, 5·55; B, 5·45; C, 4·01; and D, 2·49. Find the average land-tax per acre of cultivable area in all the four districts taken together.

(15) A house is let for only 'x' months in the year and for the remaining months it is vacant. Also, during the period it is let out, no rent could be recovered from the tenant for 'y' months. If the house-owner realised a monthly rent of Rs. 'z' on the average, find the monthly rent he charged for the house. Verify your result by putting $x = 8$, $y = 2$, $z = 10$.

(16) The total area of wet lands fed by a channel = 6750 acres and the produce per acre is 14 kalams of paddy. Of this total area, 2750 acres are first class lands and yield 21 kalams of paddy per acre. What is the average yield per acre on the rest of the lands?

(17) The average sale-proceeds of Messrs. W. L. & Co., amounted to Rs. 21,000 per month in 1912, to Rs. 6,000 more a month in 1913 and to Rs. 4,500 less a month in 1914. Calculate the average monthly sale for all the three years put together.

(18) During the first 'n' hours of a journey the average speed of a train was 15 miles an hour and that during the first $(n-4)$ hours was 14 miles an hour. Find the average speed of the train during the remaining 4 hours.

(19) The leave allowances given to an officer on furlough amount to half his average salary during the three years preceding the furlough. An officer was on furlough from 16-4-17 and his salary for the preceding years was as follows:—

From 1-4-14 to 31-8-15	Rs. 450	a month.
,, 1-9-15 to 30-4-16	Rs. 500	,,
,, 1-5-16 to 31-12-16	Rs. 600	,,
,, 1-1-17 to 15-4-17	Rs. 750	,,

Calculate the monthly leave allowances he may be entitled to.

(20) Find the arithmetical mean of the following:—

$$-6 \text{ and } +10, \quad -15 \quad -12 \text{ and } +20.$$

(21) Verify if the straight line joining the middle points of the oblique sides of a trapezium is ths *arithmetical mean* of the parallel sides.

(22) The following extract taken from an S. I. R. Time-table gives the times of arrival and departure of certain passenger trains from Shencottah to Maniyachi:—

To Madras Beach. Miles.	Stations.		No. 148 P. H. M.	No. 60 P. H. M.	No. 76 P. H. M.
495	Shencottah	... D	3— 0	8—28	15—30
467	Ambasamudram	A	5—25	10—20	17—26
		D	5—43	10—28	17—34
445	Tinnevelly Bridge	A	7—54	11—56	19—12
		D	8—30	12—20	19—42
427	Maniyachi	... A	9—45	13—30	20—50

Find the *mean* speed of each train, calculated on the time it is in motion from Shencottah to Maniyachi.

CHAPTER VII.

PERCENTAGE.

47. To compare the fractions $\frac{2}{5}$ and $\frac{3}{4}$ we reduce them to a common denominator. We may also reduce them to the same denominator 100 in all cases. Thus $\frac{2}{5} = \frac{40}{100}$ and $\frac{3}{4} = \frac{75}{100}$. The fractions are then expressed as *percentages*.

In a town the number of school-boys increased in one year from 450 to 630. During the same year the number of school-boys increased from 500 to 650 in another town. The rate of increase in the first town is $\frac{180}{450}$ or $\frac{40}{100}$ and that in the second is $\frac{150}{500}$ or $\frac{30}{100}$. The no. of school-boys therefore increased more rapidly in the first town than in the second. Thus the rates of increase may be compared by expressing them as a percentage, *i.e.*, as a fraction with denominator 100. $\frac{40}{100}$ may otherwise be written as 40 *per cent.* or 40 %
(Read forty per cent.) The numerator in the fraction expressing the percentage gives us the *rate per cent.*

Ex. 1. Convert $\frac{5}{16}$ as a percentage.

Assume $\frac{x}{100} = \frac{5}{16}$.

Then $x = \frac{5}{16} \times 100 = 31\frac{1}{4}$ or 31·25.

$\therefore \quad \frac{5}{16} = \frac{31{\cdot}25}{100} = 31{\cdot}25$ per cent.

Ex. 2. Convert 25 % into a vulgar fraction.

$25\ \% = \frac{25}{100} = \frac{1}{4}$.

Similarly, $50\ \% = \frac{1}{2}$ and $75\ \% = \frac{3}{4}$.

EXERCISES.

1. Out of a total of 60 marks in an examination a boy scored 42. What per cent. of the total did he score?

2. $\frac{5}{6} = \frac{x}{100}$. Find the value of x.

3. The number of houses in my village was 75 last year This year it is 72. Find the percentage of decrease.

4. The population of our town was 2500 in 1901 and 2697 in 1911. What is the percentage of increase?

5. Out of 1250 candidates that appeared for an examination 15 passed in the 1st class, 650 in the 2nd class and the rest failed. Find the percentage of failures.

6. In a question paper 15 questions are given. Full marks are awarded for answering $\frac{4}{5}$ of the paper. A boy works 9 questions correct. What per cent. of the maximum may he expect to score?

7. An agriculturist has wet lands worth Rs 12,500 from which he gets an annual income of Rs. 800. A money-lender starts business with Rs. 5000 and gets Rs. 400 yearly. Who gets a greater rate of return for his money?

8. In which of the following is the rate of profit greatest?—

(*a*) A cloth merchant invests Rs. 900 and gets a profit of Rs. 100 per year.

(*b*) A shepherd buys milch-cows for Rs. 275 and earns a net income of Rs. 15 per month.

(*c*) A rice merchant invests Rs. 300 and gains Rs. 50 for a year.

9. The population of the Tinnevelly District was 1,976 900 in 1901 and 2,096,950 in 1902, while that of the Trichinopoly District was 1,865,800 in 1901 and 1,985,002 in 1902. Find in which district the population has increased at a graater rate.

10. One boy measures a line 7·5" long and reads the length to be 7·6". Another boy measures another line 4·5" long and reads its length as 4·55". Find the percentage of error committed by each boy. Which of them has committed a greater percentage of error?

11. My weight was 120 lbs. and my friend's weight 130 lbs. last New Year's Day. This New Year's Day my weight is 136 lbs. and my friend's 148 lbs. Reckon whose weight has increased at a greater percentage.

Ex. 3. The price of a mosquito-net is now 15 % less than what it was last year. The current price is Rs. 4-4 as. Find what its price was last year.

Let Rs x = the price of the net last year,

Then $4\frac{1}{4} = x - \frac{15}{100}x.$

$\therefore \quad \frac{85}{100}x = \frac{17}{4}.$

$\therefore \quad 85x = \frac{17}{4} \times 100.$

$\therefore \quad x = \frac{17}{4} \times \frac{100}{85} = 5.$

∴ Last year the price of the net was Rs. 5.

[Note the following working if the current price be given to be 15 % greater than the price last year :—

$$4\tfrac{1}{4} = x + \frac{15}{100}x = \frac{115}{100}x.$$

$\therefore \quad 115x = \frac{17}{4} \times 100.$

$\therefore \quad x = \frac{17}{4} \times \frac{100}{115}.$

Similarly, if Rs. $4\frac{1}{4}$ was obtained by selling an article at a gain of 15 % on its cost, the cost price = Rs. $4\frac{1}{4} \times \frac{100}{115}$. If Rs. $4\frac{1}{4}$ represented the sale price at a loss of 15 % on the cost, the cost price = Rs. $4\frac{1}{4} \times \frac{100}{85}$].

12. If the price of teak now is 30 °/₀ more than what it was last year and if the current price of a teak beam be Rs. 45-8, find what it would have cost last year.

13. The price of ghee was 2 as. per ollock 25 years ago. Now it is 8 as. per ollock. How much per cent. has the price risen?

14. In a school, only 370 pupils pay full fees. They form 92·5 °/₀ of the total strength. What is the total number of pupils in the school.

15. A fountain pen which was selling at Rs. 7-14 as two years ago is now selling at Rs. 5-4 as. Find by how much °/₀ the price has been reduced and to what °/₀ it is reduced.

16. In a clearance sale the prices have been reduced by 15°/₀. (*a*) If a clock was selling at Rs. 15 before reduction, find its price in the clea rance sale. (*b*) If the price of a doll was Rs. 17 in the clearance sale, find its price before reduction.

17. A firm manufactures 1000 sheets of paper at a cost of Rs. 5. It sells the paper to a wholesale dealer at a gain of 10 °/₀. The wholesale dealer sells the paper at a gain of 25 °/₀. If the retailer sells the paper at 4 as. per quire, find his gain per cent. (Note that gain or loss is calculated on the cost of the paper to the dealer).

18. If a cycle manufacturer makes a profit of 20 °/₀, the wholesale dealer a profit of 25 °/₀, and the retail shop-keeper a profit of 20 °/₀, what was the cost of manufacture of a cycle sold by a retail dealer for Rs. 225 ?

19. A clock was bought by A. He sold it to B at a loss of 10 °/₀. B sold it to C at a loss of 15 °/₀. If C paid Rs. 19-2 as. for it, how much did it cost A ?

20. The following statistics refer to a district :—

	(*a*) Population of school-age.	(*b*) Scholars in schools.	(*c*) Percentage of (*b*) to (*a*)
Males	105846	( )	( )
Females	( )	7391	( )
Total	210901	( )	25

Fill up the missing columns above.

[Answer to the nearest unit.]

21. The following table shows the value of the imports into India of the articles specified therein during May 1914 and the increase or decrease per cent. on the figures for the corresponding month of the previous year :—

Articles.	May 1914 cwt.	Increase plus, decrease minus. May 1913.
Cotton, raw	1,838,900	+ 12·8 per cent.
Wheat	3,837,695	— 47·6 ,,
Rice	2,418,273	+ 25·1 ,,
Linseed	1,267,177	— 27·1 ,,
Jute	488,152	+ 60·1 ,,
Tea	61,483	+ 20·6 ,,

Find how much of each of the above articles was imported during May 1913.

22. In a certain Taluq, 5 °/₀ of the gross land revenue was remitted in consequence of a drought, 10 °/₀ of the remainder was not collected during the year and after deducting from the annual collections 5°/₀ as cost of collecting, the net proceeds remaining were Rs. 4,87,350. What was the gross land revenue ?

23. Messrs. Tawker & Sons allow their customers 10 °/₀ discount on their list prices and a further discount of 10 °/₀ on this reduced price for cash paid at least within two months after purchase. (*a*) A customer bought from the Company jewels worth Rs. 25,000 according to the price list, gave Rs. 15,000 in a month's time and cleared the dues a month after. What should he then pay the company ? (*b*) Another customer bought a nose-screw from the company and paid Rs. 162 cash. Find its list price.

24. One year the birth-rate of monkeys in a jungle was 15 °/₀ and the death-rate 8·5 °/₀. The percentage of monkeys leaving the jungle was 8 and thst of monkeys coming in was 7. At the end of the year there were 738,500 monkeys in the jungle. Find the number of monkeys in it at the beginning of the year.

25. A clerk's pay is Rs. 20 per month. Monthly, he has to spend Rs. 12 for rice, Rs. 5 for house-rent, Re. 1-4 as. for oil, Re. 1 for milk and Rs. 5-12 as. for other expenses. What per cent. of his pay does he spend on each item of expenditure and what per cent. of his expenditure does he incur as debt ?

26. In a month a man earned Rs. 450 and spent Rs. 300. Next month his earnings increased by 12 °/₀, and his expenditure by 15 °/₀. By what per cent will his savings increase.

27. A certain press publishes and sells a certain text-book on these terms;—To a professional book-seller 10°/₀ discount and one extra-copy gratis for 12 bought at a time. The price of a copy is labelled as Re. 1. A book-seller buys copies on these terms and sells at Re. 1-2 as. per copy. Find the percentage of his gain. [Assume that copies are bought only in dozens.]

28. A rice merchant bought 2 kinds of rice: 1st class Nellore rice at Rs. 10 a bag and 2nd class rice at Rs. 9-5-4 a bag. He mixed them so that only 40 °/₀ of the mixture was 1st class rice. If he sold the mixture at Rs. 12 a bag, find his gain per cent.

29. From a vessel containing 15 measures of spirit, 2 measures are taken and replaced by water. From the mixture 2 measures are again taken and replaced by water. Find the percentage of water in the final mixture.

30. A merchant bought coffee seed at Re. 1-12 as. a viss and sold them at Re. 1-12 as. per measure. He assumed that a measure of coffee-seed weighed $\frac{2}{3}$ of a viss; whereas, in reality, it weighed $\frac{3}{4}$ of a viss. What profit per cent. did he think he would make and what profit per cent. did he actually make?

31. The speed of a motor-car is 75 °/₀ more than that of a cycle. If travelling be done by the motor instead of by the cycle, by how much per cent. is the time of travelling reduced?

32. A Kashmere shawl costs 11·25 °/₀ more than a similar German shawl but lasts 33⅓ °/₀ longer. Which would be more economical to buy, cost and durability being considered?

33. A chatram is run on the hospitality of four landlords A, B, C and D. A allows it 12 °/₀ of his income, B, 10 °/₀, C 5 °/₀, and D 15 °/₀ of his income. If B earns 10 °/₀ more than A, A 20°/₀ less than C, and C 10 °/₀ more than D, and D earns Rs 1,250 a month, how much does the chatram get every month?

34 The sum for which one contractor offered to do a work was 4 °/₀ greater than the sum for which another contractor offered to do the same work. The former's estimate of the cost was 8 °/₀ greater than the latter's estimate and the latter expected to gain 25 °/₀. What per cent. did the former estimate to gain?

35. If 'p' is increased to 't', by how much per cent. is it increased? If 'p' is 'q' per cent. less than 't', by what must 'p' be multiplied to get 't'?

36. In an examination a boy's marks are as follows :—

	Maximum for the subject.	Marks scored by the boy.
English	75	40
Mathematics	80	43
History	60	33
Vernacular	50	44

In which subject has he done best and in which worst?

37. An increase in the price of coffee reduces the quantity that could be got for the same amount to 80 p.c. What is the percentage of increase in the price?

38. A reduction in the price of sugar increases the quantity got for the same amount by 20%. By how much per cent. is the price reduced?

CHAPTER VIII.

AREAS OF SIMPLE RECTILINEAR FIGURES.

SECTION I.

The Rectangle.

48. The rectangle ABCD in fig. 67 is 5 units long and 3 units broad. It contains 3 rows of 5 square units each. Its area is hence 3×5 or 15 square units.

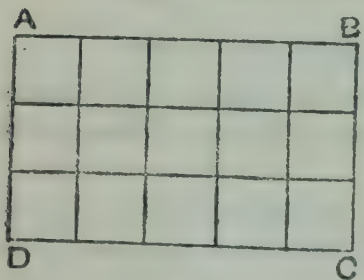


Fig. 67.

If AB = l", AD = b", *the area of the rectangle* = '$l \times b$' *sq. ins.*

If ' A ' represents the area in sq. ins.

$$A = l \times b,\ l = \frac{A}{b} \text{ and } b = \frac{A}{l}.$$

If A = 30, l = 6, then $b = \frac{30}{6}$ or 5.

The above principle is involved in constructing the tables of square measure (Refer to Appendix).

[Distinguish between '5 *ins. square*' and '5 *sq. ins.*' Any kind of figure may have an area of 5 sq. ins. but a fig. 5 ins. square should be a square, each side of which is 5 ins. and whose area = 25 sq. ins.]

In the metric measures of area, 1 sq. metre = 100 sq. decimetres, 1 sq. dm. = 100 sq. cm., and 1 sq. cm. = 100 sq. mm.

The *unit* of metric land-measure is the *Are* (=a Dm. sq.) which contains 100 sq. m. One Hectare = 100 Ares, and one centi-Are = $\frac{1}{100}$ Are.

EXERCISES.

1. How will you find the area of your class-room? If the floor is 25 ft. by 15 ft., find its area.

2. Fill up the vacant columns in the following :—

Length of rectangle.	Its breadth.	Its area.
26'6"	12'	( )
( )	8 chains	8 acres
6 yds.	( )	198 sq. ft.
( )	12·75 chains	20 acres 40 cents.
18'9"	( )	187 sq. ft. 72 sq. ins.
80 yds.	( )	1 acre.

3. A railway line is 11 miles long and 70 ft. wide. How many acres of land does it cover?

4. Draw on squared paper the plan of a rectangular field 2·5 chains long and 1·7 chains broad. What scale do you adopt? What area in the plan represents the field? What actual area is represented in the plan by 1 sq. inch?

5. In a plan 100 yds. occupy 1 inch. What will be the area in the plan of an estate containing 250 acres?

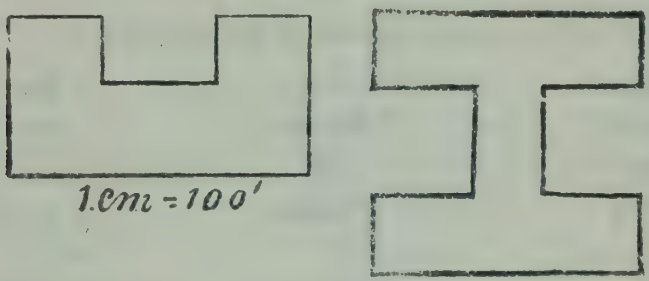
[If a plan is drawn to a scale of 1 inch to 100 yds., any line in the plan is $\frac{1}{36 \times 100}$ of its real length. This fraction is called the **Representative Fraction** (R.F.) of the plan]

6. The sides of a rectangle are thrice the corresponding sides of another rectangle. If the area of the first be 48 sq. ins., find the area of the second.

7. Find the areas of India, Australia and Ceylon by placing area paper over the respective maps in your atlas. Calculate from the scale given, the R. F. of the maps.

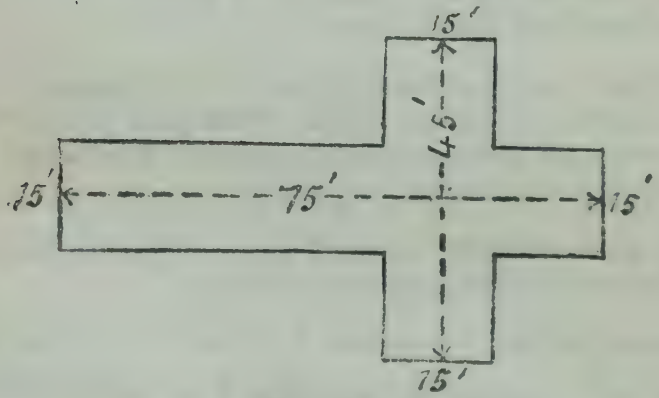
8. Measure the dimensions of a rectangular sheet of paper in inches and in cm. Hence find the number of sq. cm. in a sq. inch.

9. Find the actual areas of the figures whose plans are given below :—



(*a*) Ground-plan of a college.

(*b*) Ground-plan of a bungalow.



(*c*) The floor of a church.

Fig. 68.

10. In a rectangular garden 120 ft. by 90 ft. there is a cross-shaped well with each of its edges 13 ft. in length. Find the area of the garden excluding the well.

11. The figure drawn is a plan of a street drawn to a scale of 1 cm. to 100 ft. Find the area of the street-road.

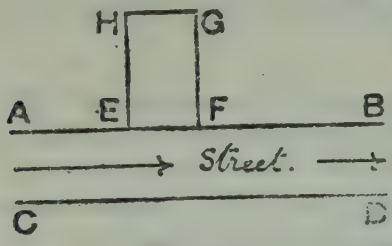


Fig. 69.

EFGH is a rectangular plot of building-land on one side of the street. What is the length of its frontage EF and that of its depth FG? If the land is valued at Rs. 50 a cent., estimate the worth of the plot.

12. A strip of dry land facing a canal has a frontage of 12·5 chains and a depth of 20·75 chains. If the land is bought for Rs. 2,490, find the price paid per acre.

13. PQRS is the sketch of a school-room and the shaded portion around it represents a verandah 8 ft. wide. If PQ=36', QR=18', and if the room accommodates 80 pupils, find how much of floor space is allowed to each pupil. (Include the floor-space of the verandah also in the calculation).

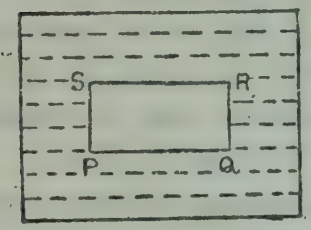


Fig. 70.

14. Draw the plan of a room 20' by 10' and show it covered with slabs 1'8" × 10". Find the number of the slabs required.

15. Out of 30 yards of long-cloth 2' 6" wide, kerchiefs are cut 15" square. Find how many kerchiefs could be obtained.

16. Out of 3½ yds. mull, 4½ ft. wide, square kerchiefs are cut which are 1 ft. 6 in. long. Find the number of kerchiefs that could be obtained.

17. The waiting-room in a bungalow is 18' × 12'. Find the cost of matting the floor at 6 as. a sq. yd.

18. A court-yard, 28' × 18', has to be paved with Cuddapah slabs, each 1' 6" × 1' 2". Find the cost of the slabs required at Rs. 37-8 as. per 100.

19. The dining-hall in a hostel is 40' by 20'. If its floor has to be covered with bricks 8" square, find the cost of the bricks required at Rs. 15 per 1000.

20. A marriage pandal is in the form of a rectangle 100'×30'. The floor has to be fully covered with mats and then a portion in the centre has to be covered with carpets. How many mats, each 6' by 2' 6", will be required for the purpose. If 4 carpets, each 4 yards by 9 feet, be spread in the centre, find the area left uncovered by the carpets.

Ex. A hall is 25′ by 15′. Draw a plan of it to a scale of 1 inch to 1 ft. and show it covered with carpet 3 ft. wide. Find the length of the carpet required to cover the floor.

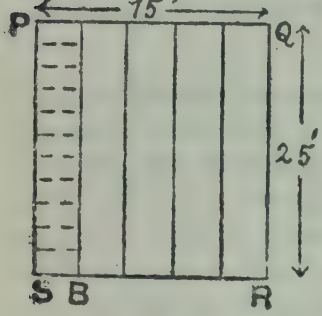


Fig. 71.

If SR = 15′, RQ = 25′, and SB = 3′, five strips of carpet, each of the form shaded in fig. 71 would be required to cover the floor.

∴ the total length of the carpet required = 25 × 5 or 125 ft.

Also, the total area occupied by the carpet = that of the floor.

Hence the length of the carpet required

$$= \frac{\text{the no. of sq. ft. in the area of the floor}}{\text{the no. of feet in the breadth of the carpet}}.$$

21. In the front of a shop a curtain is to be put. If the curtain is to be 18 ft. long and 10 ft. wide and the curtain-cloth is 1 ft. 8 in. wide, find the length of the cloth required and the cost of it at 8 as a yd.

22. A floor, 14 ft square, has to be covered with coir carpet. If the carpet is 2 ft. 4 ins. wide and costs 6 as. a foot, find the cost of carpet required to cover the floor.

Ex. A square tank with each side 340 ft. long is surrounded by a path 5 ft. wide. Find the cost of paving the path at 2 as. a sq. ft.

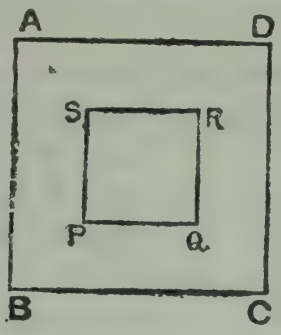


Fig. 72.

[The area of the pathway
= the sq. ABCD — the sq. PQRS.
= $350^2 - 340^2$ sq. ft.
= $(350 + 340)(350 - 340)$
or 6,900 sq. ft.]

23. A square tank with each side 300 ft. long has a mantapam 10 ft. square in the centre. If the remaining portion of the tank be under water, find the area of the water surface.

24. A tennis court is 26 yds. by 12 yds. Two verandahs, $4\frac{1}{2}$ ft. wide, are marked inside the court along each lengthwise edge. What area of ground is allowed for the balls to hit when playing singles? (The verandah is not allowed in playing singles.)

25. On each page of a book a margin of $\cdot 8''$ is left on the right-hand side, of $\cdot 4''$ on the left-hand side and of $1''$ on the top and $1''$ at the bottom. What is the area allowed for printing if the length of the top edge $= x''$ and that of the adjacent edge $= y''$?

26. The bridge over a river is half a mile long and 32 ft. wide. On either side along the length there is a raised foot-path $4\frac{1}{2}$ ft. wide and the remaining portion is set apart for the carts to run. What area on the bridge is set apart solely for the foot-passengers and what area for the carts?

27. In a rectangular garden 200 ft by 175 ft. there is a pathway 4·5 ft. wide running all round it inside. Also, a pathway of the same width runs from the middle of each side to that of the opposite side. Thus the garden is divided into four equal beds. Find the total area of the pathways and of each bed.

Ex. Two rectangular halls are of equal width but their lengths differ by 10 ft. The cost of flooring the bigger hall is Rs. 160 and that of flooring the smaller Rs. 120. Find the length of each hall.

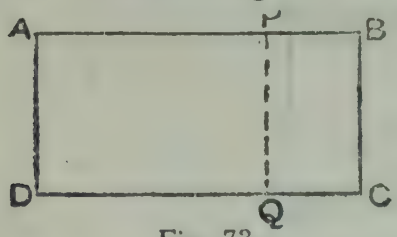


Fig. 73.

Suppose ABCD represents the floor of the bigger hall and APQD a portion of it is equal to the floor of the smaller hall.

Then the cost of flooring the portion PQCB along a length (PB) of 10 ft. = Rs. 40.

∴ the cost of flooring along a length of 1 ft. = Rs. 4

∴ Rs. 160 is the cost of flooring along a length of $\frac{160}{4}$ or 40 ft. ∴ AB = 40 ft. and hence AP = 30 ft.

28. The cost of carpeting a room is Rs 120 and that of carpeting another of the same length but 2 ft. wider is Rs. 144. Find the width of each room.

29. The cost of matting the floor of a room is Rs. 14 and that of matting another equally long but 5 ft. less wide is Rs. 10-4 as. Find the breadth of each room.

30. A rectangular field contains 2,000 sq. yds. more than another. If one field is as wide as the other but is 40 yds. longer, find the breadth of each field.

Ex. A rectangular room is 18′ long 12′ broad and 10′ high. Draw a plan of each of its four walls and find their total area.

If a cardboard model of the walls be made and stretched out, a figure like the one below would be formed, with the walls I and III equal in area as well as the walls II and IV.

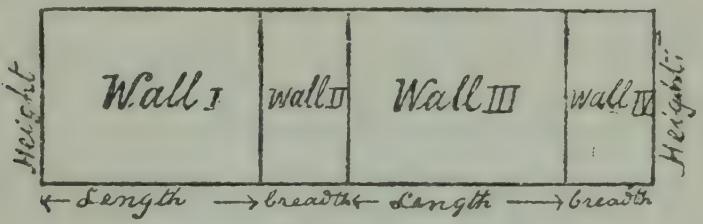


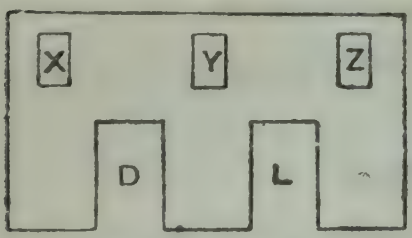
Fig. 74.

∴ *the area of the four walls*
= *twice the areas of the walls I and II*
= 2 × *Height* × (*Length* + *Breadth*).

If 'A' be the area in sq. ft. of the four walls of a room 'l' feet long, 'b' feet wide and 'h' feet high, then

$$A = 2h \times (l + b)$$

Hence, $h = \frac{A}{2(l+b)}$, $l = \frac{A}{2h} - b$ and $b = \frac{A}{2h} - l$.



[Scale 1 cm = 5'].
Fig. 75.

31. The accompanying figure represents a wall with two doorways D, L and three windows X, Y and Z. Find the cost of painting one face of the wall at 1 anna a sq. ft.

32. Fill up the vacant columns in (*a*), (*b*), (*c*) and (*d*) below:—

	(*a*)	(*b*)	(*c*)	(*d*)
Length of room	18 ft.	( )	15 ft.	( )
Its breadth	15 ft.	14 ft.	( )	25 ft.
Its height	10 ft.	12 ft.	( )	12 ft.
Area of the four walls	( )	720 sq. ft.	522 sq. ft.	( )
Area of the floor	( )	( )	210 sq. ft.	625 sq. ft.

33. From the formula $A = 2h(l + b)$,

find 'h' when $A = 496$, $l = 18$, $b = 13$;
'l' „ $A = 780$, $h = 10$, $b = 14$;
'b' „ $A = 990$, $h = 9$, $l = 33$;
and 'A' „ $h = 12$, $l = 31$, $b = 19$.

34. Along two adjacent walls of a class-room 21 ft. by 18 ft. wall-boards are put up to a width of 4 ft. 6 ins. Find the cost of putting up the wall-boards at 10 as. a sq. yd.

35. A school-garden is 30 ft. by 15 ft. It requires fencing to a height of 4 ft. all round. If thatties, each 4 ft. by 1 ft. 8 ins. are to be used, find the cost of fencing at 4 as. a thatty and at an additional cost of Rs. 5-8 as. for laying them.

36. A rectangular hall is 32 ft. long, 25 ft wide and 15 ft. high. There are 8 windows each 3 ft. by 2 ft. 6 ins. and 6 doorways, each 6 ft. by 3 ft. 6 ins. Find the inner surface area of the walls. How much would it cost to paint the inner surface at one anna a sq. foot?

Ex. The cost of painting the walls of a room 17 ft. long is Rs. 37-8 as. at 9 as. a sq. yd. and that of carpeting the floor at Re. 1-2 as. per sq. yd. is Rs. 27-10 as. Find the breadth and height of the room.

The area of the walls $= \frac{37\frac{1}{2} \times 16}{9}$ sq. yds. or 600 sq. ft.

and that of the floor $= \frac{27\frac{5}{8}}{1\frac{1}{8}}$ sq. yds. or 221 sq. ft.

The length of the room = 17 ft.

$\therefore$ its breadth $= \frac{221}{17}$ or 13 ft.

$\therefore$ $2h(l + b) = 600$, and $l + b = 30$.

$\therefore$ $h = \frac{600}{2 \times 30}$ or 10.

$\therefore$ the height of the room = 10 ft.

37. The cost of painting the walls of a room at 8 as. per sq. yd. is Rs. 68 and that of carpeting the floor at Re. 1-4 as. a sq. yd. is Rs. 90. If the rooms be 27 ft. long, find its breadth and height.

38. Find the cost of lining with lead an open rectangular box whose internal dimensions are 6 ft. long 3 ft. wide and 1 ft. 6 ins. high at Re. 1 per sq. foot.

39. A leather portmanteau is rectangular in shape and is a" by b" by c". If leather costs 3 ps. a sq. inch and workmanship Rs. 2 on the whole, find the cost of the portmanteau.

40. A well in my studio is 14' by 10'6". I want to paste on it pictures from the "Illustrated Times of India." If each picture is 1'9" by 1'2" and a dozen pictures cost Re. 1, find the total cost of the pictures required.

SECTION II.

The Right-angled Triangle.

49. Construct a paper-rectangle ABCD having AB = 2" and BC = 3·5." Fold and cut along a diagonal AC. The two triangular pieces ABC, ACD may be placed one over the other so as to exactly coincide. [How?]

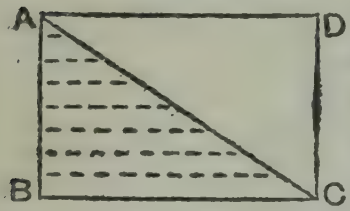


Fig 76.

∴ the area of the right-angled triangle ABC
= half that of the rectangle ABCD
= $\frac{1}{2}$ AB × BC.
= *half the product of the sides containing the right-angle.*

Hence, if AB = 2", BC = 3·5", the area of the rt. ∠ed △ ABC = $\frac{1}{2}$ × 2 × 3·5 or 3·5 sq. ins.

EXERCISES.

41. Find the areas of the shaded portions in the following from the measurements given:—

(*a*)

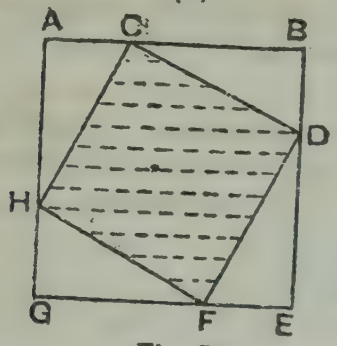


Fig. 77.

AC = GH = FE = DB = 1″
BC = ED = GF = AH = 2·5″
∠A = a rt. ∠.

(*b*)

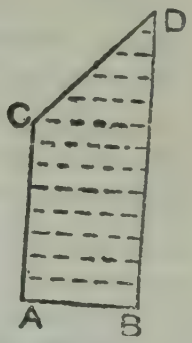


Fig. 78.

AC = 4 yds.
AB = 2·4 yds.
DB = 6·4 yds.
∠A = ∠B = 90°

(*c*)

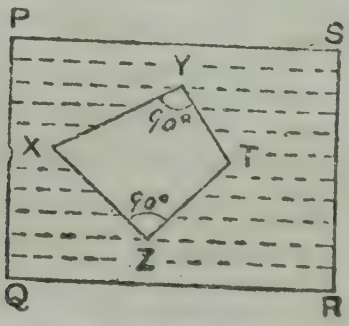


Fig. 79.

PQ = SR = 15 chains.
PS = QR = 20 chains.
TY = TZ = 8 chains.
XY = XZ = 10 chains.

42 Find the number of square inches of plate in your nickel set squares.

43. Construct two quadrilaterals ABCD, PQSH having each side of both = 1·4". $\angle ABC = \frac{3}{4}$, $\angle PQS = 90°$. Join the diagonals in each and measure the angles at their points of intersection, Compare the areas of the quadrilaterals.

50. Construct a right-angled triangle ABC in which AB = 3 cm., BC = 4 cm. and $\angle B = 90°$. Draw squares externally on the three sides. Compare the square on the hypotenuse AC with the sum of the squares on AB and BC. (The side opposite to the right angle is called the *hypotenuse* of the right-angled triangle). If we do this with reference to any right-angled triangle, it will be found very nearly that **the square on the hypotenuse = the sum of the squares on the other two sides.**

The followfng method of dissection will prove the truth of the statement :—

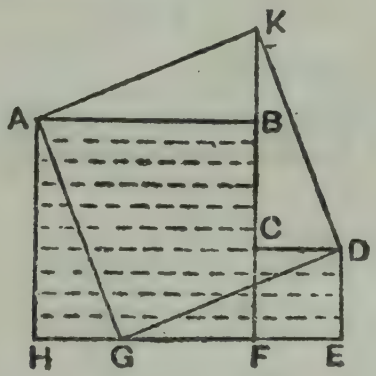


Fig. 80.

Cut out a piece of cardboard in the form shaded in the above figure so as to contain two squares AHFB, CFED. Mark HG = DE. Cut along AG, GD and place the triangular pieces AHG, GED in the positions ABK, DCK. The square AGDK would be formed. It hence follows from the figure that the square on AG = the sum of the squares on AH, HG, or $AG^2 = AH^2 + HG^2$.

Verify by measurement if the same relation exists between the edges of your set-squares. Note that in the 45° set-square the edges are in the ratio 1 : 1 : 1·4 and in the 60° set-square they are in the ratio 1 : 1·73 : 2.

EXERCISES.

44. Construct a triangle with sides 5 cm., 12 cm., and 13 cm., and another with sides 7 cm., 24 cm., and 25 cm. Find by measurement whether the triangles are right-angled.

[If, in any triangle AGH, $AG^2 = AH^2 + HG^2$, the $\angle AHG$ would be a rt. $\angle$,]

45. Construct triangles whose sides ara as follows :—

9 cm., 12 cm., 15 cm.;

8 cm., 15 cm., 17 cm.;

$2n$, $n^2 - 1$, $n^2 + 1$;

$2x + 1$, $2x^2 + 2x$, $2x^2 + 2x + 1$;

and $l^2 + m^2$, $l^2 - m^2$, $2lm$.

Verify if they are right-angled.

46. Construct a right-angled triangle with hypotenuse 10 cm. and one side 8 cm. Calculate the length of the other side and verify by measurement.

47 *Given a triangle, show how by using only a measuring tape, you can find whether it is right-angled or not.*

48. A straight road is to be newly opened across a rectangular

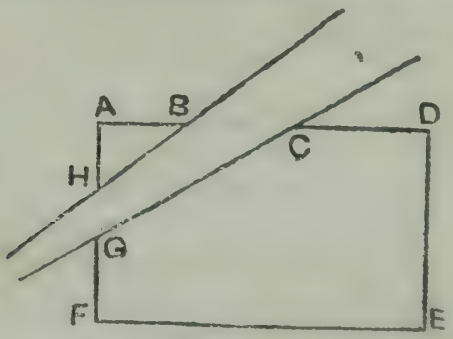


Fig. 81.

field as shown in the figure. If AB = 40 yds., AH = 30 yds., HB = 18 yds., BC = 50 yds., find what area in the field would be occupied by the road. Calculate the lengths of HB and GC.

49. Having found the North-to-South line by looking at the stars, the ancients found the Eastern direction by the following method:—

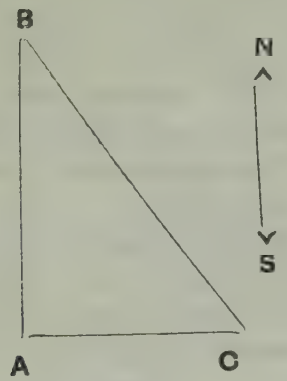


Fig. 82.

From a point A, AB is taken 6 ft. long Northwards and a rope 12 ft. long was used with the ends fixed at A and B to form two sides of a triangle ACB so that AC was due East. How did they do it?

SECTION III.

The Parallelogram.

51. ABCD is a parallelogram. Its opposite sides are parallel and equal. Its opposite angles are also equal. DP is ⊥ to BC and AD.

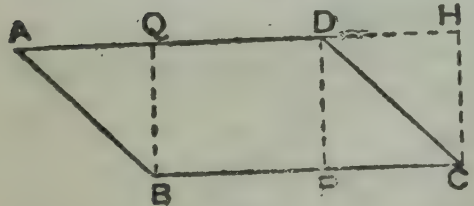


Fig. 83.

If BC be the *base* of the ||m, DP is its *altitude*.

Draw BQ ⊥ to AD. Cut out the triangle ABQ and fit it in the position CDH.

Then the area of the ||m ABCD
= that of the rect. BCHQ
= BC × DP
= *base* × *altitude*.

EXERCISES.

50. Two straight roads, 40 ft. and 45 ft. wide, cross each other at an angle of 60°. Find the area of land common to the two roads.

51. A ||m = a rect. 5" by 3·5". One side of the ||m = 6·3". Find the length of the altitude corresponding to that side. If an adjacent side of the ||m = 3 3", construct the ||m. Show how you would divide the ||m in the ratio 2 : 3.

52. A garden is bounded by four straight fences, two of which run from North to South and are each 430 yds. long and the other two run from North-West to South-East and are each 250 yds. long. Find the area of the garden in acres.

SECTION IV.

The Triangle.

52. ABC is a triangle with base BC and altitude AP. Complete the ||m ABCD. Cut along AC and place the triangle ABC over the triangle CDA. The two triangles can be placed so as to exactly coincide. [How?]

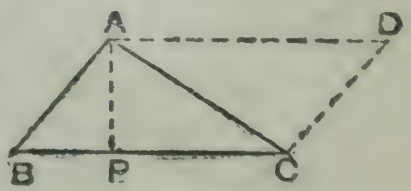


Fig. 84.

$\therefore$ the area of Δ ABC $= \frac{1}{2}$ that of the $\|^m$ ABCD

$= \frac{1}{2}$ BC $\times$ AP

$= \frac{1}{2}$ *the base* $\times$ *the altitude.*

Fig. 85 shows how a triangle ABC could be cut and fitted into a rect. EGQP. D and H are the middle points of AB and AC; DE, AF and HG are perpendicular to BC. The triangles BED, APD are congruent as well as the triangles CHG, AHQ. (Why?). Hence the area of the triangle ABC = that of the rectangle EGQP = EG $\times$ EP $= \frac{1}{2}$ BC $\times$ AF.

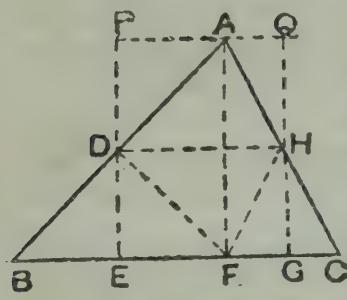


Fig. 85.

[Note that in the above fig., the Δ ABC = 2 $\times$ the rect. EGHD = 4 $\times$ the ΔDFH.]

EXERCISES.

53. Draw any three triangles ABC, DEF, PQR. Choosing each side as the base, draw the corresponding altitude of each triangle and thus find its area in three different ways. Verify if the results agree in respect of the same triangle.

54. Construct a triangle PQR, having PQ=5 cm., PR=3 cm. and $\angle$RPQ=120°. Find by measurement the length of the altitude when PQ is the base and thence calculate the area of the triangle. Verify the result by finding the area with PR and QR as bases.

55. Two triangles have their altitudes in the ratio 5 : 4. If their areas are equal, compare their bases.

56. The area of a triangle=5 sq. chs. One side of it=5 chs. Find the length of the altitude corresponding to that side.

57. Compare the areas of the triangle ABC, PBC and DBC in Fig. 86 where DP is parallel to BC.

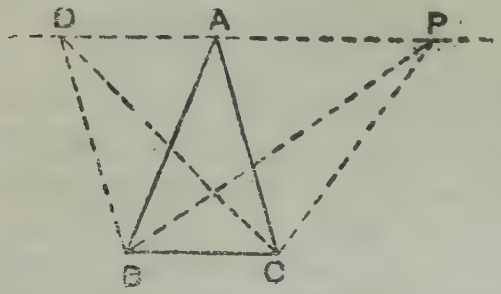


Fig. 86.

[The vertices of triangles standing on the same base and of equal areas or of equal heights lie in a straight line parallel to the base.]

58. In the accompanying figure AB=a''. BC=b'', DB=l''. DC is joined and AE is drawn parallel to DC to meet BC produced at E. Show that the triangles ABC and DBE are equal in area. Show also that, if the angle ABC=a rt. $\angle$. BE=$\frac{a \times b''}{l}$. Hence deduce a geometrical construction for finding $\frac{3{\cdot}4 \times 4{\cdot}6}{2{\cdot}5}$, $\frac{(3{\cdot}4)^2}{(5{\cdot}2)}$, $\frac{a \times b}{c}$ and $\frac{a^2}{c}$.

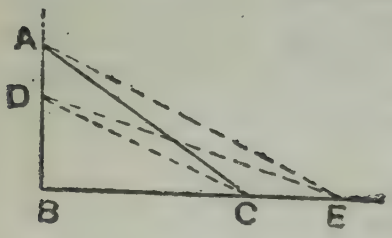


Fig. 87.

[Note that if in Fig. 87 DB represents 2 units, the no. of linear units in BE is the no. of sq. units in the area of the $\triangle$ ABC.]

59. ADFC is a $\|^m$, BHE is $\|$ to AD and GHK is $\|$ to FED. Show that the $\|^{ms}$ AH and HF are equal in area.

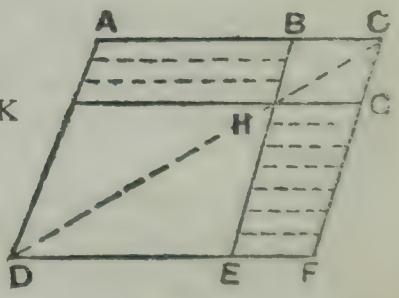


Fig. 88.

60. Construct a $\|^m$ ABHK in which AB = 66 units, AK=10 units and $\angle$KAB is any angle. Produce AB to C so that BC=12 units. Join CH to meet AK at D. Draw DE parallel to AC and CG parallel to AK. Let DE and CG produced meet at F. Produce BH and KH to meet DF at E and CF at G. Verify if HE=55 units. Hence deduce a method for finding the value of 'x' so that $x \times a = b \times c$ or $\frac{x}{b} = \frac{c}{a}$, where a, b and c are constants.

61. Find the area of an isosceles triangle ABC having BC = CA = 2" and AB = 3".

62. Construct triangles (1) with sides, each 3" and (2) with sides 6 cm., 10 cm. and 12 cm. Find their areas.

63. Find by splitting into triangles, the areas of the figures (89—93) bounded by the non-dotted lines below:—

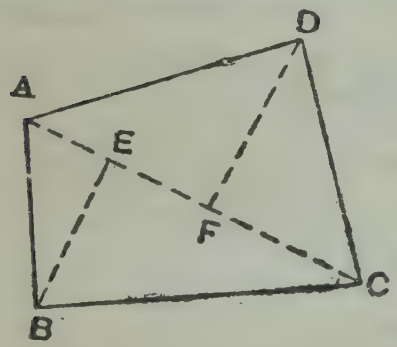


Fig. 89.

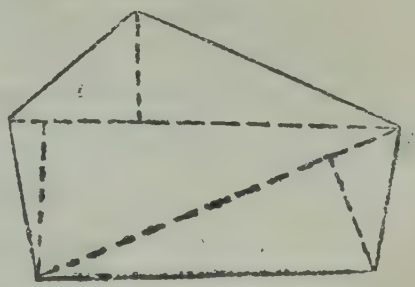


Fig. 90.

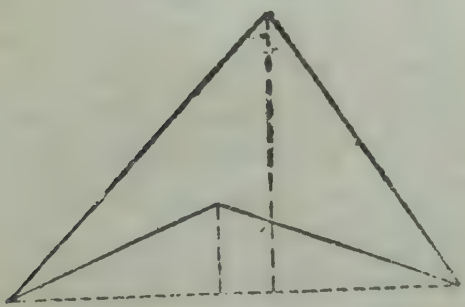


Fig. 91.

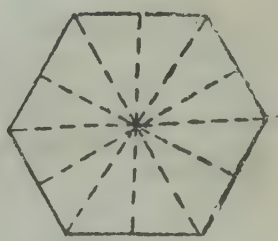


Fig. 92.

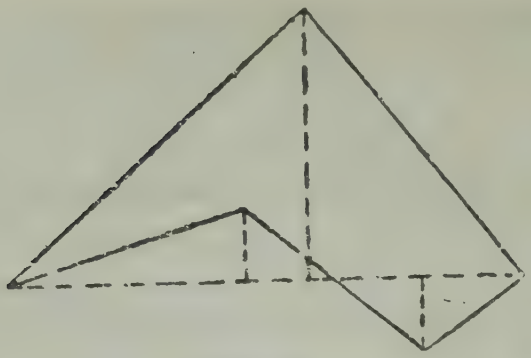


Fig. 93.

64. *Show that the area of a convex quadrilateral ABCD = a diagonal AC × half the sum of the off-sets BE, DF.* (Refer to fig. 89). Show also that if AC = d", BE = l", and DF = b" and the area of the quadrilateral = 'A' sq. inches, then $d = \frac{A2}{l+b}$, $l = \frac{2A}{d} - b$, and $b = \frac{2A}{d} - l$. Hence construct a quadrilateral ABCD having 3 square inches of area, a diagonal AC = 2", an off-set DF = 2". $\angle ACD = \angle BCF = 60°$.

SECTION V.

The Trapezium.

53. ABCD is a quadrilateral in which two of the sides AD and BC are ∥ and the other two are not ∥, *i.e*, oblique. Such a quadrilateral is called a *trapezium*. If E and F be the middle points of AB and CD, the figure in the margin shows how a trapezium may be cut and fitted into a rectangle PQRS in which QR = ½ (AD + BC) and RS = the distance between the ∥ sides.

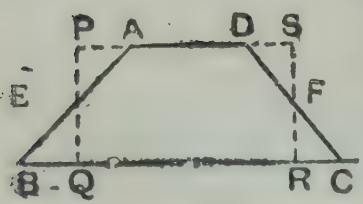


Fig. 94.

In Fig. 95, two trapeziums, each equal to ABCD, are placed so as to form a $\|^{m}$ ABPQ in which the base BP = the sum of the || sides of the trapezium and the corresponding altitude = the dist. bet. them. Hence the area of the trapezium ABCD = $\frac{1}{2}$ that of the $\|^{m}$ ABPQ = $\frac{1}{2}$ BP × the dist. bet. AD, BC = $\frac{1}{2}$ (AD + BC) × the distance bet. AD, BC.

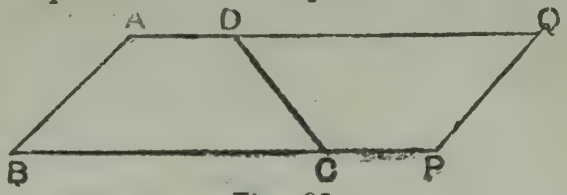


Fig. 95.

In either way it is clear that *the area of the trapezium* = $\frac{1}{2}$ *the sum of the parallel sides* × *the distance between them.*

EXERCISES.

65. If 'A' be the area in sq. ins. of a trapezium whose || sides are a" and b" with the distance between them d", show by joining a diagonal that $A = \frac{1}{2}(a + b) \times d$; $d = \frac{2A}{a+b}$; $a = \frac{2A}{d} - b$ and $b = \frac{2A}{d} - a$.

Hence fill the vacant columns in the following:—

A	d	a	b
28	4	10	( )
32	( )	6	18
( )	8	9	15
228	12	( )	9

66. My village is in the form of a trapezium ABCD in which BA and CD run from N to S, BC from S.-W. to N.-E. and AD from N.-W. to S.-E. AB = BC = 2000 yds. Draw a plan of the village to scale and find its area in acres. (Assume the surface to be flat).

67. The platform in a railway station is a trapezium with an area of ·515 of an acre. If its parallel edges are 10 chs. 78 lks. and 9 chs. 82 lks. long, calculate the shortest distance across the platform.

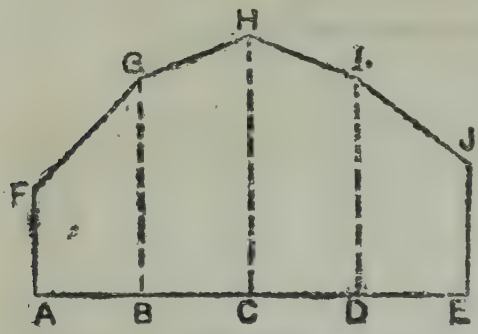


Fig. 96.

68. The figure drawn is a rough sketch of a play-ground. AB = BC = CD = DE = 50 yds. 2FA = GB = ID = 100 yds. 2 JE = HC = 120 yds. Estimate the worth of the ground at Rs. 1210 per acre.

CHAPTER IX.

GRAPHS (INTRODUCTORY)—PLOTTING.

54. FIXING THE POSITION OF POINTS IN A PLANE.

Let ABCD represent a rectangular field. Suppose the position of a well W in it has to be determined. One way of determining the position is by measuring the distances of the well from any two adjacent edges of the field. Draw WP, WQ ⊥ to AB, AD respectively. Suppose on measuring it is found that AP = 5 units and PW = 3 units. Then the well W could be reached by starting from a fixed point, *e.g.*, the corner A of the field, then proceeding 5 units along the edge AB and then going 3 units ⊥ to AB, *i.e.*, parallel to AD along PW. Thus the two measurements AP, PW help to fix the position of the well in the field. Two similar measurements would help to fix the position of any other spot in the field. [Note that there could be marked in the field only one spot W which is at a distance of 5 units from AD and 3 units from AB.] Slmilarly to fix the position of a seat in a rectangular class-room it is enough if its distances from two adjacent walls be given. Also in fixing the position of a point within the angular space of a right angle XOY it is enough if its distances from the arms of the angle be known.

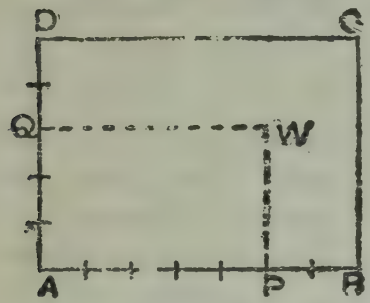


Fig. 97.

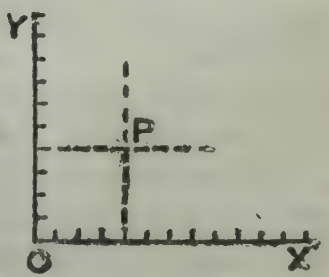


Fig. 98.

55. THE CO-ORDINATES OF A POINT.

Through a fixed point O draw two ⊥ st. lines X'OX (a horizontal or East-to-West line) and YOY' (a vertical or North-to-South line) so as to divide the plane of the paper into four angular spaces. Suppose it is given that a point is 5 units from YOY' and 3 units from XOX'. Unlike in the previous cases, the position of the point could not be uniquely determined with the conditions now given. For, four points P, Q, R, S could be found, one for each of the four angular spaces, satisfying the given conditions. Starting from O, the pairs of measurements

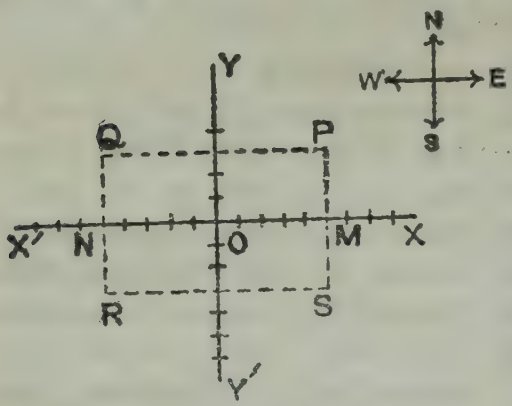


Fig. 99.

OM (5 units East), MP (3 units North);
ON (5 units West), NQ (3 units North);
ON (5 units West), NR (3 units South);
OM (5 units East), MS (3 units South)

would specify the position of the points P, Q, R, S respectively.

It is usual to represent such measurements by +ve and −ve numbers adopting the convention that the lengths measured to the right in the direction OX as well as those measured upwards are represented by +ve numbers, while those measured to the left in the direction OX' and those measured downwards are represented by −ve numbers. Thus OM, MP will be represented by (+ 5 + 3), ON, NQ by (− 5 + 3), ON, NR by (− 5, − 3) and OM, MS by (+ 5, − 3).

The two measurements above described which help to fix the position of a point in a plane with reference to two ⊥ st. lines XOX′, YOY′ are known as the **co-ordinates** of the point.

The lines of reference are called the *axes* of co-ordinates; XOX′ is called the x-axis and YOY′ the y-axis. The point of intersection of the axes is called the *origin*.

The four angular spaces XOY, YOX′, X′OY′, Y′OX into which the plane of the paper is divided by the axes of co-ordinates are respectively called the 1st, 2nd, 3rd and 4th *quadrants*.

Of the two co-ordinates of a point, that which is measured along or is parallel to the x-axis is called the *abscissa* (or x co-ordinate or 'x') of the point and that which is measured along or is parallel to the y-axis is called the *ordinate* (or y co-ordinate or 'y') of the point. A point whose abscissa is $+5$ and ordinate is -3, is indicated by the symbol $(+5, -3)$ where the abscissa is written first and the ordinate next. Thus $(+5, +9)$, $(-8, +7)$, $(-8, -8)$, $(+9, -8)$ would respectively represent the points P, Q, R, S in Fig. 100.

Ex. Write down the co-ordinates of points marked '×' in the fig. below (1) taking $\frac{1''}{10} = 1$ unit along either axis and (2) taking $\frac{2''}{10} = 1$ unit along either axis.

The method of locating a point with the help of given co-ordinates is known as *plotting* the point. The use of squared paper would help much in plotting points.

[NOTE 1. The method of determining the position of a point in a plane corresponds to that of fixing the

position of a place on the surface of the earth by means of its latitude and longitude.

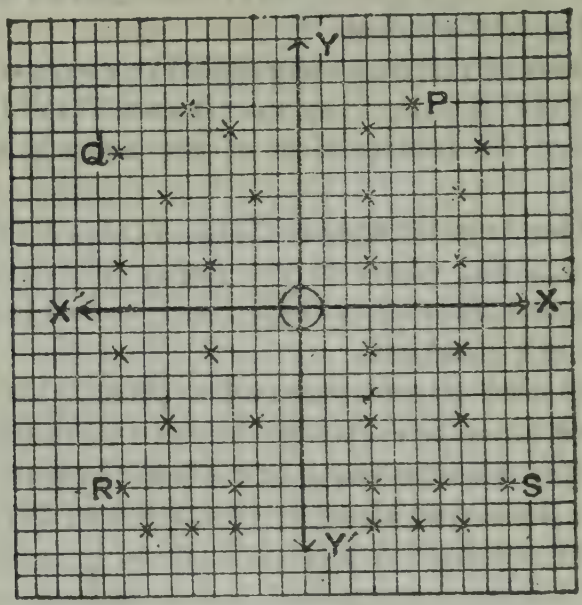


Fig. 100.

NOTE 2. The position of a point in a plane may also be fixed as follows :—

(1) by means of its distances measured parallel to, or along, two axes which may not be at right angles, *e.g.* (OQ, QP_1), (OR, RP_2) in the marginal fig.

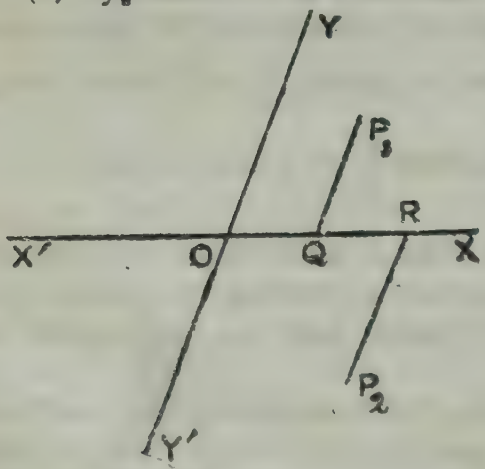


Fig. 101.

(2. by means of its distances from a fixed origin and its direction from the point as in the accompanying figure where P_1 is 5 units from O and in the direction 27° North of East and P_2 is 5 units in the direction 74° South of West.

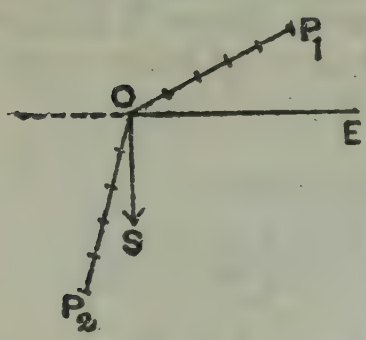


Fig. 102.

(3) by means of its distances from two fixed points as in Fig. 103 where P is 5 units from O_1 and 3 units from O_2, both the distances being taken as + ve if P be above or to the right of the line O_1 O_2 and — ve if P is below or to the left of O_1 O_2.]

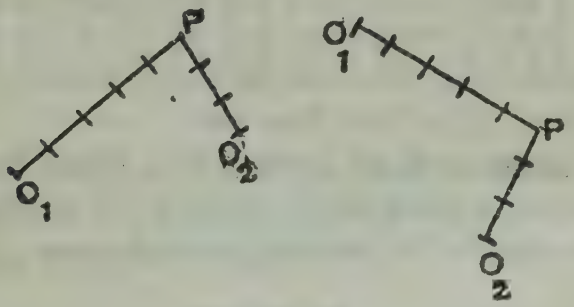


Fig. 103.

56. Examples like the following are best worked with squared paper as shown below :—

Ex. Plot the points (+ 8, + 5), (— 12, — 10). Join them. Find the distance between them.

Let "1" represent 1 unit along both the axes. Then P_1 represents (+ 8, + 5) and P_2 represents (— 12, — 10). With centre P_2 and radius P_2P_1 draw an arc cutting the

horizontal line through P_2 at M. From the figure it may be read that $P_2M = 25$ units. [Draw $P_1Q \perp$ to P_2M. Then $P_1P_2^2 = P_1Q^2 + P_2Q^2 = 15^2 + 20^2 = 625$.

$\therefore P_1P_2 = 25$].

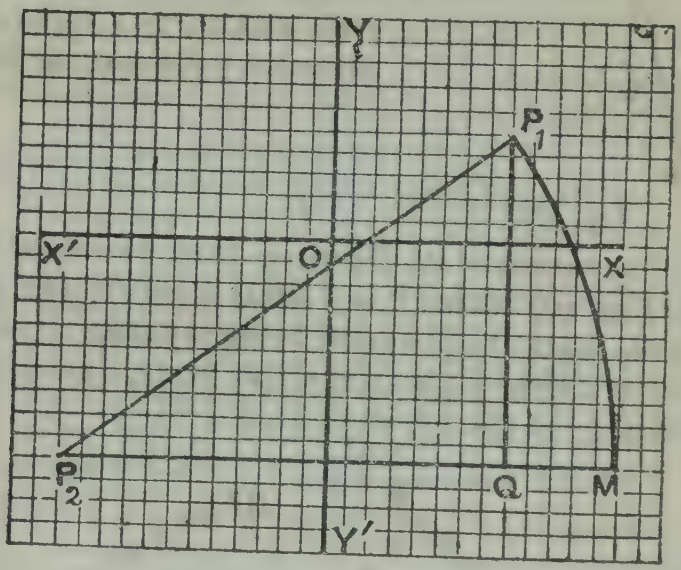


Fig. 104.

It is useful to make a point by a small cross ' × ' or by drawing a small circle round it. Usually the same scale is adopted in marking co-ordinates along either axes.

EXERCISES.

1. Plot the points:—(+ 16, + 5), (+ 5, + 16), (+ 18, + 8), (+ 12, + 12), (+ 5, + 5), (+ 10, + 15), (— 15, — 6), (— 6, —6), (— 12, — 12), (— 9, — 10), (— 2, — 12), (+ 16, — 2), (— 15, + 2), (+ 12, — 8), (— 12, + 8) and (— 12, — 8).

2. Mark six points in each quadrant. Read their co-ordinates.

[Note that in quadrant I both the co-ordinates are + ve, in quadrant III both are negative, in quadrant II the abscissae are —ve while the ordinates are + ve and in quadrant IV the abscissae

are + ve and the ordinate — ve. Hence the nature of the co-ordinates of a point would determine in which quadrant the point lies.

3. Mark any six points in each axis. Read their co-ordinates.

[Note that the abscissa of every point in the Y-axis is '0' and the ordinate of every point in the X-axis is '0.']

4. Plot the points (+ 18, + 8). (—8,—18). Join them. Find the middle point of the st. line joining them. Read its co-ordinates.

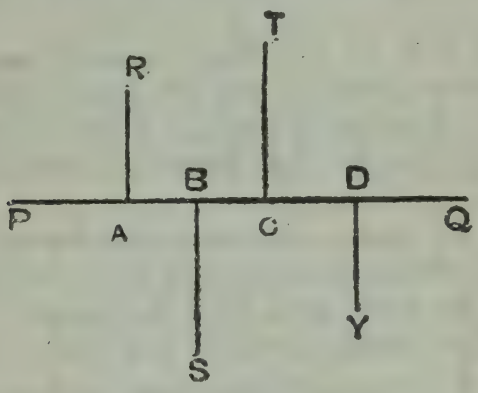


Fig. 105.

5. The straight lines drawn in the above figure represent the roads in a certain area. The points P, A, B, C, D, Q, R, S, T, Y represent certain villages. If the above figure is drawn to a scale of 1" to 10 miles, and if AR represents North, express the position of each village in terms of its distances from B as measured along the roads.

6. Two stratght lines PQ, PR intersect at a right angle. A point S is ·9" from PQ and ·2" from PR. Find its position.

7. A point P is 1·5" from a given st. line AB. It is also 3" from A. Find its position.

8. P and Q are two towns situated 16 miles apart along a straight road. Find the distance from the road of a village 'S' situated 9 miles from P and 16 miles from Q.

9. ABCD is a court-yard 20 yds. square. Inside it is a well 8 yds. square. One corner P of the parapet wall of the well is

14 yds. from AB and 2 yds. from BC and Q the opposite corner is 3 yds. from CD, and 15 yds. from AD. Draw a plan of the court-yard showing the well in it.

10. EFGH is a rectangular wall 15' long and 12' high. In the wall provision has to be made for a door-way PQRS, 6 ft. by 3 ft. and for a window LMNO. 3 ft. by 2 ft. The points L, M, Q, R are all at a height of 6 ft. from the floor FG and the extreme points L, R in the line LMQR are respectively 4' from EF and 3' from GH. Make a sketch of the wall showing the door-way and the window.

11. A place P is in 15° West Longitude and 12° North Latitude. Another place Q is in 15° East Longitude and 17° South Latitude. Represent P and Q in a diagram drawn to scale, chosing two $\perp$ lines for the Greenwich meridian and the Equator.

12 The teacher marks a dot on the blackboard. He allows you to make any measurements you like which would help you to mark the dot in the same place after the original dot has been rubbed off. What measurements will you take?

13. A point P on squared paper is on the 15th parallel from the bottom line and on the 18th parallel from the vertical line on the extreme left. Mark the position of the point.

14. Instead of finding the distance between the points $(0, +18)$ and $(-12, 0)$, a boy found out the distance between $(+18, 0)$ and $(0, -12)$. Show why his result will not be incorrect.

15. The distance between $(+15, +4)$ and $(+3, -12)$ is the same as that between $(+4, +15)$ and $(-12, +3)$. Verify the truth of the above statement.

16. Find the co-ordinates of the middle points of the lines joining $(+10, +8)$ and $(+4, +22)$: $(-10, -8)$ and $(+4, -22)$: $(+5, +6)$ and $(-5, -6)$.

17. Draw a circle with the origin as centre and with radius 5 units. Mark points on the circumference and read out their co-ordinates. Verify if the 'x' and 'y' of every such point satisfy the equation $x^2 + y^2 = 5^2$.

18. Plot the points $(-3, +5)$, $(-7, -15)$, $(+1, -1)$, $(+4, +9)$. Find the perimeter of the quadrilateral formed by joining them in order. Draw a circle with $(+7, +5)$ as centre and the distance of $(-3, +5)$ from it as radius. Does it pass through the vertices of the quadrilateral?

19. Join the points $(+3, +4)$ and $(+7, +8)$. Draw the parallel through $(+12, +15)$ to the st. line formed. Where does the parallel cut the axes? Find the lengths of the intercepts made on each axis by the pair of parallels.

20. Taking the West Coast of South India to be roughly inclined at 60° N. of W. from Cochin to Calicut a distance of 90 miles represent the following movements of a ship in a diagram drawn to scale:—A ship leaves Cochin, moves 30 miles West, then moves 50 miles parallel to the Coast, again moves 50 miles West and reaches an island. Find how far and in what direction the island is from Cochin and Calicut.

21. A and B are two places lying 50 miles apart in the same Latitude and A is East of B. A person starts from A, walks 12 miles East, then goes 15 miles North, then goes 5 miles East and reaches P. Find how far P is from A and B?

Express the distance as so many miles to the East or West and as so many miles to the North of A or B.

22. Draw a st. line passing through the points $(+5, 0)$ $(0, +8)$. Produce the st. line and mark 9 points in it. Read out their co-ordinates. Draw a st. line patallel to it so as to pass through the origin.

23. Plot the points:—$(+5, +6)$, $(+10, +4)$, $(+10, -4)$, $(+5, -6)$, $(-6, +5)$. Join them in order so as to form a five-sided figure. Find the area of the figure formed.

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CHAPTER X.

SIMPLE INTEREST.

57. Just as bandy fare is paid for the use of another's bandy, interest is paid for the use of another's money. The money paid as interest is calculated at so much per cent. per month or per year on the sum borrowed. [Unlike in the previous questions on per-centage, the element of time is involved in reckoning interest.]

Ex. 1. A borrows Rs. 500 from B and repays it with interest at 9% per annum 4 months hence. How much will B then get?

In the above question Rs. 500 is the money lent. It is called *the principal.* The rate of interest is Rs. 9 for Rs. 100 for 1 year. It is called *the rate per cent.* The time is 4 months or $\frac{1}{3}$ of a year.

The interest on Rs. 100 for 1 year = Rs. 9

$\therefore$ ..... $\frac{1}{3}$ „ = Rs. 3

$\therefore$ .....Rs. 500..... = Rs. 15.

Rs. 15 = the interest (often called *simple interest* as distinguished from *compound interest*, see Chap. XII).

At the end of 4 months B will have to be paid Rs. 500, the original sum borrowed, + Rs. 15 (interest) or Rs 515. This sum is called the *amount*.

Ex. 2. Given amount = Rs. 515, Principal = Rs. 500, Rate = 9 %, find the time.

Interest = Rs. 15

Rs. 45 = Interest on Rs. 500 for 1 year

$\therefore$ Re. 1 = ... $\frac{1}{45}$ „

$\therefore$ Rs. 15 = ... $\frac{15}{45}$ „

or $\frac{1}{3}$ year or 4 months.

Ex. 3. Given amount = Rs. 515, Principal is Rs. 500, time = 4 months, find the rate per cent of interest.

The interest on Rs. 500 for $\frac{1}{3}$ year = Rs. 15

The interest on Rs. 500 for 1 year = Rs. 45

$\therefore$ ..... Rs. 100 ..... = Rs. 9

$\therefore$ the rate is 9 %.

Ex. 4. Given amount = Rs. 515, time = 4 months, rate = 9 %, find the principal.

The amount on Rs. 100 for 4 months = Rs. 103.

If Rs. 103 be the amount, the principal = Rs. 100.

$\therefore$ ... Re. 1 ..... = Rs. $\frac{100}{103}$

$\therefore$ ... Rs. 515 ..... = Rs. $\frac{100}{\not{1}\not{0}\not{3}} \times \overset{5}{\not{5}\not{1}\not{5}}$ = Rs. 500.

[Note 1. To find an algebraic expression for the interest and amount on a given sum at a given rate in a given time :—Let 'P' be the principal in Rupees, Rs. 'r' the interest on Rs. 100 for 1 year, 'n' the no. of years, Rs. 'I' the interest and Rs. 'A' the amount.

The int. on Rs. P for 1 year = $P \times \frac{r}{100}$ rupees.

$\therefore$ *The int. on Rs. P for n years* = $P \times n \times \frac{r}{100}$ rupees.

$\therefore$ $I = \frac{P \times n \times r}{100}$

and $A = P + I = P + \frac{Pnr}{100} = P\left\{1 + \frac{nr}{100}\right\}$.

If, of the four quantities I, P, n, r, or A, P, n, r, any three be given, the fourth could be found.

Also 'n' and 'r' being constant, I as well as A varies as P;

P and r being constant, I varies as 'n' but 'A' does not vary as 'n';

P and 'n' being constant, I varies as 'r' but A does not vary as 'r';

Any two of I, A and P being constant, 'n' varies inversely as 'r' and 'r' varies inversely as 'n.'

Note 2. In Ex. 1 worked above, the following method may also be followed :—

9 per cent. per year = $\frac{3}{4}$ per cent. per month.

Interest on Rs. 500 for 4 months = Interest on Rupees twenty-hundreds for one month.

$\therefore$ the interest required = Rs. $20 \times \frac{3}{4}$ or Rs. 15.

Note 3. When interest has to be calculated from one specified date to another, one only of the two dates specified, namely, either the date on which the loan was borrowed or that on which it was repaid is included in the calculation, *e g.*, it is 10 days from May 12 to May 22. If the period for calculation be given in months, take 1 year = 12 months. If it be given in days, take 1 year = 365 days (even if the 29th day of February in a leap year be included).

If it be in weeks, 1 year = 52 weeks. Unless otherwise stated, 'rate per cent.' means 'rate per annum.'

Results to the nearest pie or penny would suffice in the final answers to be given.]

EXERCISES.

1. Calculate the interest and amount in the following :—

(*a*) On Rs. 250 for 8 months at 12% per year.

(*b*) ,, Rs. 200 ,, 2 years ,, $7\frac{1}{2}$% ,,

(*c*) ,, £550 ,, $1\frac{1}{4}$,, ,, 9% ,,

(*d*) ,, Rs. 750 ,, 29 months ,, 8% ,,

(*e*) ,, £4,225 ,, $2\frac{1}{2}$ yrs. ,, $7\frac{1}{2}$% ,,

2. In the formulæ $I = \frac{Pnr}{100}$ and $A = P\left(1 + \frac{nr}{100}\right)$ express each quantity in terms of the rest.

3. Fill up the missing columns below :—

	P	n	r	I	A
(*a*)	—	—	12	Rs. 50	Rs. 450
(*b*)	—	$\frac{575}{365}$	15	Rs. 759	—
(*c*)	—	$2\frac{1}{2}$	10	—	Rs. 75
(*d*)	Rs 2,400	2	—	Rs. 160	—
(*e*)	Rs. 400	1·5	—	—	Rs. 448
(*f*)	—	—	10	Rs. 50	Rs. 600
(*g*)	Rs. 400	—	12	Rs. 72	—
(*h*)	Rs. 750	$1\frac{3}{4}$	8	—	—
(*i*)	—	$1\frac{1}{2}$	—	Rs. 60	Rs. 560
(*j*)	—	$1\frac{1}{2}$	10	Rs. 174	—
(*k*)	—	$\frac{5}{12}$	12	Rs. 34-12	—

4. Which pays a greater rate of interest :—2 pies per Re. per month, 6 pies per Rs 100 per day or 12 per cent. per annum ?

5. An unlet bungalow which could have been sold 6 months ago for Rs. 8,750 is now sold for Rs 9 000. What was the gain or loss incurred by postponing the sale ? (Assume money to fetch interest at the rate of 9 p. c per annum).

6. I bought a house for Rs. 2,000 and it gets me a rent of Rs. 13 per month. I also bought a garden for Rs. 10,000 and it fetches me Rs. 975 per annum. What rate of interest do I get from each ? Which pays a higher rate of interest ?

7. On the Dipavali eve a villager bought cloth worth Rs. 120 from a merchant. He paid Rs. 70 cash and promised to return the balance with 18 per cent. interest at his convenience. If he cleared his debt by a payment of Rs. 77, find when he rapaid the debt.

8. A borrows Rs. 300 from B at $4\frac{1}{4}$ p.c. interest. On the same day he spends Rs. 50 and lends out the balance at 15% int. When the amount on the sum he lent is equal to that on the sum he borrowed, he clears his debt. Find the time taken to repay the debt.

9. What must I now invest so that I may receive Rs. 500 ten months hence at 5% interest ?

10. Reckoning simple interest at $7\frac{1}{2}$ per cent. per annum, what sum paid now will discharge a debt of Rs. 1,620 due 10 months hence ?

11. A buys rice from B and promises to pay him Rs. 636 ten months hence. Two months after, A offers to settle accounts by a payment of money on that date. If a fair arrangement be made, how much may A expect to get ? (Interest in the market is at 9 %.)

12. A merchant bought salt for Rs. 530 with six months' credit and sold it on the same day for Rs. 618 on three months' credit. Find the gain % on present value assuming money to fetch interest at 12% per annum.

13. When I went to borrow Rs. 125 from a merchant he deducted the interest on it for two months at 2 ps. per rupee per month and paid me the balance. If I had to pay Rs. 125 at the end of the period, what rate of interest did he get for his money?

14. At what rate per cent. per annum simp. int. will a sum of money double itself in $8\frac{1}{3}$ yrs.

15. On 17-3-15 a retail dealer bought goods worth Rs. 620 and promised to pay Rs. 651 after a certain period. If simple interest is calculated at $8\frac{1}{3}$% per annum, find on what date the amount falls due.

16. A bookseller buys books whose nominal value is Rs. 250 but he has to pay only $\frac{3}{4}$ of the nominal price on Rs. 100 worth of them and $\frac{4}{5}$ of the nominal price on the remainder. He agrees to pay a sum of Rs. 200 at the end of 3 months. What rate of interest does he pay ? If he is able to realise at the end of six months a sum of Rs. 210 by the sale of books (the remaining books cannot be sold), what rate of interest does he get for his money ?

17. On the 28th of March last year Rs. 1,600 was deposited in a bank which allows 5% simple interest. On the 21st of August of the same year the depositor withdrew the principal and interest. What sum did he then get ?

18. A sum of Rs. 750 was borrowed on 14-12-1911 and repaid with simple interest at 10% on 12-3-1912. Find the amount repaid.

Ex. 5. Symmonds & Co. sell a motor-bike for Rs. 1,200 *cash*, or Rs. 250 *cash* and five monthly instalments of Rs. 200. What rate of interest does a man pay who buys the motor-bike on the instalment system?

The cash price of the motor-bike = Rs. 1,200 and the ready cash paid according to the instalment system = Rs. 250. Hence, instead of paying Rs. (1,200—250) or Rs. 950 *ready cash* the buyer has to pay Rs. 200 at the end of each 5 months.

∴ the amount that the company could get on Rs. 950 in 5 months may be assumed to be equal to the sum of the amounts which it could get on Rs. 200 in 4, 3, 2, 1 and '0' months.

∴ if 'r' be the rate % per annum of simple interest,

$$950\left(1+\frac{5}{12}\times\frac{r}{100}\right)=200\left\{\begin{array}{l}1+\frac{4}{12}\times\frac{r}{100}\\ +1+\frac{3}{12}\times\frac{r}{100}\\ +1+\frac{2}{12}\times\frac{r}{100}\\ +1+\frac{1}{12}\times\frac{r}{100}\end{array}\right.$$

$$\therefore 950+\frac{95r}{24}=1000+\frac{20r}{12}. \quad \therefore \frac{11r}{24}=10$$

$$\therefore r=\frac{240}{11}=21\tfrac{9}{11}$$

∴ the rate of interest paid = $21\frac{9}{11}$ % per annum.

19. The subscription of a member of the Indian Mathematical Society is either Rs. 150 in a lump or Rs. 15 per year. Allowing simple interest at 6% per annum, show that if a member continues for 12 years or less he loses by paying in a lump instead of by instalments, while if he continues for 13 years or more he gains by doing so.

20. In a certain society the subscription for membership is Rs. 50 per year or Rs. 900 for life. If a member continues for 26 years he finds it just as good to pay in a lump as to pay every year. Find the rate of interest for money.

If the subscription be Rs. 48 per year or Rs. 900 for life and the number of years 24, show that the interest should be $3\frac{1}{2}$ %.

21. A Life Insurance Company promises to pay you Rs. 1,000 at the end of 19 years if you pay Rs 39 1 a. per year. What rate of interest (simple) will this bring?

22. A house-building agency gets a house built for me for Rs. 3,807 which is to be repaid in three equal annual instalments, the first to be paid at the end of the first year. Calculating simple interest at $5\frac{3}{4}$ %, find the amount of each instalment.

If the amount of each instalment be Rs. 1,457, find the rate of interest.

23. The Post-Office five-year cash certificates were issued on the following terms :—

For Rs. 10, Rs. 20, Rs. 50, Rs. 100 and Rs. 500, payable 5 years after issue, issue price = Rs. 7-12 as., Rs. 15-8 as., Rs. 38-12 as., Rs 77-8 as, and Rs. 387-3 as. What rate of interest is thereby allowed for money invested?

24. A bicycle may be bought either for Rs. 250 cash or for Rs. 50 cash and 21 monthly instalments of Rs. 11 each. What rate of interest per annum does this bring?

25. In 1916 March I was offered Rs. 1,750 for my house. But hoping to sell it at a higher rate after the railway through our town began to work, I waited for 2 years and as no railway seemed likely to work, I sold my house only for Rs. 1,700. If I had been getting a rent of Rs 125 per annum, (payable at the beginning of each year) what rate of interest *on an average* does this amount to on the price of my house in 1916?

CHAPTER XI.

CONTRACTED METHODS.

58. SIGNIFICANT FIGURES.

We express the results of our measurements and calculations in terms of some chosen unit. The unit varies with circumstances. In estimating incomes, we say, for example, that a labourer gets Rs. 9, a pleader Rs. 500 and a merchant Rs 8,000. Here the units chosen are Re. 1, Rs. 100 and Rs. 1000 and the incomes are expressed to the nearest unit though the actual earnings may be a little more or less. In so expressing, it is only some figures that state how many times a chosen unit is contained in a result, while the rest do not *matter*, being considered negligible when 'compared with the unit.' The former are called 'significant figures.' The following would illustrate the use of the term further :—

(i) When we estimate the population of a District to be 3,000,000 (3 millions) we imply thereby that we use a million as the unit and express the population *to the nearest million* though it may be in fact 3,189,821. In so estimating, the figure '3' in the millions' place is alone considered '*significant*' and the rest are not so considered, being negligible when compared with a million. If a greater degree of approximation to the real population be required, we use a smaller unit, say 100,000 and state the population to be 32,00,000 (32 hundred-thousands) *to the nearest* 100,000. In this case, the two figures 2, 3 are alone '*significant*.' And of these two, the figure of the higher place-value is the first significant figure and that of the next lower place-value is the second significant figure.

(ii) When a man's property is estimated to be worth Rs. 6,00,000 it is not meant that the value reckoned *to the nearest rupee* is Rs. 6,00,000. The convenient *standard* adopted for estimating is Rs. 1,00,000 and the value is expressed *to the nearest Rs. 1,00,000*. Here, there is only one '*significant*' figure '6' which indicates 6 lakhs of rupees. The valuation may amount to Rs. 6,07,945 *to the nearest rupee*, where all the six figures, 6, 0, 7, 9, 4, 5 are *significant*. If instead of a rupee we take Rs. 1,000 as the unit, the value could be expressed to be Rs. 608,000 (608 thousands) in three significant figures 6, 0, 8 *to the nearest Rs. 1,000*. Here *the cipher* in 608 is *significant* as it defines the place-values of 6 and 8 and enables the whole sum being expressed in terms of the assumed unit, whereas the last three ciphers in 608,000 are *not significant* as they are used only to indicate that the assumed unit is Rs. 1,000.

So, to express a given quantity approximately to a required number of significant figures we must first ascertain in terms of what unit the result would thereby have to be expressed and then find the number of times that unit is contained (approximately) in the quantity given. The latter result would express the quantity correct to the required number of significant figures. Thus when we say that a plot of land contains ·58 of an acre it is not meant that the *exact* area is so much. We only mean that the area is expressed approximately in two significant figures 5, 8 with ·01 of an acre as the unit of estimate. If instead, we adopt ·1 of an acre as the unit, the area may be expressed as ·6 of an acre, correct to one significant figure only.

[Note that in ·0034 there are only two significant figures 3, 4; in ·0304 and ·0340 there are three significant figures; and in ·3004 and ·3400 there are four significant figures.]

EXERCISES.

1. Find, correct to two significant figures, the values of the following and state the unit chosen in each case :—

2·025 lbs. ; 423·67 lbs. ; £·0321 ; Rs. 60·46 : ·4765 cwt. : 48235 sq. miles : and 3987 gallons.

2. The value of A's property is Rs. 864.818 and that of B's Rs. 786.466 to the nearest rupee Express the values correct to 4. 3, 2 significant figures respectively.

3. The population of a district is given to be 2,040,786, Express the population correct to 2, 3, 4, 5, 6 significant figures.

4. The distance round the earth is given to be 24899 miles, to the nearest mile, Express the distance correct to 1, 2, 3 significant figures.

5. 1 gram = ·0352736 oz. Express the equivalent of 1 gram in oz. correct to one significant figure.

6. 1 cubic centimetre = ·001759 pint. Express 1 cub. cm. in pints correct to one significant figure.

59. CONTRACTED METHODS.

The results of our calculations need not in practice be expressed to a large number of decimal places. For example, in finding the area of a rectangle to be 15·4518 sq. ins., the figures beyond the second decimal place are usually negligible. (Why ?) In expressing the value of a sum of money in decimals of a Re. or £, the figures beyond the third decimal place are superfluous as the value correct to three decimal places would give a result to the nearest pie or penny. (Refer to Ex. 21, page 35). How calculations may be made to any required degree of approximation without superfluous working is shown in the contracted methods worked out below :—

(*a*) Contracted Addition and Subtraction.

Ex. 1. Find the sum of

·6533596,
·833348,
·63332846,
·7485851

correct to three places ofdecimals.

Express each as a decimal correct to four decimal places and work as follows:—

·653	4	The sum = 2·869, corrrct to three
·833	3	places of decimals.
·633	3	Verify by working out in full.]
·748	6	[Note that, in making such approxi-
2·868	6	mations in respect of each decimal, errors in execess and in defect *generally*

cancel each other when the decimals are added.]

A similar method may be followed in subtraction.

Ex. 2. Find to the nearest million the difference between 1,567,845,438 and 949,275,899.

1,567	8	The difference = 619,000,000
949	3	to the nearest million.
618	5	Verify by working out in full.

(*b*) Contracted Multiplication.

Ex. 3. Find the product of ·065382974 × ·1896482, correct to two places of decimals.

The product required = 0·65382974 × 1·896482, by expressing the multiplier *in its standard form*.

And its rough estimate = ·7 × 2 or 1·4. By the rule for approximations, the partial products may be found correct to three places of decimals in order that their sum may be correct to the second decimal place. Hence

multiplication may be begun with digits marked with like letters :—

d c b	*a*
0 ·6 5	3 [8 2 9 7 4]
	a b c d
	1 ·8 9 6 [482]

·6 5	4* (4* ∴ 1 × ·0008 gives approximately. carry
·5 2	2* ·001. 1 × ·003 = ·003 — ·001 = ·004).
·0 5	9 (2* ∴ 8 × 3 = 24. carry 2, 8 × 5 = 40,
·0 0	4 + 2 = 42).
1 ·2 3	9 (The digit of the lowest place value in each partial product is obtained similarly).

∴ the product = 1·24, correct to two places of decimals.

[The same result would be obtained if the multiplication be worked out in full by the ordinary method. Note the superfluous digits in each step in the above working].

Ex. 4. Multiply 8004·002 by ·0426997 correct to four significant figures.

The product reqd. = 8·004002 × 42·6997 = 8 × 40 or 320, roughly.

∴ the digit of the highest place-value in the product should be in the hundreds' place and hence the result correct to four significant figures is, in this particular case, the same as that to the nearest tenth.

d c b	*a*
4 2 ·6	9 [97]
	a b c d
	8 ·0 0 4 [002]

3 4 1 ·5	9
·1	7 The product = 341·8 correct to four
3 4 1 ·7	6 significant figures.

[Note that 8·004002 is chosen as the multiplier instead of 42·6997 as it contains *fewer effective* digits (*viz.*, 8, 4 and 2) and so would lead to fewer partial products.]

EXERCISES.

7. Find the products in the following :—
   (*a*) 129·94121 × 17·956 and ·6544 × 88·128 correct to two places of decimals;
   (*b*) 1888·46826 × 55·557 and 1777·04 × ·0042 to the nearest tenth:
   (*c*) 1·41421 × 1·41421 and 1·732 × 1 732 to the nearest unit,
   (*d*) 15·684 × 78·44 and 1568·6765 × 78·44, 106871 × 9982 and ·014825 × 16·2345 correct to four significant figures.

8. If $a = ·8192$, $b = ·9659$. $c = ·5736$, $d = ·2588$. find correct to two significant figures the values of $ab - cd$, $ab + cd$, $ab + bc$, $bc - ad$, $ac + db$ and $ac - db$.

9. If $x = ·7547$ and $y = ·5561$, find to the nearest tenth the value of $x^2 + y^2$.

10. Find the values of the following to the nearest ten thousandth :—

(*a*) $\frac{1}{8} + \frac{1}{8^2} + \frac{1}{8^3} + \frac{1}{8^4}$ ..... etc.

(*b*) $1 + \frac{1}{2} + \frac{1}{2 \times 3} + \frac{1}{2 \times 3 \times 4} +$ ..... etc.

(*c*) $\frac{1}{2} + \frac{1}{3 \times 2^3} + \frac{1}{5 \times 2^5} + \frac{1}{7 \times 2^7} + \frac{1}{9 \times 2^8} +$ ... etc.

(*d*) $\frac{2}{3} + \frac{3}{40} + \frac{4}{500} + \frac{5}{6600} + \frac{7}{70000} +$ ....., etc.

11. If 1 metre = 3·2809 ft., express a sq. metre to the nearest hundredth of a sq. ft. Also, express a speed of 3533249 cm. per hour as one of feet per second to the nearest tenth.

12. The radius of the earth is 3249·691 miles. The distance of the sun from the earth is 23333 times the radius of the earth. Find, correct to four significant figures, the distance in miles between the sun and the earth.

13. The ancients calculated only 365 days for the year instead of 365·242216 days. Find to the nearest day what the error comes to in 3845 years.

14. A gallon of water contains 277·274 cub. ins. and a cubic inch of water weighs 252·458 grains. If 1 lb. = 7000 grains, find correct to two significant figures the weight in lbs. of a gallon of water.

15. The area of an ellipse $= \pi \times a \times b$ sq. metres. If $\pi = 3{\cdot}14159625$, $a = 3963{\cdot}296$ and $b = 1000$ find the area of the ellipse in sq. metres correct to two significant figures.

16. The population of British India was 294,361,056 in 1901 and ·62 per mile of the population were successfully vaccinated during the year. Find their number to the nearest 10,000.

17. Taking Re 1 = £·0556, express the value of Rs. 998·875 in £ *s*. *d*. to the nearest penny.

18. If 1 ton = 27 222 maunds express 18,444,655 tons in maunds correct to four significant figures.

19. A firm finds that it is charged customs duty at Re. ·45626 per lb. of tea imported into British India If it imported $7{\cdot}19876 \times 10^6$ lbs. of tea during a year, find, to the nearest Rs. 100, the amount of customs duty it paid during the year.

20. The area of the entire surface of the globe $= 196{\cdot}788442 \times 10^6$ sq. miles. For every 10000 sq. miles of the area there are 2436·55 sq. miles of land. If the rest be covered by water, find, correct to four significant figures, the area in sq. miles covered by water.

21. If I invest Rs. 100 in a bank now, it would fetch me Rs. 124·2296875 at the end of three years. Find, to the nearest pie, what I may expect to get three years hence if I invest Rs. 976·75 in it now.

22. The average daily expenditure incurred by a country in waging a war was £451724. Find, to the nearest million pounds, the expenditure to the country on behalf of the war if it lasted from 4-8-14 to 13-10-18, both days inclusive.

(*c*) Contracted Division.

Consider the following process of long division :—
Divide 14285789 by 7426.

The quotient $= \frac{14285{\cdot}789}{7{\cdot}426} = \frac{14000}{7}$ or 2000. $\therefore$ the digit of the highest place-value in the quotient should be in the thousands' place. A special feature of the following working is that instead of bringing down a new figure from the dividend at each step the digit of the lowest place-value is rejected one after another in the divisor.

1923 = Quotient

7426) 14285*789*

7426

742 [6]) 6860 [o $\because$ 5 in the dividend is corrected into 6.]

6683 [3 $\because$ 9 × 6 gives carry 5; 9 × 2 = 18, + 5 = 23.]

74 [2]) 177

148

7 [4]) 29

22

7

A further step in division is not possible as '7' will have to be rejected in the next divisor to be written and there is no digit to the left of it in the divisor. Hence the quotient could be worked out only to as many figures as there are in the divisor. Therefore, if the quotient has to be found correct to 'n' significant figures, it should be worked out as far as the $(n+1)^{\text{th}}$ significant figure and in the divisor there should be $(n+1)$ figures from the left. To obtain them, either the superfluous figures to the right may be rejected in the divisor or the necessary figures made up by adding '0s' to the right without altering the value of the divisor as in Note 2, page 25. Also, it would be enough if as many digits be kept in the dividend as are necessary for the first step in division.

Ex. 5. Divide 712·49684 by 504·4687 and find the quotient correct to two places of decimals.

$\frac{712\cdot49684}{504\cdot4687} = \frac{7\cdot1249684}{5\cdot044687}$, by expressing the divisor *in its standard form* $= \frac{7\cdot0}{5}$ or 1·4, roughly.

Hence, to get the quotient correct to two places of decimals it must be worked out as far as the fourth significant figure. Also, the digit of the higher place-value in the quotient is in the units' place.

Detailed working :—

```
                1·412 = Quotient
5·044[687])7·1249684
           5·045(a)
5·04[4]    )2·080(b)
           )2·018(c)
5·0[4]     ) ·062
           ) ·050
5·[0]      ) ·012
             ·010
             ·002
```

(*a*) '5' ∵ ·0006 × 1 gives carry ·0001 ; ·004 × 1 = ·004, + ·001 = ·005.

(*b*) '0' ∵ ·124 in the dividend is corrected into ·125.

(*c*) '8' ∵ ·004 × ·4 gives carry ·002 ; ·04 × ·4 = ·016, + ·002 = ·018.

Contracted working by the Italian method :—

```
                  p q r s
s r q p           1·4 1 2
5·044[687 ])7· 1 2 4 [9684]
            2· 0 8 0
                 6 2
                 1 2
                   2
```

∴ the quotient = 1·41 correct to two places of decimals.

[NOTE:—Figures marked with like letters *p*, *q*, *r*, *s* in the quotient and in the divisor are multiplied together to get the digit of the lowest place-value in the product at each step].

[After the quotient is worked out as far as 1·41, it could be corrected to two places of decimals as follows:—The divisor at the next stage is 5 and the dividend is 1 hundredth. Since 1 is less than half of 5 the remaining portion of the quotient would be less than half a hundredth. Hence the quotient may be determined to be 1·41 to the nearest hundredth, without working out a further step in division.]

Ex. 6. The distance between the sun and the earth $= 14{\cdot}849 \times 10^7$ kilometres. If light travels at $29{\cdot}7854 \times 10^4$ kilometres per second, find to the nearest second the time taken by sunlight to reach us.

The time in seconds is given by $\frac{14{\cdot}849 \times 10^7}{29{\cdot}7854 \times 10^4}$

i.e., by $\frac{1484{\cdot}9}{2{\cdot}97854}$ (how ?) and is hence roughly equal to $\frac{1500}{3}$ or 500.

$$
\begin{array}{r}
498{\cdot}6 \\
2{\cdot}978\overline{[54]}\,)1484{\cdot}9 \\
\underline{1191{\cdot}4} \\
293{\cdot}5 \\
\underline{268{\cdot}0} \\
25{\cdot}5 \\
\underline{23{\cdot}8} \\
1{\cdot}7 \\
\underline{1{\cdot}7}
\end{array}
$$

∴ the time reqd. = 499 seconds, to the nearest second.

EXERCISES.

23. Find the quotients required below:—

8187465·74 ÷ 10421·38, to the nearest unit; 765·4784 ÷ 869·765 as far as the third decimal figure; 2 ÷ 1·4142, 3 ÷ 1·732, 1 ÷ 3·14159625 correct to four significant figures: ·184842 ÷ ·0123 correct to two decimal places; and 12·146 ÷ 11·16176 correct to five significant figures.

24. In a year 842,798 tons of sugar valued at £9,998,658 were imported into India. Find, to the nearest penny, the value per cwt. of the sugar imported.

25. In a certain year the total agricultural produce of a country amounted to Rs. 826,566,888 and the portion consumed by its people to Rs. 620,404,666. Find to the neareast tenth, what per cent. of the total produce was consumed by the people.

26. Assuming $p = 3{\cdot}14159$ and $e = 2{\cdot}7182818285$, find correct to three significant figures the values of :—$\frac{1}{p}$, $\frac{1}{e}$, $p + \frac{1}{p}$, $e + \frac{1}{e}$, $\left(p - \frac{1}{p}\right)$ $\left(e - \frac{1}{e}\right)$, $e^2 + p^2$, $(p + e)$ $(p - e)$ and $\frac{1}{2}\left(p - \frac{1}{p}\right)$.

27. If $x = y \div 2{\cdot}30258509$, find, correct to five places of decimals, by what 'y' may be multiplied to get 'x'. If $y = {\cdot}69314718$, find the value of 'x' correct to four significant figures. Also, find correct to four significant figures the values of y when $x = {\cdot}845093$, ${\cdot}071882$ and ${\cdot}0253059$.

28. In a district with a population of 1,200,468 the number of births was found to be 15496 during a certain year. Find to the nearest hundredth the birth-rate per 1000 of the population.

29. A bar of silver weighs 20·01 oz. (Troy) when weighed in air and 18·1054 oz. when weighed in water. Find correct to three significant figures the value of the ratio of the weight of the bar in air to its loss of weight in water.

30. The present area under wheat cultivation in our land is 4,987,636 acres and the value of the annual produce is estimated at Rs. 74,987,666 exclusive of the money spent in cultivation. Find to the nearest Re. ·001 the probable average annual produce of wheat per acre.

31. Of £245,824,333 sterling of British products exported abroad in 1894, India was a customer for goods to the value of £29,300,069. Find correct to two significant figures what per cent. India bought of the total British products exported.

32. The supply of petroleum in Burma was 8,698,495 gallons in 1892 and 10,276,435 gallons in 1893. Find to the nearest tenth the increase per cent. in the supply during the latter year when compared with that in the former.

33. A province has an area of 141,189 sq. miles and contains 35,630,440 inhabitants. If its annual revenue be Rs. 163,142,417 and its annual expenditure Rs. 83,998,974, find (1) its average population to the sq. mile, (2) the average expenditure per head of population, (3) the percentage of revenue spent, and (4) the average revenue per sq. mile. [Answer (1) and (3) fo the nearest unit, (2) to the nearest pie, and (4) to the nearest rupee.]

34. Mercury takes 87·969 days and Neptune 6012672 days to travel once round the sun. How many complete revolutions will the former make when the latter makes 5 revolutions.

35. The earth takes 365·2564 days in making a complete revolution of 588,825,756 miles round the sun. Find correct to two significant figures the rate in miles per minute at which the earth travels on the average.

Ex. 7. Find the value of—

·015876584 × ·1727857 × 564·347 correct to two places of decimals.

Expressing each multiplier in its standard form, the product = ·15876584 × 1·727857 × 5·64347 = ·2 × 2 × 6 or 2·4 roughly.

Since the final product should be found to three places of decimals, the product of the first two factors should be found to four places of decimals and the working is therefore as follows :—

$$
\begin{array}{r}
0{\cdot}1587\ [\overline{6584}] \\
1\ 7278\ [\overline{57}] \\
\hline
1{\cdot}588 \\
{\cdot}1111 \\
{\cdot}0032 \\
{\cdot}0011 \\
{\cdot}0001 \\
\hline
1{\cdot}274\ [\overline{3}] \\
5{\cdot}643\ [\overline{47}] \\
\hline
1{\cdot}372 \\
{\cdot}164 \\
{\cdot}011 \\
{\cdot}001 \\
\hline
1{\cdot}548
\end{array}
$$

$\therefore$ the product = 1·55 correct to two places of decimals.

Similarly if there be three multipliers in the product, each expressed in standard form and if the result be required correct to two places of decimals, there must be three places of decimals in the final product and the product of the first three factors should be worked to four places of decimals and that of the first two to five places of decimals.

[NOTE:—If the value of $1848 \times 78{\cdot}486 \times 298498$ has to be found correct to three significant figures, the following method may be adopted :—

$1848 \times 78{\cdot}486 \times 298498$

$= 1{\cdot}848 \times 7{\cdot}8486 \times 2{\cdot}98498 \times 10^9$, by expressing each factor in its standard form.

Since $1{\cdot}848 \times 7{\cdot}8486 \times 2{\cdot}98498$ is roughly equal to $2 \times 8 \times 3$ or 48, the value of the product required, correct to three significant figures, may be obtained by first finding out the value of $1{\cdot}848 \times 7{\cdot}8486 \times 2{\cdot}98498$ to the nearest tenth and then multiplying the result by 10^9.]

Ex. 8. Find the value of $\frac{0{\cdot}76543 \times 0{\cdot}8976367}{0{\cdot}496487}$, correct to two places of decimals.

The value required $= \frac{{\cdot}76543 \times 8{\cdot}976367}{4{\cdot}96487}$

$= \frac{{\cdot}8 \times 9}{5}$ or 1·4 roughly.

∴ to get the value correct to two places of decimals (*i.e.*) correct to three significant figures, four figures should be retained in the divisor and hence four or five figures should be retained in the dividend. Hence the value of the dividend should be found correct to five significant figures. Since the dividend is roughly equal to ·8 × 9 or 7·2, it must be worked correct to four places of decimals, (Why ?). Hence the following working :—

$$
\begin{array}{rl}
 & 0{\cdot}7654\ [\underline{3}] \\
 & \quad 8{\cdot}9763\ [\overline{67}] \\
\hline
 & 6{\cdot}1234 \\
 & \cdot6889 \\
 & \cdot0536 \\
 & \cdot0046 \\
 & \cdot0002 \\
\hline
4{\cdot}964\ [\overline{87}] &)6{\cdot}8707(1{\cdot}384 \\
 & \underline{4\ 965} \\
 & 1{\cdot}906 \\
 & \underline{1{\cdot}489} \\
 & \cdot417 \\
 & \underline{\cdot397} \\
 & \cdot020 \\
 & \underline{.020} \\
\end{array}
$$

∴ the value reqd. = 1·38, correct to two places of decimals.

EXERCISES.

36. Find the values of the following correct to three significant figures:—

$39{\cdot}76857 \times 0{\cdot}643781 \times 7{\cdot}65879122.$

$45{\cdot}4684 \times {\cdot}000463274 \times 286{\cdot}14237.$

$\cdot 002040505 \times 7789{\cdot}425 \times 5{\cdot}2468.$

$1758 \times 54566 \times 8987.$

$14{\cdot}001218 \times {\cdot}007637 \times 4{\cdot}8912.$

$$\frac{28{\cdot}66461 \times 17{\cdot}5432}{19{\cdot}6547}. \qquad \frac{21{\cdot}6517}{8{\cdot}18967 \times 7{\cdot}65461}.$$

$$\frac{217{\cdot}6517 \times 14{\cdot}32}{56{\cdot}528 \times 226{\cdot}6606} \text{ and } \frac{{\cdot}4146744 \times 88{\cdot}12564}{3{\cdot}142857 \times 7{\cdot}42582}.$$

37. Find the value of 'x' in the following, correct to three places of decimals:—

$$\frac{x}{1{\cdot}41421} = 1{\cdot}765 \times 4{\cdot}4968:$$

$$x \times {\cdot}5566 = 1{\cdot}\dot{7}\dot{2} \times 5{\cdot}44\dot{3}:$$

$$\frac{1}{x} - \frac{1}{1{\cdot}\dot{4}\dot{2}} = 1{\cdot}\dot{4}1\dot{4}:$$

$$\frac{x}{1{\cdot}41421} \times 2{\cdot}41421 = 14{\cdot}\dot{2}8527\dot{1}:$$

$$\text{and } \frac{x}{1{\cdot}732} \times 1{\cdot}4142\dot{1} = 4{\cdot}\dot{6}20\dot{4}.$$

38. Express, correct to two significant figures:—

(*a*) a velocity of 11000 ft. in 1297 seconds into one of miles per hour.

(*b*) a pressure of 50 lbs. per 7289 sq. inches into one of tons per sq. mile.

39. If $\frac{1}{M} = \frac{1}{S} + \frac{1}{Y}$, find correct to two places of decimals the value of 'M' when $S = 29{\cdot}5305887$ and $Y = 365{\cdot}2564$.

40. Given that 1 ft. = 30·48 cm. and 1 lb. = 453·6 grams, express 1 cm. in ft. and 1 gram in lbs., correct to three significant figures. Also, express a pressure of 15 lbs. to the sq. inch into one of kilograms per sq. metre.

41. If a barometer reads x inches at a place, the height of the place above sea-level is roughly equal to $\frac{30-x}{30+x} \times 52428{\cdot}445$ ft. If a barometer reads 24·6 inches on the summit of a hill, find to the nearest yard its height above sea level. What would be the reading in the barometer in a balloon when it is just 3·875 miles above sea-level?

42. The distance from pole to pole along a meridian is 20,014,800 metres If the distance be divided into 180 equal degrees, express a degree in English miles correct to two significant figures. (Assume a metre = 39·37079 inches).

43. The population of India was 253,896,330 in 1881, and 315,132,537 in 1911. If from 1911 to 1941 the population may be expected to grow at half the rate as from 1881 to 1911, find to the nearest hundred-thousand what the population would be in 1941.

44. Owing to the manufacture of synthetic indigo in Germany the area under indigo cultivation in India has fallen from 1,249,766 acres to 131 725 acres during the past twenty years. If the same rate of diminution in the cultivated area should continue, find correct to three significant figures how many acres would be under indigo cultivation twenty years hence.

45. A metre is supposed to be ten-millionth part of the distance from the North Pole to the Equator, whereas, in fact, it is only $\frac{1}{10,007,400}$ of the above distance Find correct to two significant figures by what per cent. the present standard length of the metre falls short of the length which would make the usual supposition correct.

46. A fathom $= \frac{1}{1000}$ of a nautical mile and 60 nautical miles = 69·165667 ordinary miles. Express a fathom in feet correct to two places of decimals.

47. Taking the earth's distance from the sun as unity, Mercury's distance from the sun may be represented by ·38268 and the distance of Venus by ·70711. If Venus be 6576×10^4 miles from the sun, find to the nearest 10^5 miles the distance of Mercury from the sun.

48. From the formula $V = \frac{H \times B}{6}(P^2 + Q^2 + R^2)$, find to the nearest unit the value of 'V' when H = 5, B = 3·1416, P = 4·3456, Q = 6·54107 and R = 3·2456.

49. If the radius of the earth = $6{\cdot}37 \times 10^8$ cm. and that of the moon = $1{\cdot}74 \times 13^8$ cm, find correct to three significant figures their surface areas in sq. metres. (Assume that the surface area of each = 4 × 3·14159325 × the sq. of its radius.)

50. The star Sirius is $1{\cdot}375 \times 10^6 \times 924{\cdot}67000$ miles from the earth. If light travels at $365{\cdot}256 \times 158976 \times 10^6$ miles in a year, find to the nearest year the time taken by light to reach us from the star.

51. When the height of the mercurial column in a barometer is 'x' inches in a certain place, the pressure of the atmosphere per sq. foot = $\frac{x}{12} \times 13\,596 \times 62{\cdot}28125$ lbs. Find to the nearest hundredth of a lb. the pressure of the atmosphere per sq. inch when the height of the mercurial column is 29·8 inches in the same place.

52. If a rectangle is 4·56″ by 2·45″, the measurements being taken to the nearest ·01″ show that its perimeter should lie between 2 (4·555 + 2·545)″ and 2 (4·565 + 2·545)″ and its area between 4·555 × 2·535 sq. ins and 4·565 × 2·545 sq ins Hence find to any reliable degree of accuracy its perimeter and area.

53. If in a triangle ABC, AB = 2·54″. ∠A = 59° an ∠B = 49°, the relation between the sides is given by

$$\frac{AB}{\cdot 9511} = \frac{AC}{\cdot 7547} = \frac{BC}{\cdot 8572}$$

Find to the nearest ·01″ the lengths AC, BC. Also, if the area of the triangle be given by ½ AB × AC × ·8572, find it to the nearest ·01 sq. inch.

If 'p' radians = 180 degrees, where 'p' = 3·1416, express correct to four significant figures the size of each angle in the above triangle in radians.

54. If $\frac{l}{g} = \frac{t^2}{p^2}$, find '$g$' correct to three significant figures when $l = 218{\cdot}6$, $p = 3{\cdot}1416$, $t = 1{\cdot}483445$.

55. A boy finds approximately that the length and breadth of a room are 3·17 ft. and 12·6 ft. while the real dimensions are 13·69 ft. and 12·64 ft. Express the error in each measurement as a percentage of the actual lengths. (Answer correct to two significant figures). By how much will the areas according to his calculation fall short of, or be in excess of, the true area?

3·17 : 12·6 :: 13·69 and 12·64

$\frac{3{\cdot}17}{12{\cdot}64} \times \frac{100}{13{\cdot}69}$ = 2540 = 6·3

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CHAPTER XII.

COMPOUND INTEREST.

60. Suppose A borrows Rs. 1,000 from B at 10% interest per year. Then at the end of a year A owes B Rs. 1,000 + an interest of Rs. 100 for the year. If A does not then pay even the yearly interest it is usually compounded with, or added on to, the original principal. Thus for the second year the principal will be considered to be Rs. 1,100. At the end of the second year A owes B Rs. 1,100 + an interest of Rs. 110 for the year. If A clears the debt then, he may have to pay Rs. 1,210. Since the original principal = Rs. 1,000, and the amount at the end of two years = Rs. 1,210, *the interest for the period* = Rs. 210. This interest is called **compound interest.**

[Note that the simple interest for the period is Rs. 200 only. Why is there an excess of Rs. 10?]

Ex. 1. A invests Rs. 2,000 in a Bank which pays 6% interest per annum. If interest be added on to the principal yearly, find the amount standing in his name at the end of two years.

Rs.	2000	= Principal for year I.
„	120·00	= Interest „ „
„	2120	= Principal „ II.
„	127·20	= Interest „ „
„	2247·2	= Amount at the end of two years
		= Rs. 2,247-3-2 (to the nearest pie).

[NOTE.—In the above decimal method, multiplication by 6 and division by 100 are done in a single line by writing each digit in the product got by multiplying by 6 two places to the right of where it ought to be written otherwise.]

If the time contains a fraction of a year, *e.g.*, $1\frac{1}{2}$ or $1\frac{3}{4}$ years, simple interest is usually calculated for the fractional part. Thus, the amount on Rs. 2,000 for $1\frac{3}{4}$ years in the above example = Rs. 2,120 + interest on Rs. 2,120 for $\frac{3}{4}$ of a year at 6% per annum = Rs. 2,120 + Rs. $2120 \times \frac{3}{4} \times \frac{6}{100}$ or Rs. 2215·4.

At times interest may be payable more than once a year, *e.g.*, half-yearly, quarterly, etc. If interest at 6% per annum be payable half-yearly the amount at compound interest on Rs. 2,000 for two years may be calculated as follows for four periods at 3% per period :—

Rs.	2000	= Principal	for Period	I.
,,	60·00	= Interest	,,	,,
,,	2060	= Principal	,,	II.
,,	61·80	= Interest	,,	,,
,,	2121·8	= Principal	,,	III.
,,	63·654	= Interest	,,	,,
,,	2185·454	= Principal	,,	IV.
,,	65·5636	= Interest	,,	,,
,,	2251·0176	= Amount after four periods		

= Rs. 2,251-0 as 3 ps. (to the nearest pie).

[NOTE 1. In the actual working, approximations are effected in the fourth decimal place, (*e.g.*, in writing 65·5635 instead of 65·56362 as interest in rupees for period IV above) so that the final result may be expressed correct to the third decimal figure to give the amount to the nearest pie or penny. (Refer to Art. 59, Chapter XI.)

NOTE 2. When interest at 6% per annum is payable half-yearly, Rs. 2,000 would *actually* fetch in one year an interest of Rs. 121-8, *i.e.*, of 6·09%.].

61. The amount at compound interest on a sum of money for a given period may be found by means of a formula :—

Let Rs. 'P' amount to Rs. 'A' in 'n' years at 'r' % per annum compound interest.

Also, let Re. 1 amount to Rs. 'R' in 1 year.

i.e., let $R = \left(1 + \frac{r}{100}\right)$.

Since Rs. 'P' amounts to Rs. 'P × R' in one year, Rs. 'P × R' is the principal for the second year.

∴ the amount on Rs. 'P' in two years

= Rs. 'P × R × R' or 'PR^2';

and that at the end of 'n' years = Rs. $P \times R^n$.

$$\therefore A = P \times R^n = P\left(1 + \frac{r}{100}\right)^n.$$

∴ the compound interest on Rs. P for 'n' years

= Rs. $(P \times R^n - P)$ or $P(R^n - 1)$.

Thus, if $P = 1500$, $n = 2$ and $r = 6$, $A = P \times R^2$

$= 1500 \times (1\cdot06)^2$ or $1500 \times 1\cdot1236$,

∴ Rs. 1,500 would in two years at 6% per annum (compound interest) amount to Rs. 1,685-6 as. 5 ps. (to the nearest pie).

Ex. 2. I bought a house from Sankaran who wanted Rs. 1500 for it. I paid him only Rs. 500 cash and remitted to him Rs. 200 annually from the date of the first payment. Calculate how much I owe him at the beginning of the third year. (Reckon compound interest at 9 % per year).

	Rs. 1000	= the amount I owe at the beginning of the first year.
	,, 90	= Interest for the year.
	,, 1090	= Amount due at the end of the year.
Deduct	,, 200	= the sum that I pay.
	,, 890	= the amount I owe at the beginning of the second year.
	,, 80·1	= Interest for the year.
	,, 970·1	= Amount due at the end of the second year.
Deduct	,, 200	= the sum that I pay.
	,, 770·1	= the amount I owe at the beginning of the third year.

∴ I owe him Rs. 770 1 a. 7 ps. at the beginning of the third year.

[Note that out of Rs 200 I pay at the end of the first year, Rs. 90 forms the interest due for the year and Rs. 110 is part of my original debt. Similarly, out of Rs. 200 I pay at the end of the 2nd year, Rs. 80·1 forms the interest and Rs. 119·9 forms part of my original debt.]

Ex. 3. Reckoning interest at 9 % per annum, payable yearly, what must a man now invest in a bank in order that he may be eligible to receive Rs. 1500 at the end of each of three years from to-day?

First Method:—

Suppose an investment of Rs. 'P' now made fetches Rs. 1500 at the end of each of three years from to-day.

Since Re. 1 now invested would amount to Rs. 1·09, Rs. $(1·09)^2$, Rs. $(1·09)^3$, at the end of one, two and three

years respectively, a payment of Rs. 1500 at the end of one, two and three years would fetch Rs. 1500 × $(1{\cdot}09)^2$ + Rs. 1500 × 1·09 + Rs. 1500 at the end of the third year, and Rs. 'P' now invested would fetch Rs. P × $(1{\cdot}09)^3$ in three years.

$$\therefore P \times (1{\cdot}09)^3 = 1500 \times (1{\cdot}09)^2 + 1500 \times (1{\cdot}09) + 1500 = 1500 \times \{(1{\cdot}09)^2 + (1{\cdot}09) + 1\}.$$

$$\therefore P = \frac{1500}{(1{\cdot}09)^3} \times \{(1{\cdot}09)^2 + (1{\cdot}09) + 1\}$$

$$= 3797, \text{ to the nearest Re.}$$

∴ the value of the investment reqd. = Rs. 3797, to the nearest Re.

[NOTE 1. A loan of Rs. 3797 at 9% per annum compound interest could be cleared by a payment of Rs. 1500 at the end of each of three years.

Note 2. The value of $(1{\cdot}09)^2 + (1{\cdot}09) + 1$ = that of $\frac{(1{\cdot}09)^3 - 1}{(1{\cdot}09) - 1}$; similarly $x^3 + x^2 + x + 1 = + \frac{x^4 - 1}{x - 1}$ *etc.*

NOTE 3. If Rs. 'P' now invested fetches Rs 'K' annually for 'n' years 'r' % per annum and if Rs. 'R' be the amount on Re. 1 for 1 year, it may be shown that $P \times R^n = K \times \frac{R^n - 1}{R - 1}\Big]$.

Second Method:—

An investment of Rs. 100 now made would fetch the man Rs. 9 at the end of each of three years and an additional sum of Rs. 100 at the end of the third year.

Since Re. 1 now invested would amount to Rs. $(1{\cdot}09)^3$ at the end of three years, a payment of Rs. 100 three years hence = one of Rs. $\frac{100}{(1{\cdot}09)^3}$ now made.

$\therefore$ an investment of Rs. $\left\{100 - \frac{100}{(1{\cdot}09)^3}\right\}$ now made would fetch him Rs. 9 at the end of each of three years.

$\therefore$ in order to fetch him Rs 1500 at the end of each of three years, Rs. $\frac{1500}{9} \times \left\{100 - \frac{100}{(1{\cdot}09)^3}\right\}$ should be now invested.

$\therefore$ the value of the investment required

$$= \text{Rs. } \frac{500 \times 100}{3}\left\{1 - \frac{1}{(1{\cdot}09)^3}\right\}$$

$$= \text{Rs. } \frac{50000 \times {\cdot}295029}{3{\cdot}885087} = \text{Rs. } 3797, \text{ to the nearest Re., using contracted methods.}$$

EXERCISES.

1. Find the amount at compound interest on :—
   (*a*) Rs. 750 for 2 years at 8°/₀ per annum :
   (*b*) £900 for 2 years at 6°/₀ per annum ;
   (*c*) Rs. 1,500 for $1\frac{1}{2}$ years at $7\frac{1}{2}$ °/₀ per annum ;
   (*d*) £750 for $1\frac{1}{2}$ years at 10°/₀ per annum, interest payable half-yearly : and
   (*e*) Rs. 1350 for 2 years at $5\frac{1}{2}$ °/₀ per annum, interest payable half-yearly.

2. If $A = P\left(1 + \frac{r}{100}\right)^n$, find '$n$' when A = 1210, P = 1000 and $r = 10$; find 'r' when P = 2000 A = 2205 and $n = 2$; find P when A = 1331, $r = 10$ and $n = 3$.

3. What is the least sum that a man must now invest in order that the amount on it accumulated at 5°/₀ compound interest may meet an expense of Rs. 2205 for his daughter's marriage two years hence ?

4. On 1-8-16. A borrowed Rs. 725 at 8°/₀ simple interest and lent it on the same day to B at 8°/₀ compound interest. If both the transactions were closed on 1-8-18, find his gain.

5. The prop erty of a man has ben steadily increasing in value by 10°/₀ per annum. If it is worth Rs. 50,000 now, estlmate its worth two years hence. Also, find its worth two years ago,

6. A sued B for Rs 3,000 which was the amount at compound interest on a sum lent two years ago at 12°/₀ per annum. Find the amount originally borrowed by B.

7. Chothmal Sowcar lends money at 11°/₀ per annum, interest payable half-yearly, while Krishna Lal Sowcar lends at 11¼°/₀ per annum, interest payable annually. I want to borrow Rs. 220 which I could repay with compound interest at the end of two years. From whom is it cheaper to borrow, and what will be the difference when I repay ?

8. A sum of money was lent at 10°/₀ per annum two years ago to be repaid now with compound interest, but a dispute arises as to whether the interest should have been compounded yearly or half-yearly. If the difference between the amounts in dispute be Rs. 220 4 as., find the sum lent.

Exercise (9) to (17) given below could be worked by a principle similar to that involved in Examples (1) and (2) worked in this chapter :—

9. Owing to the increase of automobiles in a city the number of carriages drawn by animals fell from 5120 at the beginning of 1912 to 4480 at the beginning of 1913. Assuming a constant annual rate of decrease, find the number of such carriages in the city at the beginning of 1915.

10. The value of a bungalow increases each year by 20°/₀. If it is now valued at Rs. 10,000. find its worth (*a*) two years ago, and (*b*) two years hence,

11. The price of brass vessels bought for a hostel in July 1914 was Rs 500. In July 1918 an exactly similar set was bought for Rs. 1,620. Assuming a uniform rate of increase in the price of brass, find what the vessels cost in July 1916.

12. The value of a silver vessel depreciates by 10°/₀ each year. It is now valued at Rs. 1 50, find its worth (*a*) two years ago, and (*b*) two year hence.

13. The annual birth-rate in a Taluk is 15 per 1000 and the annual death-rate is 2 per cent. of the population. If the total population now be 820,000. find what it was two years ago and what it would be two years hence.

14. A ball, allowed to bounce vertically rises to heights which gradually decrease in such a way that the height in any one bound is 40% of the height in the preceding bound. Find the height of the fourth bound if the ball be dropped from a height of 4 ft.

15. At the beginning of 1914 the population of a town was 4490. Since then the town has attracted people from outside at the rate of 35 per thousand every year. If the birth-rate per annum had been 4·2% and the death-rate 2·5% of the population, find the population at the beginning of 1918.

16. In a certain College the income in 1913-14 was Rs, 33,000 and the expenditure Rs 30,000. The income every year rose by 10% when compared with that for the previous year while the expenditure rose by 9%. Find the surplus in 1917-18.

17. Owing to the import of foreign sugar into a country, the indigenous industry fell by 40% every year. Find what percentage of the industry *would die out* in four years.

18. A man is entitled to receive Rs. 1,100 at the end of each of two years from the present day. What immediate payment would be a fair equivalent, compound interest being reckoned at 10% per annum ?

19. A Municipality borrows Rs. 2,45,000 for providing fresh-water supply to its rate-payers. If the loan is to be cleared in three equal annual payments, find the amount of each payment reckoning compound interest at 5% per year.

20. I invest on the first day of every year my annual savings of Rs. 300 in a Bank paying 10% interest per annum. If the Bank adds on the interest to the capital every half-year, find the amount of my savings at the end of $1\frac{1}{2}$ years from my first investment.

21. A school is started with an endowment of Rs. 10,000 which is invested in a Bank to accumulate at 5% per annum compound interest. If the school draws from the Bank Rs. 2,500 annually to meet the deficit for the year, find the amount to its credit at the end of four years.

22. A man invests Rs. 2,106-3 as. in a Banking Company and receives Rs. 500 at the end of every year. If compound interest be calculated at 6 °/₀ per annum, for how many years will he receive the *annuity*?

23. Prove that, if interest be payable 'q' times a year, the formula $A = P\left(1 + \frac{r}{q}\right)^{nq}$ gives the amount Rs. (A) at comp. int. on Rs. (P) in 'n' years.

24. A man invests Rs. 2,550 in a Banking Company which pays him Rs. 600 at the end of every year. If comp. int. be calculated at 9 °/₀, find the amount to his credit after the fourth payment.

25. Natarajan bought an estate from Jafir for Rs. 45,000. He paid Rs. 35,000 cash and agreed to pay the balance in two equal *annual* instalments. If 4½ °/₀ comp. int. be reckoned, find the amount of each instalment.

26. Suppose your ancestor invested 1 pie to your credit in 1600 to accumulate at 6 °/₀ comp. int. Find to the nearest Rs. 100 your share in 1900, given that $(1{\cdot}06)^{200} = 10^3 \times {\cdot}389$ very nearly. [Assume that no accidents happened in the meanwhile.]

27. Suppose your father invested Rs. 10 to your credit on 1-4-1901 to accumlate at 3 °/₀ compound interest in the Post Office Savings Bank. What amount could you expect to draw on 1-4-1941? [Assume $(1{\cdot}40)^{40} = 3{\cdot}251$ very nearly.] 8

28. When a *zemin* was taken up for management by the Court of Wards it had a debt of Rs. 2,32,050 borrowed at 10 °/₀ per annum, interest payable yearly. Instead of paying the interest as it fell due, the Court of Wards repaid Rs. 73,205 from the income at the end of each year. Show that in four years the loan would be completely repaid.

29. I promise to be a contributor to an auction-chit-fund to last for 6 years, the nominal amount of each chit being Rs. 6000, to be paid by six contributors at a nominal rate of Rs. 1000 per year at the beginning of each year. On the first day of each year the lowest bidder gets the chit by auction and has to pay Rs. 1,000 per year from that time. If I bid at the first chit for Rs 4,890 and invest the sum in a Bank which agrees to contribute to the chit on my behalf, show that, allowing interest at

9 °/₀ per annum, the Bank would gain only 8 as. 7 ps. by the whole transaction.

30. A house-building agency gets a house built for me for Rs. 2400, which is to be repaid with interest in three equal annual instalments, the first instalment to be paid at the end of the first year. If 5 °/₀ comp. int. be reckoned, show that (x) the amount of each instalment in Rupees is given by

(a) $2400 \times (1{\cdot}05)^3 = x \times \{(1{\cdot}05)^2 + (1{\cdot}05) + 1\}$

or by (b) $\{(2400 \times 1{\cdot}05 - x) \times 1{\cdot}05 - x\} \times 1{\cdot}05 = x$.

CHAPTER XIII.

THE CUBE AND THE CUBOID.

62. VOLUME OF A RECTANGULAR SOLID.

Place three inch-cubes so as to form a rectangular block. Place two such blocks so as to form a layer. Arrange five such layers one above the other as in Fig. 106. A *rectangular solid* would be formed whose length is 3″, breadth 2″ and height 5″.

It is called a *cuboid*. Its *volume* is equal to $3 \times 2 \times 5$ cubic inches.

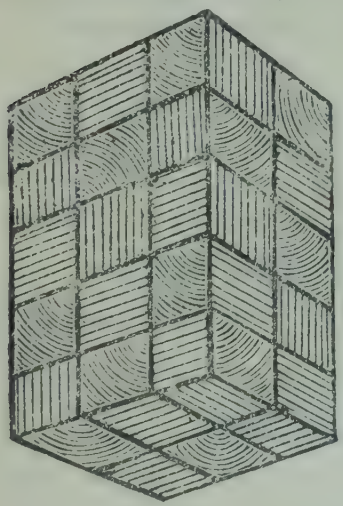


Fig. 106.

It may be similarly seen that if a cuboid be 'l″' long, 'b″' broad and 'h″' high and if 'V' be its volume in cubic inches

$$V = l \times b \times h.$$

Also, since '$l \times b$' = the area of the base in sq. ins., the volume of the cuboid in c. ins. = the no. of sq. ins. in the area of the base × the no. of inches in the height.

The volume of a cube 'b″' long = b^3 c. ins. This principle is involved in building up the table of British Cubic Measure (refer to Appendix) as also in the following metric measures of volume:—1 cubic metre = 10^8 cub. dm.; 1 cub. dm. = 10^8 cub. cm; 1 cub. dm. is called a *litre* and 1 cub. m. is called a *stere*.

The surface area of a wooden inch-cube, b'' long $= 6b^2$ sq. ins. and that of a cuboid l'' by b'' by $h'' = 2(lh + lb + bh)$ sq. ins. The figures below representing the *nets* of a cube and of a cuboid would make this clear:—

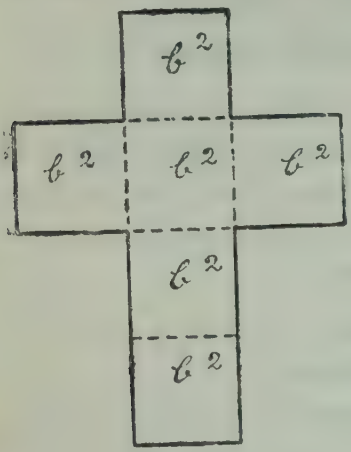


Fig. 107.
Net of a cube.

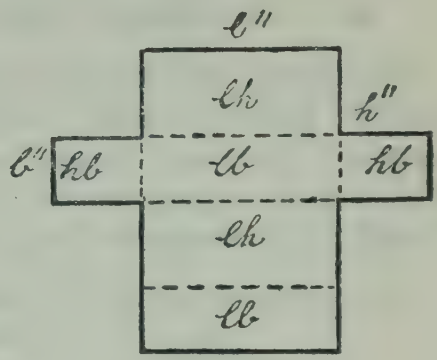


Fig. 108.
Net of a cuboid.

EXERCISES.

1. Find the valumes of the following:—
   (*a*) a brick, 8″ long, 4½″ wide and 2½″ thick.
   (*b*) a blackboard, 1 metre long, 55 cm. wide and 3 cm. thick.
   (*c*) a wall, 25 ft. long, 10 ft. high and 1 ft. 6 ins. thick.

2. Show with reference to any rectangular block that

$$V = l \times b \times h$$
$$l = \frac{V}{b \times h}$$
$$b = \frac{V}{l \times h}$$
$$\text{and } h = \frac{V}{l \times b}.$$

where 'l' ft. is the length of the block, 'b' ft. its breadth, 'h' ft. its height and V cub. ft. its volume.

Verify numerically.

3. If 'A' is the area in sq. ft. of the base of a rectangular prism and 'b' its height in ft., show that its volume $= A \times b$

c. ft. Also, if $V = A \times b$, show by giving numerical values that $A = \frac{V}{b}$ and $b = \frac{V}{A}$.

4. A class-room is 18 ft. long, 15 ft. broad and 12 ft. high. Find the volume of air in the room. If 29 pupils and one teacher are seated in the room, find how many c. ft. of air-space is allowed to each person on an average.

5. A dealwood box measures *internally* 2 ft. 9 in. in length, 2 ft. 3 in. in breadth and 1 ft. in height. How many measures of wheat flour will it hold? [1 measure occupies 100 c. in.]

6. The gallon-tin in my house is roughly in the form of a cuboid whose internal length=6″, = breadth = 4″ and height = 11·55″. Find the no. of c. in. in a gallon.

A kerosene tin is $13\frac{7}{8}$ in. high and its bottom has an area of 80 sq. in. How many gallons could it hold?

7. A beam of wood is 22 ft. long 1 ft. 5 in. thick and 1 ft. high. Find its weight if 1 c. ft. of the wood weighs 52 lbs.

8. A black-board is made by fitting three planks each of which is 'x' ft. long, 'y' inches wide and 't' inches thick. How much of air-space will the board occupy? Also, give the numerical result when $x = 4{\cdot}5$, $y = 14$, and $t = {\cdot}75$.

9. A rectangular tank is 120 ft. long, 84 ft. wide and 20 ft. deep. How many gallons of water could it hold? [$6\frac{1}{4}$ gallons= 1 c. ft.]

10. A new hall has to be constructed in our school so as to accommodate 80 pupils. If, $9\frac{1}{2}$ sq. ft. of floor-space and 95 c. ft. of air-space should be allowed to each pupil, explain how you would choose the dimensions of the hall.

11. In a rectangular tank, 100 yds. long, 80 yds. wide and 40 ft. deep, the level of the water is 10 ft. below the top. A sluice is kept open which discharges on the average 40 c. ft. of water per second. How long should the sluice be kept open in order that the depth of the water may be 25 ft.?

12. In a certain place there was a rainfall of 2·7 inches on a certain day Calculate how much of water fell on a rectangular field, 1 acre in area.

13. In a river valley there was a rainfall of 25″ during December 1914. If the valley contains 400 sq. miles and if only 75% of the rainfall reached the river, estimate how much of rain-water flowed into the river.

14. A c. ft. of lead is made into a sheet 60 ft. long and 4 ft. wide. Find the thickness of the sheet. [Express your result as the decimal of an inch.]

15. The road from P to Q is $1\frac{1}{2}$ miles long and 30 ft. wide. It is to be covered with gravel to a height of 6″. How many cart-loads of gravel will be required, if each cart holds 18 c. ft. of gravel?

16. In a lake the sheet of water has a surface area of 20 acres and is 20 ft. deep on the average. If the lake has to irrigate 500 acres of land at ·015 c. ft. of water per acre per second, how long would the water last? [Neglect the quantity of water evaporated].

17. The inner surface of the flat roof of a rectangular room 21′ by 18′ is covered with wood 2″ thick. Find the volume of wood required.

18. A rectangular vessel 2 dm. long and 1 dm. broad is partly filled with water. A steel ball is dropped into the vessal and the level of the water then rises by 1 cm. Find the volume of the ball, if it be completely immersed in water.

19. A rectangular jar measuring on the inside 10″ high, 8″ long and 5″ wide, contains water to the height of 6″. Four blocks of marble each 2″ by 1·5″ by 1″ are put into it. Find to what height the water would rise in the jar.

20. A c. dm. of steel is drawn into a uniform rectangular bar 5 m. long. Find the area of the rectangle forming the cross-section of the bar.

21. The cross-section of a pipe is a rectangle l' by w'. Water flows through its full bore at r' per minute. Find the quantity of water discharged continuously in 'h' hours.

22. A sheet of metal has an area of 10 sq. dm. Its weight is 2·1 kg. What is its thickness? [1 cub. cm. of the metal weighs 11·2 grams].

23. A rectangular cistern measuring internally 'x' ft. long 'y' ft. broad and 'z' ft. high contains water to a height of 'h' ft. What would be the height of the water in the cistern when 'n' bricks, each $9'' \times 4\frac{1}{2}'' \times 2\frac{1}{2}''$, are thrown into the water?

[Each brick being porous absorbs one-tenth of its own volume of water].

Ex. A room measures externally 25 ft. long, 18 ft. broad and 14 ft. high. It measures internally 23 ft. long, 16 ft. broad and 14 ft. high. Find the amount of space occupied by the walls.

The space required

$= 25 \times 18 \times 14 - 23 \times 16 \times 14$ c ft. (Why ?)

$= 1148$ c. ft.

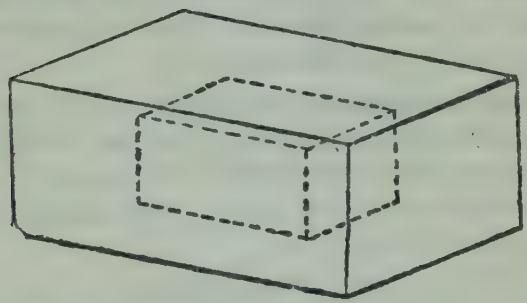


Fig. 109.

24. A closed teakwood box is measured *externally* and its length is found to be 2 ft., breadth 1 ft. 4 in., and height 1 ft. 3 in. The thickness of the wood is $\frac{1}{8}$ an in. Find (1) how much the box could hold ; (2) the quantity of wood in the box, and (3) the weight of the box, given that 1 c inch of the wood weighs $\frac{1}{2}$ oz.

25. How many c. in. of wood are there in a closed dealwood box whose *external* dimensions are 4 ft. by 2 ft. by 1 ft. 8 in., the wood being 1 inch thick ? What is the *capacity* of the box?

26. A chalk-box measures *externally* 8″ by 5″ by 6″. Each side is 4″ thick The box contains 144 pieces of crayon chalk packed with saw-dust If there are 10 c. in. of saw-dust, find the space occupied by each chalk-piece.

27. An iron-safe, made of plate 2″ thick, is 2′ long, 1′ 6″ wide and 3′ high. on the outside. Find the weight of the safe when it is empty, given that a cubic foot of iron weighs 480 lbs.

28. A closed box measures *externally* 'l' inches long, 'b' inches wide and 'h' inches high. It is made of wood 'k' inches thick. Show that

(i) its internal length $= (l - 2k)$ inches
" " breadth $= (b - 2k)$ "
" " height $= (h - 2k)$ "

(ii) the box could hold $(l-2k)(b-2k)(h-2k)$ c. in.
and (iii) the volume of wood in the box
$= lbh - (l - 2k)(b - 2k)(h - 2k)$ c. ins.

29. A rectangular box without a lid is made of wood $\frac{1}{2}''$ thick at the sides and 1" thick at the bottom. Its *external* length is 2′ 6", width 1′ 6" and height 1′ 4". Find the capacity of the box and the volume of wood contained in it.

30. An open rectangular box is made of tin 2 mm. thick. Its external length is 12 cm., width 10 cm. and height 5 cm. It weighs 520 grams when empty and ·9 kg. when full of rice flour. Find (1) the weight of 1 cubic cm. of tin and (2) the volume occupied by 1 kg. of rice-flour.

31. A block of wood is a'' by b'' by c''. It is overlaid with metal t'' thick. Find the volume of metal used. 1 cubic inch of the metal costs Rs. 2, find the value of the metal used. Also give the numerical results when $a = 10$, $b = 6$, $c = 5$ and $t =$ '05'

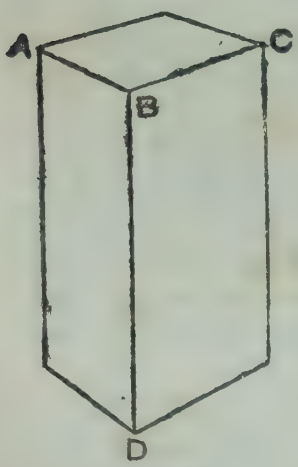


Fig. 110.

32. Find the volumes of the solids represented by the figures given below. Also calculate their surface areas.

(*a*) A rectangular solid (Fig. 110)
(i) AB = BC = DB = 10"
(ii) AB = 12", BC = 18" and DB = 20".

(*b*) A rectangular solid with a rectangular opening.

AB = 18', BE = 10', BC = 1' 6"

FH = 2'4" and HG = 5'4".

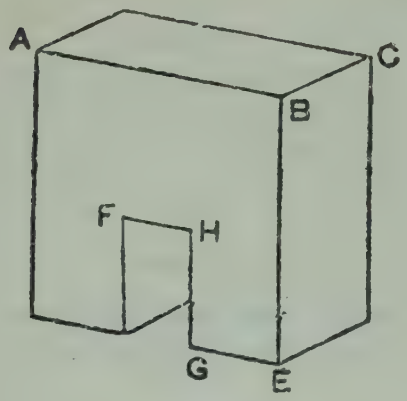


Fig. 111.

(*c*) A rectangular solid with a rectangular groove cut in it.

AR = HI = IK = RE = LI = RC = 4"

AG = HF = 1" and FG = 3".

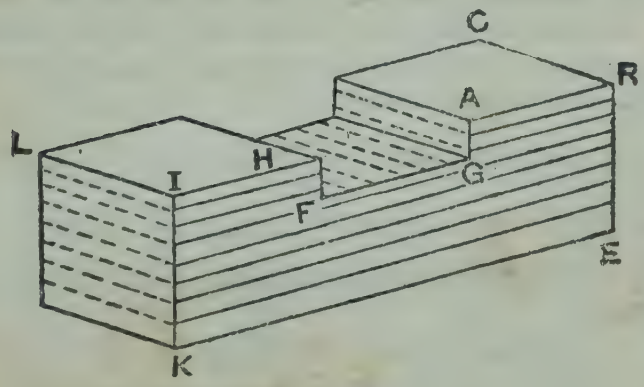


Fig. 112.

(*d*) Two rectangular pieces of wood fitted together in the form of a cross.

AD = 28", AB = 2", BC = 1", EF = 14", EG = 2", GH = 1".

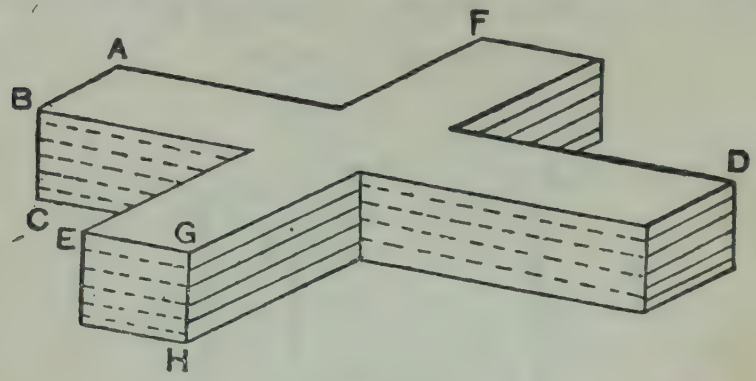


Fig. 113.

33. Find the cost of lining with lead a closed dealwood box whose internal dimensions are 3 ft. by 1 ft. 8 ins. by 1 ft. 8 ins. at Re. 1 per sq. ft.

34. Find the cost of lining with tin an open rectangular box which measures internally $4\frac{1}{2}$ ft. long, $1\frac{3}{4}$ ft. wide and $1\frac{1}{2}$ ft. high at 8 annas a sq. ft. [Include the rims at the top which are $\frac{1}{2}$ inch wide].

35. Three cubes have their edges in the ratio $1 : \frac{1}{2} : \frac{1}{3}$. Compare their volumes.

36. A lantern-post consists of a cubical structure of edge '4' ft. surmounted by another of edge '$1\frac{1}{2}$' ft. supporting a granite stone 8 ft. high, 8 ins. wide and 8 ins. thick. Find the volume of the entire structure.

37. A rectangular solid is 'l' ins. long, $\frac{'l'}{2}$ ins. broad and '$3l$' ins. high. Another is $\frac{'l'}{2}$ ins. long, $\frac{'l'}{4}$ ins. broad and $\frac{'3l'}{2}$ ins. high. Compare their surface areas as well as their cubical contents.

38. A block of wood is in the form of a cube whose edge is $(x + y)''$. Points are marked in each edge dividing it into two parts x'' and y'' in length. Through these points the cube is cut into 8 cuboids. Find the volume of each part. Hence show that $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

39. Illustrate, by using inch-cubes and separating them afterwards, that

(1) $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$.

(2) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

(3) $(a + b)(c + d)(e + f) = ace + acf + ade + adf + bce + bcf + bde + bdf$.

Verify your results by actual multiplication and by giving numerical values to a, b, c, d, e, f, x and y.

40. Give a geometrical meaning to the expressions a^3, $a^2 + b^2 + c^2$, $2(ab + ac + bc)$, $x^3 + y^3$, $6a^2$ and $a^2 + b^2 - c^2$.

41. The soil removed from a rectangular railway-cutting 100 ft. long, 40 ft. wide and 10 ft. deep is spread uniformly over a field of 2 acres. If the soil occupies $\frac{1}{7}$th part more space when dug out and spread, find by how much the level of the field would be raised.

42. A town containing 9075 inhabitants is supplied with fresh water from a rectangular reservoir which has a flat base of area $12\frac{1}{2}$ acres. If water should be stored in the reservoir so as to last the town for 200 days, allowiug each person 30 gallons per day on the average, find to what height water should be stored in ft. [Assume $6\frac{1}{4}$ gallons = 1 c ft. and neglect the evaporation.]

43. The canal leading from the Red Hills Lake is 14 ft. wide at a place of observation, where it has vertical walls. If the depth of the water there be 7 ft. and if water flows at three miles an hour on the average, calculate the quantity of water discharged from the Lake between 6 A.M. and 11 A.M. during a day.

44. I pour 'x' potfuls of water into an empty rectangular zinc tub and I find that the water is 'y' ft. deep. If a potful holds 'd' cubic inches of water, find the area of the bottom of the tub.

45. A sheet of paper is given to you of exactly the same size as a sheet of your note-book. Find the edge of the biggest cube you could make from the sheet. How much of the sheet will remain, after the cube is made?

46. The cross-section of an iron girder is of the form shown in the figure.

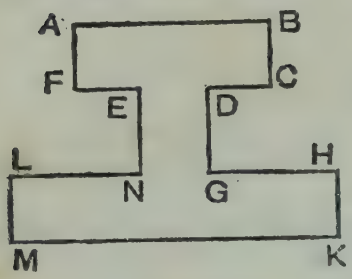


Fig. 114.

AB = 1 ft., FE = ED = DC = NG = $\frac{1}{3}$LN = $\frac{1}{3}$GH = LM = HK = $\frac{2}{3}$EN = AF = BC.

Find the weight of the girder, given that it is 24 ft. long and 1 c. ft. of iron weighs 480 lbs.

47. If the thirsty crow in Æsop's Fables found the water at the bottom of a tall rectangular jar 'h' ins. high, 'l' ins. long and 'b' ins. broad and threw 'n' pebbles into it each containing 'p' cubic inches before the water reached up to the brim, find the depth of the water in the jar before the pebbles were thrown in.

48. Two rectangular bars of silver whose dimensions are $a' \times b'' \times c''$ and $x' \times y'' \times c''$ respectively are melted to form a new rectangular bar with a cross-section 'd' sq. ins. Find the length of the bar. Also, give the numerical result when $a = 1$, $b = 2$, $c = 1$, $x = 3$, $y = 2$ and $d = 4$.

49. Examine the truth of the following :—

With a given total surface area, the greatest volume of a cuboid is got when it is a cube.

50. The size of a book is $7{\cdot}5'' \times 5'' \times 1{\cdot}5''$, 1620 copies of the book have to be packed into a rectangular box. What is the size of the box you would choose

CHAPTER XIV.

SIMULTANEOUS EQUATIONS AND THEIR APPLICATIONS.

63. Consider the following problem :—

I had 3 acres of wet land and 2 acres of dry land on which I paid an annual land-tax of Rs. 31. Recently I came to possess 4 acres of wet land and 3 acres of dry land on which I paid a land-tax of Rs. 42. Find the land-tax per acre on each kind of land.

Suppose the annual land-tax to be Rs. x per acre of wet land and Rs. y per acre of dry land.

Then the first condition given in the question gives the relation $3x + 2y = 31$ ... ... (1)

But this relation alone would not help us to find the values of x and y. For, any no. of values of x and y could be found so as to satisfy the given relation as shown in the table below :—

x	10	9	8	7	.....
y	$\frac{1}{2}$	2	$3\frac{1}{2}$	5	.....

...(A)

On examining the second condition given in the question, *viz.*, that the land-tax on 4 acres of wet land and 3 acres of dry land is Rs. 42, we get the relation

$$4x + 3y = 42 \quad \ldots \quad \ldots \quad (2)$$

The values of x and y satisfying this relation are shown in the following table :—

x	10	9	8	7	6	...
y	$\frac{2}{3}$	2	$3\frac{1}{3}$	$4\frac{2}{3}$	6	...

...(B)

From (A) and (B) above, it is seen that the values $x = 9$, $y = 2$ satisfy both the relations (1) and (2).

∴ the land-tax is Rs. 9 per acre of wet land and Rs. 2 per acre of dry land.

The above problem corresponds to finding the values of x and y which satisfy the equations, (1) $3x + 2y = 31$ and (2) $4x + 3y = 42$ at the same time.

From the above solution it follows that an equation involving two *unknown* quantities in the first degree has an unlimited no. of solutions and hence its solution is *indeterminate*. Two such *independent* equations are required in order that the solution may be *determinate*. On further examination it will be found that *one*, and *only one*, *set of values could be found for the unknown quantities which u ould satisfy both the equations at the same time*. Such equations are called *Simultaneous* equations.

The equations $3x + 2y = 31$ ... ... (1)

and $4x + 3y = 42$ ... ... (2)

have been solved above by a method of trial. Since the above method of search would be very tiresome and at times unsuccessful, the following method may advantageously be employed :—

From equation (2), $3y = 42 - 4x$

and hence $y = \frac{42 - 4x}{3}$ ... (3)

Substituting this value of y in equation (1),

$$3x + \frac{2(42 - 4x)}{3} = 31 \quad \ldots (4)$$

$$\therefore \quad 9x + 84 - 8x = 93 \quad \ldots (5)$$

$$\therefore \quad x = 9 \quad \ldots (6)$$

Since $y = \frac{42 - 4x}{3}$ in equation (3)

$$\therefore \quad y = \frac{42 - 36}{3} \text{ or } 2.$$

$$\therefore \quad \left.\begin{matrix} x = 9 \\ y = 2 \end{matrix}\right\}$$

[Check :—$3x+2y=3\times9+2\times2$ or 31 as given in (1);
$4x+3y=4\times9+3\times2$ or 42 as given in (2).
$\therefore$ the solution above obtained is correct.]

Just as step (6) in the above solution could be got from step (5), step (5) could also be got from step (6), *i.e.*, to say, the steps (5) and (6) are *reversible*.

The underlying principle in the above solution is one of *elimination*, *i.e.* to say, *to deduce a simple equation in which one only of the unknown quantities appears.*

The above principle may suggest the following methods :—

Method (a)

From equation (1), $x = \frac{31 - 2y}{3}$ ... (7)

(2), $x = \frac{42 - 3y}{4}$ ... (8)

By comparison of (7) and (8), $\frac{31-2y}{3} = \frac{42 - 3y}{4}$ ... (9)

$\therefore$ clearing the equation of fractions,

$4(31 - 2y) = 3(42 - 3y)$
or $124 - 8y = 126 - 9y$
$\therefore \quad y = 2$

$\therefore$ from equation (7), $x = \frac{31-4}{4}$ or 9.

Method (b)

Of *equalising the coefficients* :—

$3x + 2y = 31$ ... (1)
$4x + 3y = 42$ ... (2)

Multiplying either side of equation (1) by 3 and either side of equation (2) by 2, we get

$9x + 6y = 93$ ... (3)
$8x + 6y = 84$ ... (4)

Subtracting, $x = 9$ ... (5)

$\therefore \quad y = \frac{31 - 3 \times 9}{2}$ or 2.

[*How are the multipliers 3 and 2 leading to steps (3) and (4) suggested?* In the expression $3x + 2y$, 3 and + 2 are called the *coefficients* of x and y.]

When the co-efficients of 'x' and 'y' are interchanged in the two equations, the following method of 'addition' and 'subtraction' may be employed:—

Solve :— $5x + 2y = 121$ ... (1)

$2x + 5y = 103$ ... (2)

Adding, $7x + 7y = 224$

or $x + y = 32$ ... (3)

Subtracting (2) from (1), $3x - 3y = 18$

or $x - y = 6$ ... (4)

Adding (3) and (4), $2x = 38$ or $x = 19$
Subtracting (4) from (3), $2y = 26$ or $y = 13$

EXERCISES.

1. Express 'y' in terms of 'x' in the following :—

$2y + x = 15$; $x - 2y = 15$; $2x + 2y = 15$; $\frac{x}{15} = y + 10$;

$\frac{y}{10} - \frac{x}{2} = 10$; $\frac{y}{15} = x + 10$; $\frac{x}{2} - \frac{y}{3} = 6$ and $4y - 5x = \frac{1}{5}$.

2. Express 'x' in terms of 'y' in the following :—

$2y + x = 5$; $5x + 3y = 5$; $6x + 4y = 15$; $15y - \frac{1}{3}x = 6$;

$\frac{2}{3}x + 8 = y$ and $\frac{x}{5} + \frac{y}{4} = 1$.

3. (*a*) Substitute $\frac{(y-2)}{3}$ for x in the following and find the values of x and y in each case :—

$15x = 3y$; $15y = 38x + 44$; $15 - \frac{5}{8}x = y - 1$; $4x + 6y = 0$

$22x - 4y = 36$; $2x = 3y + 1$ and $\frac{x}{2} = y + 7$.

(*b*) Substitute $\frac{5 - 2x}{4}$ for 'y' in the equations given below and hence find the values of x and y :—

$2y + \frac{x}{2} = 4$; $5x + 6y = 20$; $14 - 4x = y$; $19 - 2y = 12x$
$4x - 2y = 32$.

4. Solve the following :—

(*a*) $x = 3y$;
$4y = 3x - 10$.

(*b*) $x - 4y = 0$;
$5y - x = 2$.

(*c*) $y = 3x + 2$;
$x = 4y - 19$.

(*d*) $2x + y = 10$;
$y + 4x = 16$.

(*e*) $2x + y = 4$;
$3x + 2y = 5$.

(*f*) $x - y = 1$;
$3x - 2y = 102$.

(*g*) $2x - 3y - 1 = 0$.
$2y + 3x - 5 = 0$.

(*h*) $4x + 2y + 3 = 0$;
$5x + 6y + 4 = 0$.

(*i*) $x + 2y - 4 = 0$;
$x - 2y - 2 = 0$.

(*j*) $x - 3y + 7 = 0$;
$3x + y - 9 = 0$.

(*k*) $x + y = 28$;
$x - y = 12$.

(*l*) $2x + y = 15$;
$x + 2y = 0$.

(*m*) $2x - y = 8$;
$x - 2y = 1$.

(*n*) $199x + 113y = 285$;
$199x - 113y = 511$.

(*o*) $5x + 3y = 13$;
$5x - 3y = 7$.

(*p*) $2x - 5y = 3$;
$5x - 2y = 18$.

(*q*) $x + y = 14$;
$3x + 2y = 34$.

(*r*) $x - y = 1000$;
$4x - 5y = 1000$.

(*s*) $3x + 2y = 1950$;
$5y - 2x = 125$.

(*t*) $2(x + 2y) - 4 = 10$;
$3(x - y) - 1 = 8$.

(*u*) $7y = 15x + 3$;
$5 - 2y = 4x$.

(*v*) $34x + 22y = 45$;
$22x + 34y = 39$.

(*w*) $9l + 6b = 10$;
$16l - 8b = 56$.

(*x*) $4A + B = 5$;
$25A + 5B = 30$.

(*y*) $\frac{m}{6} + \frac{n-4}{8} = 6$;
$\frac{m}{4} + \frac{2n-5}{7} = 11$.

(*z*) $y = bx$;
$b^2x + b^2y + q = 0$.

5. Solve.—

(a) $x + 3y - 20 = 0$;
$3x - y - 10 = 0$.

(b) $\frac{x-2}{y-2} = \frac{3}{4}$;
$\frac{x+2}{y+2} = \frac{7}{8}$.

(c) $\frac{x}{3} + \frac{y}{2} = 16$;
$\frac{x}{2} + \frac{y}{2} = 14$.

(d) $xy = (x + 10)(y - 6)$
$= (x - 10)(y + 9)$.

(e) $x - y = 30$;
$(x+10)(y+10) = xy+200$.

(f) $\frac{x}{11} = \frac{y}{13}$;
$y - x = 10$.

(g) $5x - 4y + 10$
$= 15x - 3y - 5$
$= 6y - 20x - 15$.

(h) $\frac{x+y}{3} - 1 = \frac{2y-x}{2}$
$6y = 7(x - 1)$,

(i) $\frac{6}{7}y = \frac{1}{2}(3 - \frac{2}{3}x)$;
$4x + 6y = 13$.

(j) $4x - 5y + 7 = 0$;
$3x + 2y = 12$.

(k) $\frac{1}{2}(5x + 3y + 7)$
$= \frac{1}{3}(4x + 5y + 6)$
$= \frac{1}{5}(3x + 7y + 9)$.

(l) $x - 4y = \frac{1}{8}(4x - 7y) - 2$
$= \frac{2}{3}(2x - 8y + 5) - 6$.

(m) $8x + 5y + 14 = 0$;
$6x + y + 5 = 0$.

(n) $\frac{1}{3}x + \frac{2}{7}y = 38$;
$x + y = 100$.

(o) $\frac{y}{6} = 22 - \frac{x}{13}$;
$\frac{x}{3} + \frac{y}{2} = 78$.

(p) $13x + 2y = 32$;
$2x + 13y = 43$.

(q) $4x - 5y + 7 = 0$;
$3x - 2y = 12$.

(r) $7x + \frac{5y + 9x}{11} = 17$;
$5y + \frac{11y + 9x}{17} = 21$.

(s) $13x + 2y = 32$;
$2x + 13y = 43$.

(t) $9y = 46 - 19x$;
$81x + 39y = 198$.

(u) $x = y - 2$;
$\frac{x}{y} = \frac{3}{5}$.

(v) $12(y - x) = 16$;
$16x = 64 - 12y$.

(w) $\frac{48}{x+y}+\frac{48}{x-y}=14$; $\frac{4}{x}=\frac{3}{y}$.

(x) $ax+by=c$; $bx+ay=d$.

(y) $\frac{a}{b+2}-\frac{a}{b+4}=\frac{1}{2}$; $\frac{a}{b}=\frac{a}{b+4}+\frac{5}{3}$.

(z) $\frac{1749}{y}=\frac{1760}{x}+57$; $\frac{1760}{y}=\frac{1672}{x}+81$.

6. The sum of two numbers is 30 and their difference 8. Find the numbers.

7. The cost of a chair and a table is Rs. 16 and that of 4 chairs and 2 tables = Rs. 44 at the same rate. Find separately the cost of a chair and of a table.

8. A and B went to play marbles and together had 31 marbles. A lost 6 marbles and B gained 9. A then had only as many marbles as B. How many marbles had each at first?

9. Five trunks and three locks cost Rs. 81. Three trunks and five locks of the same kind cost Rs. 55. Find the price of a trunk and of a lock.

10. Three measures of curd are exchanged for two measures of rice. One measure of curd and four measures of rice could be had for Re. 1-12 as. Find the price in money for a measure of curd and of rice.

11. Find two numbers whose difference = 26 so that the larger divided by the smaller gives a quotient 7 with remainder 2.

12. Some members of a Reading Room pay a subscription of 4 as. per month and the rest pay 8 as. per month. The total monthly subscription amounted to Rs. 9-8 as. Recently the rates of subscription were raised to 6 as. and 10 as. per month. The same members continuing, the total monthly subscription rose to Rs. 13. Find the number of members paying each rate of subscription.

13. In a certain school a trained teacher is paid Rs. 30 a month and an untrained teacher Rs. 20. The total monthly salary of the staff is Rs. 290. In the next year when the trained teachers receive Rs. 32 and the untrained teachers Rs. 21 per month, the total monthly salary of the staff is Rs. 308. Find the number of trained and untrained teachers in the school. [Assume that there was no change in the staff.]

14. In a certain community $\frac{1}{3}$ of the men and $\frac{1}{8}$ of the women know to read and write and their total no. is 27,000. The rest are illiterate and number 80,000. Find the number of men and women in the community.

15. A contractor undertakes to supply either 80 benches and 20 chairs or 65 benches and 40 chairs for the same sum of Rs. 950. Find his price for a chair and for a table.

16. Three rectangular fields are of equal areas. Their dimensions are represented as follows :—

Field.		Length in yds.	Width in yds.
1.	...	$(y + 4)$	$(x - 4)$
2.	...	$(x + 20)$	$(y - 12)$
3.	...	$(x + 8)$	$(y - 6)$.

Find the dimensions of each field.

17. In re-laying a rectangular garden the length is diminished by 300 ft. and the breadth increased by 200 ft. The garden is now square in shape but its area is unaltered. Find the original as well as the present dimensions of the garden.

18. A rectangular field A is exchanged without loss of area for another B, 6 yds. longer but 4 yds. less wide. The field B is exchanged for another C which is as long as A but 6 yds. wider. The party so taking up C gains 180 sq. yds. of ground. Find the dimensions of each of the fields.

19. The length in cm. of a side of an equilateral triangle may be represented by $(2x-5y+18)$, $(10y-4x+3)$ and $(4x-\frac{1}{5}y-26)$. Find the number of cm. in each side.

20. Forty bullock carts and twenty cycles paid tolls at a toll-gate on a certain day. The corresponding nos. for the next day were 100 and 40. The collections from them were Rs. 12-8 as. on the first day and Rs. 30 on the second day. Find the amount of toll levied for a bullock cart and for a cycle.

21. After selling $\frac{1}{3}$ of my wet lands and $\frac{1}{5}$ of my dry lands I find my assessment reduced in consequence from Rs. 250 to Rs. 150. If the rates of assessment on these two kinds of land are respectively Rs 15 and Re. 1 per acre, find the extent of each kind of land which I now hold.

22. Solve $\frac{1}{x}+\frac{1}{y}=5$; [Represent $\frac{1}{x}$ by X and $\frac{1}{y}$ by Y.]

$$\frac{2}{x}+\frac{3}{y}=13.$$

23. Ax^2+Bx+C has the values $1\frac{1}{2}$, $\frac{1}{2}$ and $3\frac{1}{2}$ when $x=0$, 1, 2 respectively. If A, B and C stand for nos., find them. Also find the value of Ax^2+Bx+C when $x=5$.

24. $P+Qy+Ry^2=-7, +8, +5$, when $y=-1, 0, +1$ respectively. If P, Q and R represent numbers, find the value of the expression $Px^2+Qxy+Ry^2$ when $x=1$ and $y=2\frac{1}{3}$.

25. $y=mx+c$ when $x=5$, $y=6$ and also when $x=10$, $y=8$. Find the numerical values of 'm' and 'c.'

26. If $\frac{x}{a}+\frac{y}{b}=1$ be satisfied by the values $x=7$, $y=\frac{3}{2}$ and $x=\frac{7}{3}$, $y=-\frac{1}{3}$, find the numerical values of 'a' and 'b'.

27. If $x=y-x$ and $10y-5x=6$, find the value of $x^4+x^2y^2+y^4$.

28. If $x=2t-1$ and $y+5=24t-16t^2$, find the value of y when $t-x=5$.

29. Find the values of the constants A and B in *identical* equalities given below :—

(*a*) $4x-5\equiv A(x+3)+B(x-5)$

(*b*) $\frac{5x+9}{(x+1)(x+3)}\equiv\frac{A}{x+3}+\frac{B}{x+1}$.

30. $Ax+B$ is equal to 3 when $x=-1$ and to -2 when $x=4$. Find the values of A and B.

31. The resistance (R) in pounds per ton to the motion of an automobile is given by $R=l+mv^2$ where 'v' is its speed in miles per hour and 'l' and 'm' are constants. When $v=10$, R is 6·9 and when $v=30$, R is 14·1. Find the values of 'l' and 'm'. Also, find the values of 'R' when 'v' is equnl to 20 and 40.

32. Verify the common solution of $2x-3y+5=0$ and $3x+4y-7=0$ would also satisfy $5y-9x-8=0$.

33. If $y=px+r$ and $y=qx+d$, express x and y in terms of p, q, d and r.

34. If, in $x = a + \frac{b}{y}$, $x = 5$ when $y = 1$ and $x = 3$ when $y = 3$, find the values of 'a' and 'b'.

35. Given that $\frac{1}{a} + \frac{1}{b} = 1$ and $2a = b$, find the values of 'a' and 'b'.

36. Show why the following sets of equations do not have a common solution :— (*a*) $5x + 2y - 20 = 0$;

$$4y + 10x = 100$$

and (*b*) $2x + 3y = 5$;

$$4x - 4 + 6y = 0.$$

37. Verify if the equations $2x + 3y = 5$, $13x - 3y = 20$, and $5x - 2y = 3$ have a common solution.

38. A boy was asked to frame a pair of simultaneous equations. Having a particular solution in view he obtained the equations $10y - 5x = 6$ and $10x = 20y - 21$. Show why it is not possible to find out from his equations the solution he had in view.

If he had the values $x = \cdot 4$ and $y = \cdot 8$ in view and if the second equation had been $12x = 20y - k$, find the value of k.

39. A fraction is equal to $\frac{5}{9}$ when '3' is added to both its terms. Its value reduces to $\frac{1}{5}$ when '5' is subtracted from each term. Find the terms of the fraction.

40. One vessel contains coffee 25 % of which is milk and the rest decoction. Another vessel contains coffee, 60% of which is milk and the rest decoction. How many seers should be taken from each vessel in order to obtain a third mixture containing $11\frac{1}{2}$ srs. of milk and $13\frac{1}{2}$ srs. of decoction ?

41. An alloy contains 3 parts by weight of gold to 2 parts by weight of copper. Another alloy contains 3 parts by weight of gold to 5 parts of copper. Both the alloys were fused together so as to get equal quantities by weight of copper and of gold in the resulting alloy. In what ratio were the two alloys fused ?

42. Archimedes roughly found out that gold lost $\frac{1}{19}$ of its weight and silver $\frac{1}{11}$ of its weight when weighed in water. A crown was made of the two metals and it was pretended that it was made of gold. Archimedes suspected its genuineness, weighed the crown in water and found that it lost $\frac{1}{15}$ of its real

weight or 4 Imperial tolas. He next calculated the weight of the metal used in making the crown. Find the results he got.

43. Of two mirasdars A and B, A incurs a debt of Rs. 10,000 and B Rs. 25,000, They then find their net worths equal. A contracts a further debt of Rs. 10,000 while B wins a lottery prize of Rs. 5,000. It is then found that A's net worth : B's = 8 : 11. Find the original worth of each mirasdar

44. The property acquired by a man was divided between his son and daughter as follows :— (1) movable property in the ratio 3 : 5 and (2) immovable property in the ratio 8 : 5. The son was then worth Rs. 87,000 and the daughter Rs. 70,000, Find the respective worth of the movable and immovable property acquired by the father.

45. A bangle made of gold and copper loses $\frac{7}{10}$ of a tola when weighed in water. Another bangle containing $2\frac{1}{2}$ times as much of gold but an equal weight of copper loses 1 tola when weighed in water. Find the weights of gold and copper in each bangle, given that gold and copper lose respectively $\frac{1}{19\cdot25}$ and $\frac{1}{8\,9}$ of their weights when weighed in water.

46. Between two places P and Q a transport agency charges 4 as. for transmission of parcels not exceeding a certain weight and on heavier parcels demands an additional charge for every seer above that weight. The charge for a parcel of 20 srs, is 9 as, and that for a parcel of 30 srs. is twice that for a parcel of 16 srs. Find the scale of charges.

47. Two passengers travelling from P to Q with $2\frac{1}{2}$ mds. of luggage are together charged Rs. $3\frac{1}{2}$ excess. During the return journey, one of them takes with him the luggage of both and is charged Rs. $4\frac{1}{4}$ excess, What weight of luggage is each passenger allowed to take free? What is the charge per md. of excess luggage?

48. I performed a journey of fifty miles by travelling 2 hrs. by train and 2 hrs. by motor. On the whole I paid a fare of Rs. 2-3 as. at 1 anna per mile for the motor and at 6 pies per mile for the train. Find the speed of the train and of the motor.

49. Daily a clerk attends office from his house which is 4 miles distant, partly by walking at 3 miles an hour and partly by going in a jutka which runs at 6 miles an hour. One day it took him 50 minutes to reaeh the office from his house. Find how long he walked and how long he travelled in the jutka on that day.

50. From K to M the distance is 38 miles but the road is partly level, partly up-hill and partly down-hill. A motor which runs at 15 miles per hour on level ground, 12 miles per hour up-hill and 18 miles per hour down-hill reaches M from K in 2 hrs. 40 mins, and returns from M to K in 2 hrs. 30 mins. For what distance from K to M is the road up-hill and for what distance is it down-hill?

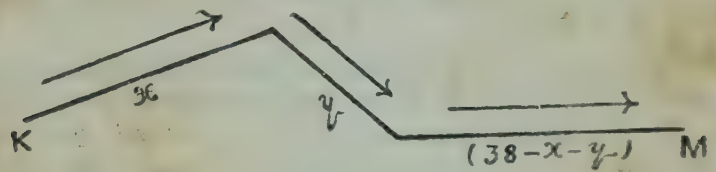


Fig. 115.

[Suppose from K to M the road is 'x' milss up-hill, 'y' miles down-hill and hence $(38 - x - y)$ miles level. Then the road from M to K is $(38 - x - y)$ miles level, 'y' miles up-hill and 'x' miles down-hill.

$\therefore$ the motor would go from K to M in $\frac{x}{12} + \frac{y}{18} + \frac{(38 - x - y)}{15}$ hours and would return from M to K in $\frac{(38 - x - y)}{15} + \frac{y}{12} + \frac{x}{18}$ hrs.

The following equations may hence be deduced:—

$$\frac{x}{12} + \frac{y}{18} + \frac{(38 - x - y)}{15} = \frac{8}{3} \quad \ldots (1)$$

and $$\frac{(38 - x - y)}{15} + \frac{y}{12} + \frac{x}{18} = \frac{5}{2} \quad \ldots (2)$$

The values of x and y satisfying the equations (1) and (2) would give the results required.]

51. In a triangle PQR the perimeter = 43 cm., PQ : QR = 5 : 6 and RP : RQ = 5 : 9. Find the lengths of PQ and PR.

52. At 5-45 a.m. a cyclist starts from P to Q and reaches it at 7-20 a.m. After remaining there for 1 hr. 38 mins., he again leaves Q for P and reaches it at 11 a m. If from P to Q he rode in good roads at 12 miles an hour and in bad ones at 8 miles an hour and during the return journey rode in good roads at 10 miles an hour and in bad ones at 6 miles an hour, find the length of bad road from P to Q.

53. Three motor-bicycles A, B and C leave Madras for Chingleput. B starts 30 mins. later than A and travels 3 miles an hour faster. C starts 45 mins. later than A and travels 5 miles an hour faster. If they all reach Chingleput at the same time, find their respective speeds as well as the distance between Madras and Chingleput along the road they travelled.

54. A gentleman has two horses A and B and a saddle worth Rs. 100. If the saddle be put on A, both the horses would be valued equally. If the saddle be put on B, its value becomes two-fifths greater than that of A. Find the worth of each horse without the saddle.

55. I bought a bandy and a pair of bulls together for Rs. 175. I sold the bulls at a gain of 20% on their cost and the bandy at a loss of 16% and thereby got Rs. 192 as their total sale-price. Find the original cost of the bandy as well as that of the bulls.

56. A cycle-dealer invests Rs. 35,000 in buying 100 ordinary cycles and 50 motor cycles. He sells the ordinary cycles at a profit of 15% and the motor cycles at a profit of 20% on their cost and gains Rs. 6,500 on the whole. Find the price he paid on an average for an ordinary cycle and for a motor cycle.

57. A and B run a race of 200 yards. At first A gives B a start of 3 seconds and beats him by 5 yds. They try again and this time A gives B a start of 45 ft. but is beaten by 1 second. At what rate per second does each run?

58. During a certain year a man spent part of his income and saved the rest. Next year his income increased by $\frac{1}{3}$ while his expenditure was reduced by $\frac{1}{4}$ and he saved Rs. 500 more than during the previous year. He then calculated that if his expenditure had been the same for the first year as during the second year he could have saved Rs. 800 during the first year. Find his income, expenditure and savings during each year.

59. A person has 5 acres of single-crop lands and 2 acres of double crop lands on which he was paying an annual assessment of Rs. 79. Recently the tax was raised by Re. 1 per acre of single-crop lands and by Rs. 3 per acre of double-crop lands. He then had to pay an equal assessment on either kind of land he possessed. What are the revised rates of assessment?

60. A boatman rows to a place 12 miles upstream and returns in 8 hrs. He is able te row 3 miles with the stream in the same time as 1 mile against the stream. Find the speed of the current and the rate at which the boatman could row in still water.

61. The distance between two places A and B is 270 miles. A train left A for B at 6 a.m. and another left B for A at 6-30 a.m. They meet at a place at 2 p.m. After crossing, the former reached its destination in 10 hours. Find the speed of each train and how far from A they met.

62. A and B travel from P to Q. A travels partly on a cycle and the remaining 70 miles in a train moving four times as fast as the cycle and thus completes the journey in $6\frac{1}{2}$ hrs. B travels the whole distance in a train and reaches Q in $3\frac{1}{3}$ hrs. Find the length of the journey and the speed of the cycle.

63. Two numbers are formed with the same two digits. The sum of the nos. is 44 and their difference is 18. Find the numbers.

64. The Lakshmi Vilasa Sabha staged a play one evening in our town. The admission tickets were as follows:—A 1st class ticket was charged Rs. $1\frac{1}{2}$ more than a 2nd class ticket, and a 2nd class ticket Re. 1 more than a 3rd class one. The no. of 1st class tickets sold was 60 less than that of the 2nd class tickets and the no. of 2nd class tickets was 200 less than that of the 3rd class tickets. The sale of the 1st class tickets fetched Rs. 120 more than that of the 2nd class tickets and the sale of these fetched Rs. 100 more than that of the 3rd class tickets. Find the charge per ticket of each class. How many tickets of each class were sold? What was the total collection that evening?

65. For a journey on transfer an officer is allowed to draw the cost of carriage of personal effects by goods train to the extent of only 6 mds. or an equal charge if carried through a cart. An officer has to carry 7 mds. from A to B, 35 miles distant. He

calculates that, with the allowance he could draw, he could transmit his goods in a country cart and gain 5 as. in addition. The charge for sending all his articles by goods train is Re. 1-5 as. more than the cart-fare. Find the cart-fare per mile and the charge per maund for carrying personal effects by goods train from A to B.

66. In the first reading of a Bill before a House, 122 members were present, part of whom voted for the Bill and the rest against it and the Bill was passed. The Bill was read a second time before the House. This time more members attended and $\frac{3}{4}$ of the members present voted for and the rest against the Bill. The number who voted for the Bill this time exceeded the corresponding no, during the first time by 10, while the no. of those who voted against increased by 20. How many members more voted on the whole during the second reading and how many voted for the Bill during the first reading ?

67. In a cycle race between two places P and Q 10 miles apart, a cyclist A gives another cyclist B a start of 3 mins. and yet beats him by $1\frac{1}{6}$ miles. During the return journey from Q to P, A gives B a start of 2 miles but is beaten by 2 mins. Find the rates at which A and B bike per hour.

68. From a piece of string t yds, r ft. p in. long I cut a piece p yds. r ft. t in. long. The piece left is 1 inch short of a yard. I cut the piece remaining into two halves and find each half $(7t - 3\frac{1}{2}p)$ in. long. Find the values of 't' and 'p.'

69. The produce of a field would be just sufficient to feed a family for a certain number of months. If 2 more members were admitted into the family the produce would last for 2 months less. If 2 members leave the family instead, the produce would last for 3 months more. Find the number of members in the family and the number of months for which the produce could ordinarily last,

70. The strength in Forms IV, V, VI of a High School is 180 and the students may in the matter of fees be divided into 3 classes :—(1) those who pay at a concession rate of Rs. 18-12 as. for the long term, (2) those who pay Rs. 26-4 as. in advance for the whole term, and (3) those who pay at the rate of Rs. 5-8 as. for each of the five instalments during the term. If the no. of pupils under class (3) be double the rest and if the total fee collections for the term be Rs. 4,725, find the number of pupils under each class.

CHAPTER XV.

THE STRAIGHT LINE GRAPH.

64. FUNCTION OF A VARIABLE.

In Chapter V, Section II, it has been shown how two quantities may vary *simultaneously* so as to have some relation between them, *e.g.*, if the quantity bought of any substance varies, the price to be paid also varies correspondingly. When two quantities vary so that the changes in the value of one of them depend on, and correspond to, the changes in the value of the other, one of them is said to be a function of the other. Thus, *when other conditions remain unaltered*, the cost of constructing a road is a function of its length, the number of days required to finish a work is a function of the number of workmen employed, the area of a triangle is a function of its base, the amount on a sum of money is a function of the time, etc. In the same sense, any expression involving 'x' such as $2x$, $\frac{3x-4}{5}$, x^2, x^2-3x+5, etc., which varies with 'x' is called a function of x. Similarly in equalities like $y = 5x$, $pv = 120$, $s = 2z + 16z^2$, $w = 130 - 12t$, y, p, s and w may be spoken of as function of x, v, z and t respectively.

65. GRAPHICAL REPRESENTATION.

When one quantity is a function of another, the various values which either may assume can be represented by means of a diagram as shown below :—

Let 'y' be a function of 'x' connected by the relation $y = 2x + 6$.

Give a series of values to 'x' and obtain the corresponding values of 'y' as in the following table :—

x	y $(=2x+6)$.
+ 5	+ 16
+ 4	+ 14
+ 2	+ 10
+ 1	+ 8
— 1	+ 4
— 2	+ 2
— 4	— 2
— 5	— 4
.....	.....
.....	.....

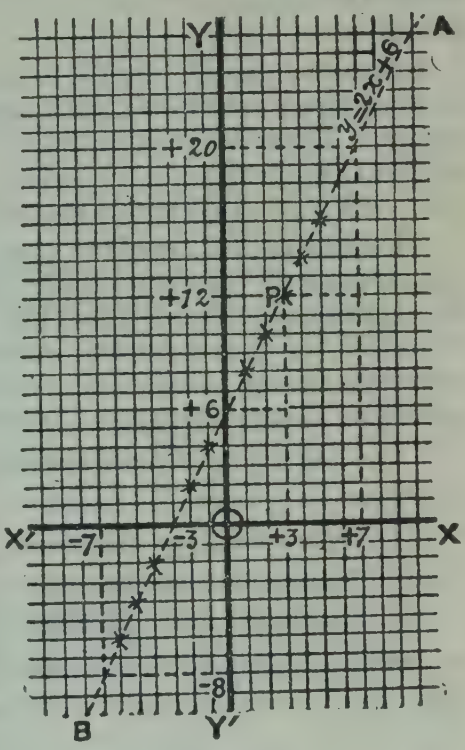


Fig. 116.

To represent diagrammatically that $y = +16$ when $x = +5$, plot a point whose abscissa is + 5 and ordinate + 16, choosing any convenient scale along either axis, *e.g.*, ($\frac{1}{10}$" = 1 unit). Similarly represent the other values of x and y in the above table by plotting the points (+ 4, + 14), (+ 2, + 10) ..... (— 4, — 2), (— 5, — 4), etc. Join the points freely by a line (AB in the above figure).

It may be seen that any psint P, *e.g.*, (+ 3, + 12) whose co-ordinates satisfy the relation $y = 2x + 6$ lies in the line AB, produced either way if necessary. Also, the co-ordinates of any point marked in the line, *e.g.* (+ 6, + 18) satisfies the relation $y = 2x + 6$ and those of points outside do not satisfy it. The line is hence called the graph of the relation $y = 2x + 6$. Since the ordinates of points in the

line give the various values of the *function* $(2x + 6)$ corresponding to various values of x represented by their abscissæ, the line is also called the *graph* of the function $(2x + 6)$.

From the graph could be read at sight the value of one of the variables corresponding to a value of the other. Thus when $x = +7$, the ordinate corresponding to the abscissa $+7$ gives $y = +20$. Also, when $y = -8$, the corresponding abscissa gives $x = -7$.

The graph in the *above* case is a *straight line*. Starting from any chosen value of x, *e.g.*, $+3$, it may be seen from the graph that for *each* unit added to the value of x, there is an increase of *two* units in the corresponding value of y. In other words, there is a *steady* or *uniform* rate of increase.

[*The following details should be attended to in working exercises on graphs*:—

(i) *Before plotting the points, draw the axes distinctly. Name them as well as the origin.*

(ii) *State the scale you choose along either axis. Choose a simple scale which would facilitate rapid and fairly accurate reading of the values decimally* e.g., *represent 1 unit by 1" or ·2" or ·5" or ·1" and avoid representing 1 unit by ·7" or ·3". (If necessary, different scales may be chosen along the axes).*

(iii) *Draw the line representing the graph distinctly and name the graph by the function or equation it represents.*

(iv) *In finding the values of the co-ordinates of points required for plotting, avoid fractional values like* $5\frac{7}{9}$, $8\frac{18}{19}$, *etc., or express them decimally,* e.g., *to the nearest tenth, if unavoidable.*]

EXERCISES.

1. Draw the graph of:—

$$y = 2x,\ y = x + 2,\ y = 2x + 1,\ y = 3x - 1.$$

Find from each graph the value of 'x' when $y = -5$ and the value of 'y' when $x = +5$. How does the value of 'y' change for each unit added to the value of 'x'?

2. Draw the graph of the functions $2x$, $3x$, $\frac{5}{2}x$, $3x-3$, $4x-1$, $2x+1$, $3x-1$. $2x+5$, $8x+5$. Find from each graph the value of the function when $x = 4{\cdot}5$ and the value of 'x' when the function $= 4{\cdot}5$. How does the value of the function change for each unit added to the value of 'x'?

3. Draw with reference to the same origin and axes of co-ordinates the graphs of :—

$y = x$, $y = x + 2$, $y = x - 1$, $y = x+3$, $y = x + 6$, $y=x-2$, $y = x - 3$.

Measure the angles which the graphs make with either axis,

Note that the lines are parallel and that the graph of an equation of the type $y = x + c$, where 'c' is any constant is parallel to that of $y = x$.

4. Draw with reference to the same origin and axes of co-ordinates the graphs of :—

(*a*) $y = 2x$, $y = 2x + 1$, $y = 2x - 1$, $v = 2x + 2$, $y = 2x - 3$, $y = 2x - 4$, $y = 2x + 5$, $y = 2x + 6$.

(*b*) $v = 3x$, $y = 3x + 1$, $v = 3x - 1$, $y = 3x + 4$, $y = 3x+5$, $y = 3x + 7$, $y = 3x - 4$. What is the general form of an equation whose graph is parallel to (i) $y = 2x$ and (ii) $y = 3x$.

5. Draw with referenee to the same origin and axes of co-ordinates the graph of $y = x$, $y = 2x$, $y = 3x$, $y = 4x$, $v = 5x$.

What is the relation between the ordinate and abscissa of every point in each graph?

Note that if a point moves so as to be equidistant from two ⊥ st. lines XOX', YOY', its *path* or *locus* is the graph of $y = x$. Similarly, define what locus is represented by the graph of each of the following :—(i) $y = 2x$, (ii) $y=3x$, (iii) $y = 4x$, (iv $y=x+1$.

6. If a point moves so that its distance from the x-axis exceeds its distance from the y-axis by 2 units, draw its locus and find its equation. Also, find what locus would be represented by (i) $y - 2x = 4$, and (ii) $y = \frac{1}{2}x$.

Note that every graph that we draw represents a locus.

7. Draw with reference to the same axes the graphs of $y=2x$, $y = -x$, $y = -2x$, $y = -\frac{1}{2}x$, $y = 6x$, $y = -\frac{2}{3}x$, $y = \frac{1}{3}x$.

Note that the above lines pass though the origin and hence that *the graph of an equation of the type $y=mx$ where 'm' is a constant, represents a straight line passing through the origin.*

8. Draw the graphs of the following by expressing them, if necessary, in the form $y = mx$.

$x = 2y$, $x + y = 0$, $x-y=0$, $x + 2y = 0$, $x - 2y=0$, $2x+y =0$, $2x-y = 0$, $x+3y = 0$, $x-3y = 0$, $y + 6x = 0$, $\frac{2}{3}x = 4y$, $\frac{2}{3}x+4y = 0$, $x = \frac{1}{4}y$, $5x = 2y$, $5y = 4x$, $5x = 4y$.

9. Draw the graphs of the following functions: –

$3x$, $\frac{1}{2}x$, $-\frac{1}{3}x$, $-\frac{1}{2}x$, $-5x$.

Note that the graph of a function of the type 'bx' where 'b' is a constant, is a st. line passing through the origin.

10. Draw the graphs of :—

$y = x + 5$, $y = x + 3$, $y = x-5$, $y = 2x + 5$, $y =3x+5$, $y = \frac{2}{3}x + 3$, $y = \frac{2}{5}x+5$, $y = \frac{1}{2}x + 3$, $y = 5-x$, $y=2-4x$, $y = \frac{1-3x}{2}$, $y = \frac{4}{5} - 6x$, $y = -\frac{1}{8}x$, $y = -2x - 1$, $y = -\frac{2}{5}x$, $y = -\frac{2}{5}x + 6$. $y = -\frac{5}{3}x + \frac{1}{2}$, $y = -\frac{4}{3}x$, $y = -\frac{4}{3}x + 3$, $y = 10x + 5\cdot5$,

Verify in each case whether the co-ordinates of any point chosen at random in the graph satisfies its equation.

Note from the above that *the graph of an equation of the type $y = mx + c$ where 'm' and 'c' are constants, is always a straight line.*

11. Draw the graphs of :—

$$\frac{x}{2} + \frac{y}{3} = 1,\ \frac{y}{4} + \frac{x}{3} = 2,\ 2x + 3y = 5,\ 2x - 3y = 6,$$

$$x + \tfrac{5}{6}y = 1.$$

$$4x + 6y + 5 = 0,\ 3y - 2x - 4 = 0,\ \frac{x}{5} - \frac{y}{4} = 1.$$

Note that the graph of an equation of the type $Ax + By = C$ where 'A' 'B' 'C' are constants is a straight line.

12. Draw the graphs of :—

$$5x - 1,\ \tfrac{5}{2}x + 1,\ \tfrac{5}{3}x - 4,\ -\tfrac{3}{4}x + 2,\ \frac{3x-4}{2},\ \frac{4-5x}{2},\ x^2,\ \frac{x^2}{2}.$$

Find from each graph the value of the function when $x = 10$, and the value of x for which the function assumes the value 10.

It will be observed that *the graph of a function of the type $Ax + B$ where 'A' and 'B' are constants, is always a straight line.*

NOTE:—An equation of the type $y = mx + c$, or $Px + Qy + R = 0$ involving 'x' and 'y' in the *first* degree is called a linear equation. Its graph is a straight line. A function of 'x' of the type $Ax + B$ involving 'x' in the first degree is called a linear function of 'x.' Its graph is also a straight line.

Since the graph of a linear equation is a straight line *it is enough if two points are plotted to draw it.* But in so doing care should be taken to see that points too close to each other are avoided and a third point is generally plotted to ensure the accuracy of the figure. Hence in drawing the graph of a linear equation,

e.g., $2x + 5y = 15$,	x	y
put $x = 0$ and obtain $y = +3$;	0	$+3$
put $y = 0$ and obtain $x = +7{\cdot}5$;	$+7{\cdot}5$	0
put $x = 5$ and obtain $y = +1$.	$+5$	$+1$

Plot the points $(0, +3)$, $(+7{\cdot}5, 0)$ and join them by means of a straight line. Verify if $(+5, +1)$ lies on the line. Produce the line on either side. The line would represent the graph of $2x + 5y = 15$. It may be observed that if the graph cuts the x-axis at A and the y-axis at B, $OA = \frac{15}{2}$ and $OB = \frac{15}{5}$. OA and OB are known as the intercepts made by the graph on the axes. What will the intercepts be when the graphs of $Px + Qy + R = 0$ and $\frac{x}{p} + \frac{y}{q} = 1$ cut the axes?]

13. Draw by the above method the graphs of:—

$\frac{x}{2} + \frac{y}{3} = 1$, $\frac{x}{2} - \frac{y}{3} = 1$, $\frac{x}{5} + \frac{y}{6} = 2$, $2x + 5y = 10$, and $3x + 6y = 42$.

14. Assign various values for 'A,' 'B,' and 'C' in the *linear relation* $Ax + By = C$ and draw the graphs of the equations obtained.

15. Draw the graphs representing the relation $Ax+By=C$ when

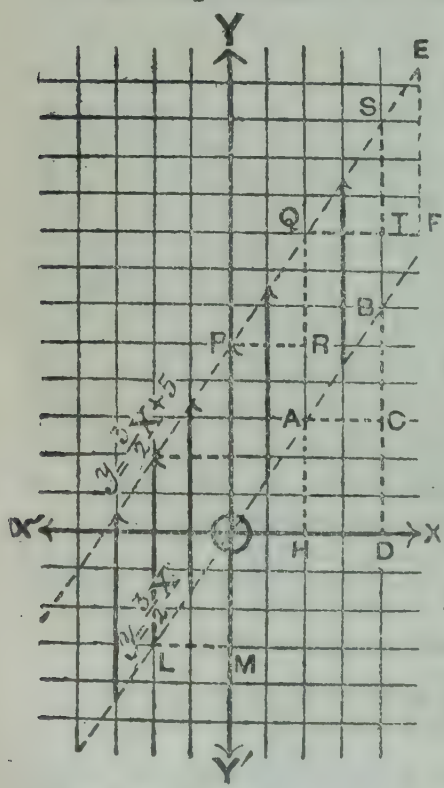
(i) $A = 0, B = 5, \quad C = 10$;
(ii) $A = 5, B = 0, \quad C = 10$;
(iii) $A = 5, B = 0, \quad C = 8$;
(iv) $A = 0, B = -5, C = 6$;
and (v $A = 1, B = 0, \quad C = 1$:

What locus does each line represent?

Note from the above (i) *that* $x = h$ *where* 'h' *is a constant represents a line parallel to the y-axis,* (ii) *that* $y=k$ *where* 'k' *is a constant represents a line parallel to the x-axis, and* (iii) *that* $x = 0$ *represents the y-axis and* $y = 0$ *represents the x-axis.*

66. GRAPHICAL INTERPRETATION OF THE CONSTANTS 'm' and 'c' IN THE LINEAR RELATION $y = mx + c$.

(i) Interpretation of 'C':—In the accompanying figure, AB represents $y = mx$ and PQ represents $y = mx + c$ when $m = \frac{3}{2}$ and $c = +5$. It may be seen that the ordinate of any point in the graph of $y = \frac{3}{2}x + 5$ is obtained by adding 5 units to that of the corresponding ordinate in the graph of $y = \frac{3}{2}x$, *e.g.*, HQ = HA + AQ = HA + 5. Thus '+ 5' in the equation $y = \frac{3}{2}x + 5$ represents how much the line $y = \frac{3}{2}x$ should be raised parallel to itself to denote $y = \frac{3}{2}x + 5$. (Refer to the arrow marks in the figure).



[Scale, $\cdot 2'' = 1$ unit].

Fig. 117.

Also when the graph of $y = mx + c$ cuts the y-axis at P, OP = $+c$. Hence

+ c represents the ordinate of point of intersection of the graph with the *y*-axis. [Where will the point of intersection be when 'c' is — ve?]

(ii) Interpretation of 'm':—In fig. 117 AB represents the graph of $y = mx$ where $m = \frac{3}{2}$. There 'm' represents the ratio $\frac{x}{y}$, *i.e.*, a ratio of the type $\frac{HA}{OH}$ or $\frac{DB}{OD}$. Also in the line PQS representing the graph of $y = mx + c$ with $m = \frac{3}{2}$ and $c = +5$, 'm' represents the ratio $\frac{y-c}{x}$, *i.e*, a ratio of the type $\frac{HQ - OP}{OH}$ or $\frac{RQ}{PR}$. Since the ratios $\frac{HA}{OH}, \frac{OM}{ML}, \frac{DB}{OD}, \frac{CB}{AC}$ are equal as well as the ratios $\frac{RQ}{PR}, \frac{TS}{QT}, \frac{FE}{QF}$, it may be seen from either graph that each such ratio indicates a rise or fall of 3 units in the ordinate corresponding to an increase or decrease of 2 units in the abscissa. **Hence 'm' represents the 'slope' or 'gradient' of the graph.** It may also be verified from the graphs that for every unit added to the value of x the value of 'y' increases by 'm' units (= $\frac{3}{2}$ units in the above case).

Thus the increase or decrease in the value of y is directly proportional to (= 'm' times) the increase or decrease in the corresponding value of x and 'm' **indicates the rate of change.** [What interpretation would you give to 'm' when it equals $-\frac{3}{4}$ or $-\frac{5}{4}$, *e.g.*, in the graphs of $3x + 4y = 0$, or $y = -\frac{3}{4}x + 2$?]

Note 1:—*The above interpretation of 'm' and 'c' would lead to the following method of drawing the graph of a linear equation* :—

Express the equation in the form $y = mx + c$, *e.g.* $y = \frac{3}{2}x + 5$. Since '$+ c$' represents the ordinate OP in Fig. 117 plot a point P, whose co-ordinates are $(0, + 5)$. Starting from that point

proceed 2 units forwards in the direction OX, then proceed 3 units upwards in the direction OY and thus reach Q; then Q is a point in the graph. Or, proceed 3 units downwards from P in the direction OY', then go 2 units in the direction OX' and thus obtain a point in the graph. In a similar manner any number of points in the graph could be obtained and the line joining them would represent the graph.

Starting from the point O, the same method would give any number of points in the graph of $y = \frac{3}{2}x$ and thus the graph could be drawn.

Note 2. From the graphs AB and PQ of equations of the type $y = mx$ and $y = mx + c$ it may be seen that in the former the value of an ordinate is proportional to the value of the abscissa, *e.g.*, $AH = \frac{3}{2} \times OH$ and in the latter the value of any ordinate, *e.g.*, QH is partly constant (*e.g.*, the part $HR = OP = 5$) and partly proportional to the abscissa (*e.g.*, the part $RQ = \frac{3}{2} \times PR = \frac{3}{2} \times OH$).

67. GRAPHICAL SOLUTION OF EQUATIONS.

Solve graphically: $3x + 2y = 34$; $4x + 10y = 60$.

By the method indicated in Note (1) Art. 61 or otherwise draw the graphs of the given equations. (See Fig. 118.)

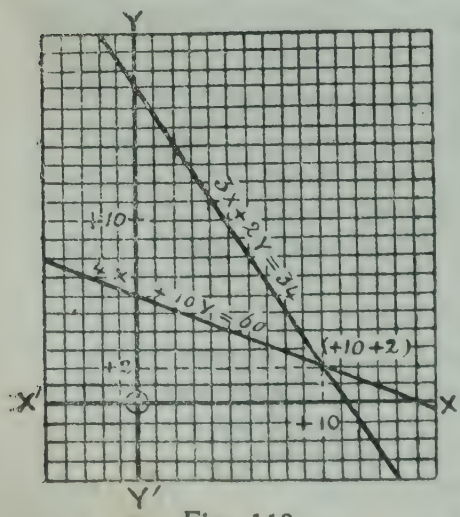


Fig. 118.

Since any number of points could be marked in the graph of $3x + 2y = 34$, their co-ordinates would give any number of values of 'x' and 'y' which would satisfy the equation. Thus the equation $3x + 2y = 34$ has an unlimited number of solutions and is hence

indeterminate. Similarly from the graph of $4x + 10y = 60$, it may be seen that its solution is indeterminate. Where the two graphs intersect the co-ordinates of the point of intersection $(+10, +2)$ give the values of 'x' and 'y' which would simultaneously satisfy the two equations. Thus $x = +10$ and $y = +2$ give the solution required in the above case. Since two intersecting straight lines could have only a single point in common, one and only one set of values could be found for 'x' and 'y' which would satisfy two linear equations. Also corresponding to the fact that two intersecting lines are required to determine a point, two independent equations are required to determine a common solution.

Ex. Show graphically (i) why the equations $x = \frac{y-5}{3}$ and $y = 3x + 2$ have no solution in common and ii) why the equations $y = 3x + 4$ and $2y - 8 = 6x$ have no particular solution in common.

[Note 1. The graphical solution of two linear equations would be only approximate for fractional values of the co-ordinates of the point of intersection of the graphs.

Note 2. Since the equation $3x + 2y = 34$ could be expressed in the form $y = \frac{34-3x}{2}$, the graph of the equation would give the various values assumed by the function $\frac{34-3x}{2}$ corresponding to various values of x. Hence the graph of the equation $3x + 2y = 34$ is also the graph of the function $\frac{34-3x}{2}$. (Refer to Fig. 118) Similarly the graph of the equation $4x + 10y = 60$ is also the graph of the function $\frac{60-4x}{10}$. Hence the abscissa of the point of intersection of the graphs of the functions $\frac{34-3x}{2}$ and $\frac{60-4x}{10}$ would determine when both the functions assume

equal value, *i. e.*, would give the value of x to be $+10$ for which $\frac{34-3x}{2} = \frac{60-4x}{10}$].

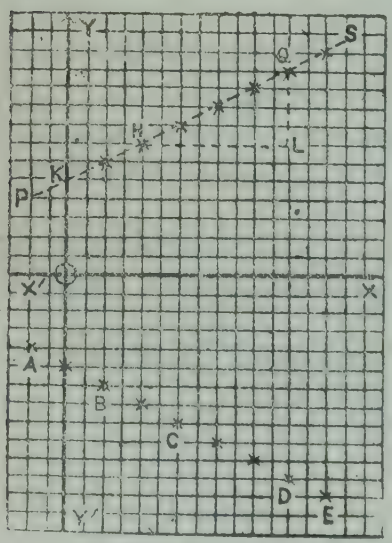
Ex Find *from the graphs* the *values* of x for which the functions differ in value by 3 units.

68. GRAPHICAL METHOD OF DETERMINING THE CONSTANTS IN A LINEAR LAW.

The following table gives the values of two variables x and y:—

x	$+12$	$+10$	$+8$	$+6$	$+4$	$+2$	-2
y	$+11$	$+10$	$+9$	$+8$	$+7$	$+5$	$+4$

Assuming a linear relation of the type $y = mx + c$ between x and y, find the values of 'm' and 'c.'



Fig, 119.

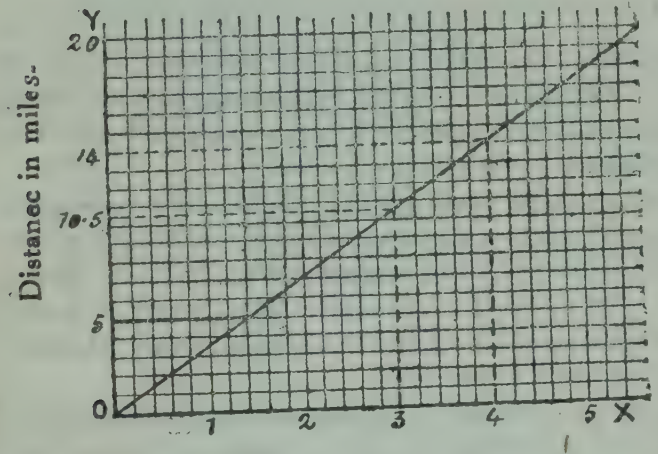
Plot the points whose abscissæ are given by the values of x and ordinates by the corresponding values of y in the above table. Draw a line so as to pass through the points, *e.g.*, PQ in Fig, 119. Let PQ cut OY at K. Then OK gives the value of 'c' to be $+5$. Name two points H, Q in the graph. Draw HL $\parallel$ to OX, and QL $\perp$ to HL. Then the value of the ratio $\frac{LQ}{HL}$ gives the value of m to be $\frac{4}{8}$ or $\frac{1}{2}$. Thus the required relation is $y = \frac{1}{2}x + 5$.

Note that the same relation could be obtained when any two points in the straight line graph are known.

Ex. If the values of x and y represent the co-ordinates of the points A, B, C, D, E in Fig. 119, find the equation of the straight line passing through them.

69. A few cases of the graphical representation of variables connected by a linear relation may appropriately be given here :—

Ex. 1. A bandy travels at a steady rate of 3·5 miles per hour. Draw a graph showing the relation between the distance travelled and the time taken.



Time in Hours.
Fig. 120.

If the distance traversed in x hours be y miles, then $y = 3{\cdot}5x$ expresses what function the distance is of time. From the relation $y = 3{\cdot}5x$ the following table could be constructed :—

Time in hours $= x$	0	2	5
Distance travelled in miles $= y$	0	7	17·5

Let the time in hours be shown as abscissæ of points along the x-axis to a scale of 1″ to 2 hours and the distance in miles as ordinates of points along the y-axis to a scale of 1″ to 10 miles. Since the distance traversed is a linear function of the time taken, the graph is a straight line and could be drawn if two points are plotted. (Refer to Fig. 120). From the graph we

could read at sight the distance traversed in a given time as well as the time taken to traverse a given distance. Thus the ordinate of a point whose abscissa represents 4 hours indicates the distance travelled to be 14 miles. Also the abscissa of a point whose ordinate represents 10·5 miles indicates the time taken to be 3 hours. Hence the graph is spoken of as a ready-reckoner.

[What do the co-ordinates of the origin indicate ?]

Note 1. The gradient of the above graph indicates a rise of 7 units vertical for every 2 units moved horizontally in the direction OX. The vertical rise indicates distance traversed and the horizontal movement indicates the time taken.

Note 2. The above graph could also be used to give the results of multiplying and dividing a given number by 3·5. Mark a point whose abscissa represents 3·2. The corresponding ordinate gives the value of 3·2 × 3·5 to be 11·2 (roughly). Next mark a point whose ordinate represents 17·5. Then the corresponding abscissa gives the value of 17·5 ÷ 3·5 to be 5.

Note 3. If milk sells at the constant rate of $3\frac{1}{2}$ measures a rupee, the above graph may indicate the quantity of milk that could be got for any given sum. In that case, represent cost in rupees along OX and quantity in measures along OY. What will the gradient of the graph then indicate ? Similarly, when ghee sells at a constant rate of Rs $3\frac{1}{2}$ a viss the graph could be used to indicate the quantity of ghee that could be got for any given sum of money. (How ?)

Hence, in general, the above graph may with slight alterations be used to represent the relation between any two variables when one of them is a function of the other connected by the relation $y = 3{\cdot}5\,x$.

Ex. 2. Two reservoirs N and R contain 25 ft. and 5 ft. depth of water respectively. Water is pumped from N to R so that the level of water in N falls at the fixed rate of 5 ft. for every 2 hours and that in R rises by 3 ft. for every two hours. Draw a graph showing the depth of water in each reservoir at any subsequent time.

In Fig. 121 the time in hours is represented along OX to a scale of ·2" to 1 hour and the depth in feet along OY to a scale of ·1" to 1 foot. It is easily seen that the depth of water is a function of the time. If 'y' be the depth of water in feet in reservoir N in (x) hours,

$$y = 25 - \tfrac{5}{2}x.$$

Similarly, $y = 5 + \tfrac{3}{2}x$ gives the depth of water (y) in feet in reservoir R in (x) hours.

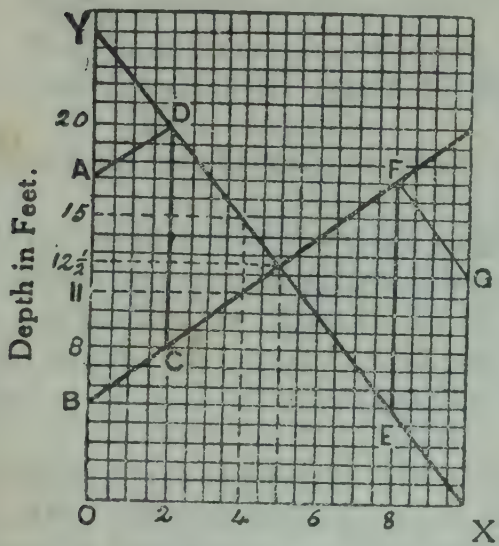


Fig. 121.

Hence the graph in either case is a straight line. Further, since the rise or fall in level is uniform in each reservoir the graph could be drawn as follows :—

(i) In the case of reservoir R :—

Plot a point (0, + 5), B in Fig. 121, to indicate that at the start the depth was 5 ft. Since the gradient should indicate a rise of 3 ft. in level for every 2 hours, proceed from B 2 units (= hours) horizontally to the right, then go 3 units (= feet) upwards and reach a point C in the graph. Similarly plot other points in the graph and thus draw BCF showing the depth of water in reservoir R at any time.

(ii) In the case of reservoir N :—

Plot a point (0, + 25) to indicate that at the beginning the depth was 25 ft. Since the gradient should indicate a fall of 5 ft. in level for every 2 hours, proceed 2 units

(=hours) horizontally to the right, then go 5 units (=feet) downwards and reach a point D in the graph. Similarly, plot other points in the graph and thus draw DEX showing the depth of water in reservoir N at any time.

From the graph the following could be read at sight:—

(i) Since the two graphs intersect at $(+5, +12\frac{1}{2})$ it is seen that at the end of the 5th hour the reservoirs would contain an equal depth of water, *viz.*, of $12\frac{1}{2}$ ft.

(ii) At the end of the fourth hour, the depth of water would be 15 ft. in reservoir N and 11 ft. in reservoir R.

(iii) The horizontal line BE indicates that the depth of water would be 5 ft. in reservoir N in 8 hours. The horizontal line through D indicates that there would be a depth of 20 ft. of water in reservoir N in 2 hours and an equal depth in reservoir R in 10 hours.

(iv) The reservoir N would be emptied in 10 hours (why?). During the time the level of water in reservoir N sinks by 25 ft., that in reservoir R rises by 15 ft.

(v) If BA represents 12 ft. and AD be drawn ‖ to BF, the vertical line DC indicates that at the end of 2 hours there would be a difference in depth of 12 ft. of water between the reservoirs N and R. Similarly, if XG represents 12 ft. and GF be drawn ‖ to XE, FE would indicate that in 8 hours the depth in reservoir N is 12 ft. less than that in reservoir R.

In a similar manner, problems involving two variables one of which is a linear function of the other, *i.e.*, to say, which are connected by a relation of the type $y = mx + c$, could be solved directly by means of graphs without the conditions given in them being expressed by means of equations.

EXERCISES.

16. Draw the graphs of :—

$$\frac{3x-6}{9}, \frac{5x+1}{5}, \frac{7x+1}{7}, \frac{3x+1}{7} \text{ and } \frac{3x-3}{5}.$$

17. Solve graphically :—

(*a*) $2x + 3y = 5$; $5x - 2y = 3$.

(*b*) $7x - 17y = 4$; $4x - 5y = 7$.

(*c*) $4x + 3y = 23$; $3x - 4y = 8$.

(*d*) $\frac{x}{3} + \frac{y}{5} = 1$, $\frac{x}{5} - \frac{y}{3} = 1$.

(*e*) $\frac{3x-4}{2} = \frac{x}{2} - 1$. (*f*) $34 - 5x = 6x + 12$.

(*g*) $177x + 133y = 8$; $133x - 188y = 15$.

Check your results otherwise.

18. One kilogram is roughly equal to 2·2 lbs. Draw a graph which would express a weight in lbs in terms of kilograms. Find from the graph how many kilograms are equivalent to 10 lbs. and how many lbs. are equivalent to 5 kilograms.

19. A speed of 'x' miles per hour is known to be equal to that of 'y' ft. per second. Draw a graph from which a speed given in miles per hour could be expressed in feet per second. Find from the graph the speed in feet per second which corresponds to one of 5 miles per hour. Also, find the speed in miles per hour equivalent to one of 22 ft. per second. (Show a speed of 30 miles per hour equals 44 ft. per second).

[Note:—Graphs of this type are called *Conversion Graphs.*]

20. Assuming Re. 1 = 1*s*. 4*d*., draw a graph from which the equivalent of a sum of money given in rupees could be read in £.

21. A distance of 8 kilometres = 5 miles (roughly). Draw a graph showing the relation between kilometres and miles.

22. A temperature of $x°$ Centigrade corresponds to one of $y°$ Reaumur. Draw a conversion graph from which a temperature stated in degrees Reaumur may be read in degrees Centigrade and *vice versa*. [Assume 100°C = 80°R.]

23. Draw a graph from which the result of multiplying any number by 1·45 could be read at sight. Show how the same

graph could be used to read the result of dividing any number by 1·45. Read from the graph values of 2·2 × 1·45 and 5·8 ÷ 1·45.

24. An income Rs. (x) per month corresponds to one of (y) gold mohurs per year. Draw a graph showing the relation between 'x' and 'y'. [Assume 1 gold Mohur = Rs. 15.]

25. Join the origin to the point (+ 5, + 2). Show that the line formed is the graph of $y = \frac{2}{5}x$. Show how from the graph the values of $\frac{3}{5}$ths of any number as well as $\frac{5}{3}$ times any number could be read.

26. Draw the graphs of $(2x + 3)$, $(3x + 5)$ and $\frac{1}{3}(5x + 9)$. Find the co-ordinates of points where they cut the axes.

27. Compare the fractions $\frac{2}{7}$, $\frac{3}{10}$ and $\frac{7}{25}$ graphically and arrange them in the ascending order of magnitude.

28. A Railway Company charges as follows for passengers' tickets:—2 pies per mile for 3rd class passengers, 6 pies per mile for 2nd class passengers and 10 pies per mile for 1st class passengers. Draw graphs which would give the charges for each class of tickets for distances up to 100 miles. Read from the graph the cost of a second class ticket for a distance of 30 miles and what distances could be travelled in the first and third classes for the same charge.

29. Coffee seed sells at Re. 1-12 as. a viss. Taking the cost of a gunny to hold it to be 4 as., draw a graph showing the relation between the coffee seed bought including the gunny and the price paid. Find from the graph how much of coffee seed could be bought for Rs. 6-6 as. and the price of 10 viss of coffee seed.

30. Show that, if F° Fahrenheit and C° Centigrade represent the same temperature, F and C are connected by the formula $F - 32 = \frac{9}{5}C$.

Draw a graph from which a temperature expressed in the Centigrade scale could be read in the Fahrenheit scale and *vice versa*. From the graph find the Centigrade readings corresponding to 10° F and 40°F and the Fahrenheit readings corresponding to —10°C and 12°C.

31. Examine if the points given below be on a straight line:—

(*a*) (+ 3, + 9), (+ 4, 12), (+ 7, + 21) and (—5, —15).

(*b*) (+ 5, + 2), (+ 8, + 3·8); (+ 11, + 5·6) and (+ 2, — ·2). Find the equation of the straight line passing though each set.

32. Two variables 'x' and 'y' are connected by the relation $x = ky$ where 'k' is a constant. When $x = +3$, $y = +5$. exhibit graphically the relation between 'x' and 'y' and find from the graph the numerical value of 'k.'

33. The following table gives the values of two connected variables 'x' and 'y':—

x	+ 4	+ 5·5	+ 7·5	+ 10·0	+ 12·5
y	+30	+ 60	+ 100	+150	+ 200

Assuming a linear relation of the type $y = mx + c$ between 'x' and 'y,' find graphically the values of 'm' and 'c.'

34. Draw the graph of $2y = 3x + 5$ and examine if (+3, +7) is a point on the graph.

35. A point moves so that the sum of its distances from two ⊥ straight lines is always 1·5". Draw its locus.

36. Plot the points (+2, + 8·6), (+ 5, +19·4) and (—2, —5·8), Test if they are collinear. Read the equation of the st. line graph passing through them.

37. A point moves so that the difference between its distances from two ⊥ st. lines = 1·2". Draw its locus,

38. Draw a straight line to pass through the points (2, + 5), (— 4, — 5). Read its intercept on the y-axis and its gradient. Verify your results otherwise.

39. Find graphically the equation of the straight line passing through (—4, —3) and (+ 6, + 9.)

40. Draw the graphs of the equations :—

(i) $x - 2y + 8 = 0$. (iii) $4x + 3y = 15$.
(ii) $2x - 3y + 12 = 0$. (iv) $5y + 2x = 20$.

Find in each case the intercepts made on the y-axis by the graph. Also, describe the gradient of each graph.

41. Through the points (+ 5, + 4), (— 3, — 2), (0, + 10) parallels are drawn to the graph of $2x + 3y = 0$. Assuming the

equation of each parallel to be $2x + 3y + c = 0$, find graphically the value of 'c' in each case.

42. Solve the following equations graphically :—

(i) $\left.\begin{matrix} 3y = 2x - 1 ; \\ 3x + 5y + 8 = 0 \end{matrix}\right\}$. (ii) $\left.\begin{matrix} y = 2x + 3 ; \\ 9x + 2y = 45 \end{matrix}\right\}$.

(iii) $\left.\begin{matrix} x + y = 2 ; \\ 5y = 7x + 34 \end{matrix}\right\}$.

Check your results otherwise.

43. Solve the following equations graphically and check your results otherwise :—

(*a*) $3x + 2 = 5x - 6$; (*b*) $\frac{1}{2}(3x + 4) = \frac{1}{6}(8x + 7)$;

(*c*) $1{\cdot}5x = \frac{3x - 9}{5}$; and (*d*) $\frac{x - 5}{6} = \frac{2x - 7}{15}$.

44. Examine if the graphs of $5y = 3x - 9$, $4x = 3y + 1$ and $y = x - 1$ meet in a point, and hence find if the three equations have a common solution.

45. Draw the graphs of relations of the type $y = mx$, when $m = +\frac{1}{3}, +3, -\frac{1}{3}$ and -3 respectively. Measure the angle which each graph makes with the other graphs as well as with the x-axis.

46. Draw the graphs of $2x + y = 4$ and $x - 2y = 4$ and measure the angles at their point of intersection.

47. Solve graphically :—

A hotel-keeper finds that if he should burn 5 kerosene lights and 2 castor oil-lamps the monthly cost of lighting would be Rs. 5 11 as. and that 3 kerosene lights and 6 castor oil-lamps would cost him Rs. 5 13 as. a month. Find how much a kerosene light as well as a castor oil-lamp costs him a month.

48. A canal feeds an empty lake at the fixed rate of 1000 c. ft. of water per minute. Draw a graph showing the quantity of water in the tank at any subsequent time.

49. A body moves at a uniform speed of 7 miles per hour. Draw a graph showing the relation between the distance traversed and the time taken.

50. If when a tank contains 500,000 c. ft. of water, its sluices are opened, they let out water at a constant rate of 600 c ft. per minute. Draw a graph showing the variation in the quantity

of water in the tank and find from it when the quantity would be reduced to half,

51. In the accompanying diagram the lines AB, CD, EF, OH represent the relation between the distance traversed and the time taken by bodies moving at different rates. If OX represents the time-axis to a scale of ·1″ to 10 seconds and OY the distance-axis to a scale of ·1″ to 10 ft., find from the gradient of each graph the speed of each moving body.

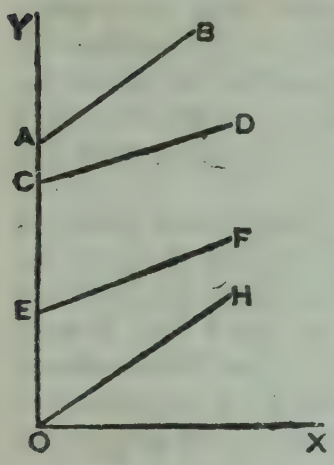


Fig. 122.

52. Find graphically which of the following speeds is the slowest and which the fastest :—

(*a*) 8 miles in 36 minutes.
(*b*) 10 miles in 44 minutes.
(*c*) 9½ miles in 43 minutes.
(*d*) 12 miles in 53 minutes.

53. If an aeroplane starts with a velocity of 'u' ft. per second and increases its velocity at a uniform rate of 'f' feet per second for every second, show that its velocity after 't' seconds is given by the formula '$v = u + ft$.'

If $u = 1$ and $f = 8$, draw a graph showing the relation between 'v' and 't'. Read from the graph the velocity of the aeroplane just after 10 seconds. Also, find when its velocity would be 88 ft. per second.

54. Stsrting from our town a cyclist goes at a uniform rate of 10 miles per hour towards Q, 25 miles away. At the same moment a jutka leaves Q and drives towards our town at a uniform rate of 5 miles per hour. Find graphically (i) when and where the cyclist would meet the jutka. (ii) when both would be 10 miles apart, and (iii) when each would be 10 miles from our town.

55. A cistern B with water 5 ft. deep is placed so that its bottom is 9 ft. higher than that of another A with water 40 ft. deep and connected with it by a siphon. If water falls 5 ft. in 3 hrs. in A and rises 4½ ft. in 2½ hours in B, find when the flow would stop and what the depth in either cistern would be at that time

56. Two cisterns, A and B, contain 25 gallons and 20 gallons of kerosene respectively. If kerosene is pumped out from A at a constant rate of 1 gallon for every 2 minutes and from B at the rate of 1 gallon for every 4 minutes, find graphically (i) the quantity of kerosene in either cistern at the end of 10 minutes, (ii) when either cistern would contain the same quantity, (iii) when the difference in quantity in the cistern would be 4 gallons, (iv) when each cistern would just contain 5 gallons, and (v) when each cistern would be just emptied.

57. In increasing the price of goods, a shop-keeper alters an original price of Rs. x into the revised price of Rs. $(2 + mx)$ where 'm' is a constant. A tin of kerosene which originally cost Re. 1 12 as. was after the revision sold for Rs. 4 2 as. Draw a graph exhibiting the relation between the original and the altered prices. Read from the graph (i) the revised price corresponding to an original price of Rs. 5 and (ii) the original price of an article selling at Rs. 6 after the revision.

58. The charge for a telegraphic message is as follows :— 12 as. for any message up to 12 words and 6 ps. extra for each word exceeding 12. Draw a graph exhibiting the variation in the charges for messages with a varying number of words. Read from the graph the charge of sending a message of 24 words and the no. of words that could be telegraphed for Re. 1 11 as.

59. A company advertises a reduction sale. It raises the price of every article by 20% and calls that the original price, and reduces this by 10% and calls this the sale price. Draw a graph to show the relation between the true original price and the sale price. Also find by how much per cent., the new sale price is above the old one.

60. A man engages a pleasure car one day by paying Rs. 2 for the use of the car and by paying an additional rate of 4 annas per mile that he rides in it. Draw a graph exhibiting the relation between the distance travelled and the cost of travelling. Read from the graph the cost of riding 40 miles and the number of miles that he could ride for Rs. 16.

61. The monthly expenses (Rs. 'y') of a hostel consist of a fixed part (Rs. 50) and a variable part proportional to (x) the number of members. If the monthly expenses amount to Rs. 470 when there are 40 members, draw a graph showing the relation

between 'x' and 'y'. Find from the graph the monthly expenses when there are 50 members and the number of members who could be maintained in a month for Rs. 680.

62. The annual income from a college is partly constant and partly proportional to the number of students. When the strength is 250 the income is Rs. 27,500 and it is Rs. 32,500 when the strength is 300. Draw a graph showing the relation between the strength and income of the college. Read from the graph the fixed part of the income and the rate of increase in the annual income for each student added to the strength.

63. The temperature of the atmosphere increases at the rate of 1°F. for every 57 ft. we proceed inside the surface of the earth. Taking the temperature at the surface to be 74·5° F., represent graphically the variation in the temperature when other conditions remain unaltered and read from the graph the temperature in a coal mine 1 mile deep.

64. The temperature of the atmosphere decreases by 1°C for every 200 metres we rise above sea-level. Taking the temperature at the sea-level to be 26° C, draw a graph showing the variation in temperature when other conditions remain unaltered. Read from the graph the temperature at a place 2500 metres above sea-level.

65. A bungalow in a town is let out at Rs. 50 per month. The house-tax. repairs, etc., amount to Rs. 100 annually. If the bungalow could be let out for only some months in the year, draw a graph showing the relation between the annual net income from the bungalow and the number of months for which it is let out. Find from the graph the anuual net income when the house was let only for 6 months in the year and the no. of months for which it was let out when the annual net income was Rs. 450.

66. For certain heights it is observed that the barometer falls roughly by 1 inch for every 1000 ft. we rise above sea-level. Taking the barometric reading to be 30 inches at sea-level, draw a graph showing the variation in the barometric resdings at different altitudes. Read from the graph the height of a hill-top where the barometer reads 28·7 inches.

67. Solve graphically Exs. 26 and 27 in page 103 and also Exs. 13 and 20 in the Chapter on Simultaneous Equations.

CHAPTER XVI.

SQUARE ROOT.

70. Square Root by Factorization :—When a number represents the product of two equal factors, each factor is called the square root of the number. Thus, in 17×17 or $17^2 = 289$, 17 is the sq. root of 289. (Refer to Art. 32, page 53). The symbol $\sqrt{289}$ or $(289)^{\frac{1}{2}}$ represents the sq. root of 289. Numbers like 1, 4, 9, etc., whose sq. roots could be *exactly* found are called **perfect squares.**

When a perfect square is expressed as the product of its prime factors, its sq. root can be easily written down by inspection, *e.g.*, $\sqrt{1764} = \sqrt{2 \times 3 \times 7 \times 2 \times 3 \times 7} = 2 \times 3 \times 7$ or 42.

Note :—(1) Since $\sqrt{2^2 \times 3^2 \times 7^2}$ and $\sqrt{2^2} \times \sqrt{3^2} \times \sqrt{7^2}$ are each equal to $2 \times 3 \times 7$, $\sqrt{abc} = \sqrt{a} \times \sqrt{b} \times \sqrt{c}$

(2) In the relation $2^3 = 8$, '2' is called **the cube root** of '8' and may be written as $\sqrt[3]{8}$ or $(8)^{\frac{1}{3}}$. Also, in $5^4 = 625$, '5' is called **the fourth root** of 625 and may be written as $\sqrt[4]{625}$ or as $(625)^{\frac{1}{4}}$. These roots also could be found by factorization in a few cases, *e.g.*, $\sqrt[3]{216} = \sqrt[3]{3^3 \times 2^3} = 3 \times 2$ or 6 and $\sqrt[4]{50625} = \sqrt[4]{5^4 \times 3^4} = 5 \times 3$ or 15.

EXERCISES.

1. Find, by means of factors :

(*a*) the sq. roots of 196 ; 625 ; 5776 ; 48400 ; 5625 ; a^2b^4 ; b^2c^4 : $a^2b^6c^8$:

(*b*) the cube roots of 1000 ; 1728 ; 512 ; 2744 ; $a^6b^6c^3$; $x^6y^9z^{15}$; x^6y^6 ;

and (*c*) the fourth roots of 6561 ; 625 and 10000.

2. Find the least number by which the following should be multiplied in order to give perfect squares :—192 ; 360 ; 3150 ; 6050 ; 14112 and 1854.

3. Find the digits in the units' place in the squares of integers ending in 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Hence, show that *no square integer can end in* 2, 3, 7 *or* 8. Show also that a square number would not end in an odd no. of ciphers nor in an odd no. of decimal places.

4. Find the langth of a square of area (*a*) 144 sq. ins, (*b*) 900 sq. ft. (*c*) 169 sq. chains and (*d*) 225 sq. yds.

5. From the formula $R=6+\cdot009\ V^2$, find the value of V when 'R'=14·1.

6. The product of two equal factors = 65,536. Find them by factorising.

71. General Method of extracting the sq. root :— The method herein described may be adopted when the prime factors of a number are not easily found.

(*a*) Ex. (i) Find the sq. root of 529. Suppose a square ABCD has an area of 529 sq. ins. and hence its side AD = $\sqrt{529}$ ins. Since 529 lies between 20^2 and 30^2, AD should be greater than 20 ins. but less than 30 ins. Let AQ represent 20 ins. (See Fig. 123). Then, if QD be known the sq. root of 529 could be found. Cut out the square AQHR and fit the remaining portion into a rect. EFCS by placing RBSH in the position EFDQ. Since the rect. EFCS contains $(529-20^2)$ or 129 sq. ins. and its length ES is slightly greater

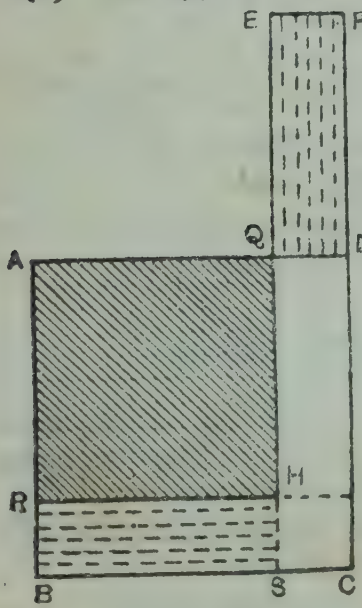


Fig. 123.

than 2×20^2 ins. (why?), its width QD should be a little less than $\frac{129}{2 \times 20}$ or $3\frac{9}{40}$ ins. Thus an approximation to the width QD is obtained to be 3 ins. Since when QD is assumed to be 3 ins. the rect. EFCS = (EQ + QH + HS) × QD = (20 + 20 or $2 \times 20 + 3$) × 3 or 129 sq. ins. AD is equal to (20 + 3) or 23 ins. Hence 23 is the sq. root of 529.

From the above it follows that **if in finding the sq. root of a number**, *e.g.*, 529, **a part 'n'** (= AQ) **be known, an approximation to the value of the remaining part 'r'** (= QD) **is suggested by dividing** $(529 - n^2)$ **by** $2n$. Also, ES = $2n + r$, EF = r, the rect. EFCS = $(2n + r) \times r$ and the sq. AQHR + the rect. EFCS = the sq. ABCD, *i.e.*, to say, $n^2 + (2n + r) \times r = (n + r)^2$.

Hence the following working :—

$20^2 =$ | $+ 3 = r$, $23 = n$ } = sq. root,
$2n = 40$ | $529 = (n + r)^2$
| $400 = n^2$
| 129 contains $(2n + r) \times r$

$\frac{129}{2n}$ suggests $r = 3$

$\therefore (2n + r) \times 3 = 43 \times 3$ = | $129 = (2n + r) \times r$.

$\therefore$ 23 is the sq. root of 529.

[Note that the place-value of the digit '5' in 529 suggests '20' to be a part of the root. Also 'n' at any stage represents the *known* part of the root and 'r' the *remaining* part.]

Ex. Find by the above method the sq. roots of 289 ; 576 ; 625 ; 529 ; 961 ; 1521 ; 11025 ; 11449 and 42025.

(*b*) **Finding the number of digits in square roots :—**

(i) Since $\sqrt{1} = 1$ and $\sqrt{100} = 10$, the sq. root of an integer lying between 1 and 100 must lie between 1 and

10. Hence the sq. root of an integer of one or two digits would have only one digit in its integral part.

(ii) An integer having 3 or 4 digits lies between 100 and 10,000 and hence its sq. root must lie between 10 ($= \sqrt{100}$) and 100 ($= \sqrt{10,000}$) and would have only 2 digits in its integral part.

(iii) An integer having 5 or 6 digits lies between 10,000 ($= 100^2$) and 1,000,000 ($= 1000^2$) and hence its sq. root would have only 3 digits in its integral part (*why* ?) ; and so on.

In general, the sq. root of an integer of (2n—1) or 2n digits would have 'n' digits in its integral part. Hence, if in an integer dots be marked over the units' digit and over every second digit to the left, the integer would be marked into periods of two digits from the extreme right and the number of dots marked would give the number of digits in the integral part of the sq. root. Thus the sq. roots of $1\dot{3}2\dot{2}25$ and $\dot{2}3\dot{4}0\dot{1}$ would have 3 digits in the integral part.

Similarly, since $\sqrt{\cdot 01} = \cdot 1$, $\sqrt{\cdot 0001} = \sqrt{\cdot 01}$, etc., it may be seen that *for every two digits in the decimal part of a number there would be one digit in the decimal part of the sq. root*. Hence, *if in a decimal* which is a perfect square *dots be marked over every second digit to the right of the decimal point, i.e.*, over the hundredths' place, ten-thousandths' place, etc., *the number of such dots would give the number of digits in the decimal part of the sq. root*.

(*c*) *Ex.* (ii) Find the sq. root of 1204·09. By dotting or otherwise, it may be seen that if $1\dot{2}0\dot{4}\cdot 0\dot{9}$ is a perfect square its sq. root would contain two digits in the integral part and one in the decimal part. Since 1204·09 lies between 30^2 and 40^2, a part of the root (N) is known to be 30. Hence the remaining part (R)

should be less than $\frac{1204{\cdot}09 - 30^2}{2 \times 30}$ or $\frac{304{\cdot}09}{60}$. [Refer to (*a*) above.] If '$R = 5$' be tried, $(2N + R) \times R$ equals 65×5 and exceeds 304·09. Hence R should be less than 5. Try if $R = 4$. Then $(2N + R) \times R = 64 \times 4$ (why?) or 256 and is less than 304·09 by 48·09. Thus a part of the root (n) is known to be (30 + 4) or 34. Also, the remainder left, *viz.*, 48·09 $= 1204{\cdot}09 - 30^2 - 2 \times (30 + 4) \times 4 = 1204{\cdot}09 - 34^2$. [*Why?*] Hence by (*a*) above, the rest of the root (r) should be less than $\frac{48{\cdot}09}{2 \times 34}$ or $\frac{48{\cdot}09}{68}$. Try if '$r = {\cdot}7$.'

Then $(2n + r) \times r = 68{\cdot}7 \times {\cdot}7$ or 48·09.

Thus the sq. root of 1204·09 = 30 + 4 + ·7 or 34·7.

Hence the following working :—

Detailed Working :—

$$\left.\begin{matrix} + & {\cdot}7 \\ + & 4 \\ & 30 \end{matrix}\right| = \text{sq. root.}$$

$N^2 = 30^2 = \left|\begin{matrix} 1204{\cdot}09 \\ 900 \end{matrix}\right. = N^2$

$2N = 60$

From $\frac{304{\cdot}09}{2N}$, R = 4

$\therefore (2N+R) \times R \left\{\begin{matrix} = 64 \\ + 4 \end{matrix}\right\} = \left|\begin{matrix} 304{\cdot}09 \text{ contains } (2N+R) \times R \\ 256 \qquad = (2N + R) \times R \end{matrix}\right.$

$2n = 68$

From $\frac{48{\cdot}09}{2n}$,

$r = {\cdot}7$

$\therefore (2n+r) \times r \left\{\begin{matrix} = 68{\cdot}7 \\ \times 7 \end{matrix}\right. = \left|\begin{matrix} 48{\cdot}09 = 1204{\cdot}09 - 34^2 \text{ (Why?) and contains } (2n+r) \times r \text{ where } n = 34. \\ 48{\cdot}09 = (2n+r) \times r \end{matrix}\right.$

Brief Working:—

$$\begin{array}{r|l} & 34{\cdot}7 \\ \hline 3 & 1204{\cdot}09 \\ & 9 \\ \hline 64 & 304 \\ & 256 \\ \hline 68{\cdot}7 & 48{\cdot}09 \\ & 48{\cdot}09 \\ \hline \end{array}$$

$\therefore$ the sq. root of 1204·09 = 34·7.

The above method may be used in finding the sq. root of *any* number in the decimal scale.

[**Note**:—In the brief working the superfluous figures, *e.g.*, the ciphers in '900', etc., are omitted. In the 2nd step one more period alone, *viz.*, '04' is written after '3'. The process $(60 + 4) \times 4$ is indicated by writing '4' to the right of '6' and then multiplying 64 by 4 orally. The digit '7' in the sq. root is suggested by dividing 480 (tenths) by 68, '$2n$,' *i.e.*, twice the part *known* at the second stage is more easily obtained by adding '4' to '64' than by multiplying 34 by 2. After sufficient practice in the above method the working could be further simplified by adopting Italian division.]

Ex. 1. Find by the above method the sq. roots of 17·9776; 930260·25; and 105·0625.

Ex. 2. Given that the sq. root of 12·0409 = 3·47, find therefrom the sq. roots of 1204·09 and 120409. Can you deduce likewise the sq. roots 120·409 and 1240·9? Why not?

(*b*) **Sq. roots of numbers that are not perfect squares :—** The method indicated in (*c*) above may be adopted in finding the sq. roots of these numbers, but it will be seen that **the roots never terminate** and hence could not be exactly found. Such roots are called **surds** and their values could be found only approximately.

Ex. (iii) Evaluate $\sqrt{50}$ as far as the third decimal place.

```
      7. 0 7 1.....
    50·00̇00̇00̇.....
       49
14·07) 1 . 0000
          9849
14·141) 15100
          14141
            959
```

$\therefore \sqrt{50} = 7{\cdot}071\ldots\ldots$, which is a *non-terminating* decimal. *Distinguish it from a recurring decimal.*

[**Note**:—The usual process is not possible in the 2nd step with dividend '1·00' and with divisor '14.' Hence the digit '0' is put in the tens' place in the sq. root and a further period '00' added to the dividend. Also, $\frac{1{\cdot}00}{14}$ or $\frac{100 \text{ hundredths}}{14}$ suggests only '7' *hundredths*, *i.e.*, '·07' in the sq. root.]

Ex. Show in a similar manner that $\sqrt{2} = 1{\cdot}414\ldots\ldots$ and $\sqrt{3} = 1{\cdot}732\ldots\ldots$ (*These equivalents may be memorised with advantage.*)

(*c*) Sq. roots of vulgar fractions :—

Ex. (iv) Find the sq. root of $\frac{45}{245}$.

$$\sqrt{\frac{45}{245}} = \sqrt{\frac{9}{49}} = \frac{\sqrt{9}}{\sqrt{49}} = \frac{3}{7}.$$

Here the fraction in its lowest terms happens to be a perfect square. Hence, in such cases *express the fraction in its lowest terms and find the sq. roots of the numerator and the denominator.*

[NOTE:— $\sqrt{5\frac{20}{45}} = \sqrt{\frac{245}{45}} = \sqrt{\frac{49}{9}} = \frac{7}{3}$.]

Ex. (v) Ffnd the sq. root of $\frac{1}{2}$.

In this case the fraction is not a perfect square and its sq. root may be found as follows :—

Method (i) By finding the sq. root of the equivalent decimal :—

$\frac{1}{2} = {\cdot}5$. $\therefore$ $\sqrt{\frac{1}{2}} = \sqrt{{\cdot}5} = {\cdot}707$ (nearly).

Method (ii).

$$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{1{\cdot}41421\ldots\ldots}{2}$$

or ·7071.....

If $\frac{1}{\sqrt{2}}$ be expressed as $\frac{1}{1{\cdot}41421\ldots\ldots}$, a step in *long division* would be involved. To save this labour, *the denominator is cleared of surds* by multiplying each term of $\frac{1}{\sqrt{2}}$ by $\sqrt{2}$.

Similarly, $\frac{1}{\sqrt{2}-1} = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \frac{(\sqrt{2}+1)}{(2-1)}$ or 2·4142 .....

[NOTE:—The following formulæ may be found useful in the simplification of surds :—

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$.
2. $\sqrt{a^2b} = a \times \sqrt{b}$.
3. $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = [a - b]$.

72. Linear Representations of the Sq. Roots of Number :—

In Art. 50, page 133, it has been shown that, in a rt.-∠d △ AHG with ∠H a rt. ∠, $AG^2 = AH^2 + HG^2$. Hence $AG = \sqrt{AH^2 + HG^2}$, $AH = \sqrt{AG^2 - HG^2}$ and $HG = \sqrt{AG^2 - AH^2}$. With the help of these equalities the sq. root of a number (n) could be represented graphically as follows:—

Method (i). Construct a rt.-∠d △ AHG so that ∠H = a rt. ∠, $AG = \frac{n+1}{2}$ and $HG = \frac{n-1}{2}$. Then $AH^2 = \left(\frac{n+1}{2}\right)^2 - \left(\frac{n-1}{2}\right)^2 = n$ (why ?)

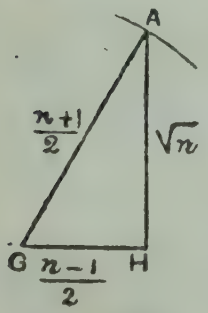


Fig. 124.

Hence AH represents $\sqrt{n}$.

NOTE.—If lines have to be bisected in representing $\frac{n+1}{2}$, etc., adopt a method like that indicated in Fig. 65, page 106 or in Art. 76 (*a*)].

Method (ii). Draw a line PQ so that its parts PH, HQ represent two nos. whose product $= n$, *e.g.*, when $n = 7$, let PH $= 7$ and HQ $= 1$ or when $n = 15$, let PH $= 5$ and HQ $= 3$. Bisect PQ at A and draw a semi-⊙ with PQ as diameter. Draw HQ ⊥ to PQ to meet the Oce at G. Then HG would represent $\sqrt{n}$.

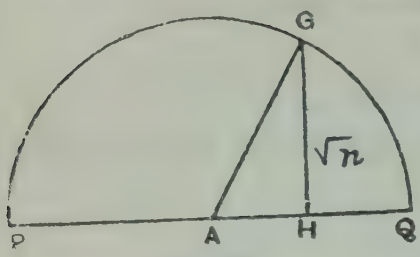


Fig. 125.

For, $HG^2 = AG^2 - AH^2$
$= (AG + AH)(AG - AH)$
$= (PA + AH)(AQ - AH)$
$= PH \times HQ$
$= n.$

$\therefore$ When PH $= 7$ and HQ $= 1$, HG would represent $\sqrt{7}$ and when PH $= 5$ and HQ $= 3$, HG would represent $\sqrt{16}$.

73. A few examples are worked below which involve the application of sq. root :—

Ex. (vi) From a Railway junction (M) two trains start simultaneously. One of them goes S.-W. at 18 miles an hour and the other goes N.-W. at 12 miles an hour. How far apart would they be in 2 hours ?

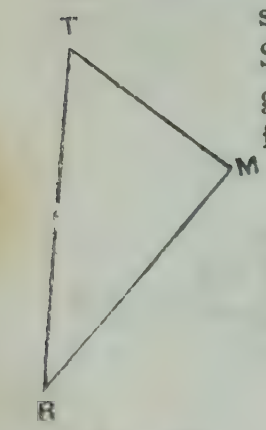


Fig. 126.

Let T and R (in Fig. 126) be the positions of the trains two hours after leaving M. Then TM represents 24 miles and MR 36 miles. Also ∠TMR = a rt. ∠.

$\therefore$ TR $= \sqrt{TM^2 + MR^2} = \sqrt{24^2 + 36^2}$
$= \sqrt{12^2(2^2 + 3^2)} = 12\sqrt{13}.$

Hence in two hours the trains would be $12\sqrt{13}$ or 43·3 miles apart to the nearest tenth of a mile.

Ex. (vii). The boy-scouts in a city are arranged in the form of a solid square and it is found that there are 11 left. If there are 740 scouts, find how many of them stand in each side of the square.

The number reqd. $= \sqrt{740 - 11}$ or $\sqrt{729}$.
$= \sqrt{9 \times 9 \times 9}$ or $\sqrt{3^2 \times 3^2 \times 3^2}$
$= 3 \times 3 \times 3$ or 27.

Ex. (viii) A ladder 18 ft. long leans against a wall at a point 16 ft. from ground-level. How far is the foot of the ladder from the wall?

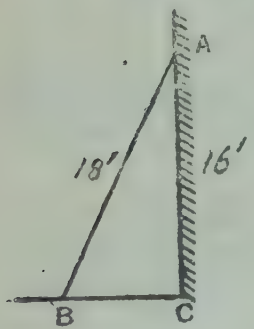


Fig. 127.

Let AC be the wall, BC the ground-level and AB the ladder.

Then BC $= \sqrt{18^2 - 16^2}$ ft.
$= \sqrt{(18 + 16)(18 - 16)}$ ft.
$= \sqrt{68}$ or 8·246 ft.

$\therefore$ the foot of the ladder is *very nearly* 8·2 ft. from the wall.

EXERCISES.

7. Find the sq. roots of :—

(*a*) 233·401 ; 210681 ; 10201 ; 7921 ; 1071225 ; 1522756 ; 18671041 ; 1874161 ; 114244 ; 93025 ; 302500 ; 1764 ; 998001.

(*b*) 325·4416 ; 3164·0625 ; 152·5225 ; 2065·7025 ; 460701·5625; 35·8801 ; 1849·086001.

(*c*) $344\frac{4}{49}$; $32\frac{196}{225}$; $254\frac{1}{156}$; $32\frac{1}{9}$; $135\frac{9289}{13456}$; $1\frac{6992}{16225}$; $\frac{595}{280}$ and $1\frac{126}{4489}$.

8. Find correct to 3 places of decimals the sq. roots of 17 ; 18 ; 1·53 ; 7 ; ·02 and ·037,

9. A square field is 1 acre in area. Find to the nearest yd. the length of its side.

10. Calculate to the nearest yd. the length of the diagonal (*a*) of a square of side 10 yds. and (*b*) of a rect. 30 yds. by 15 yds.

11. The area of two square fields are as 625 : 33124. Find the ratio of their sides.

12. The length of a rectangular field is twice its breadth. If its area = 1352 sq. yds., find the length of each side.

13. In an *exact* division the quotient and the divisor are equal. The dividend = 32,228,329. Find the quotient.

14. Two fields, each of area 2 acres, are together exchanged for a square field covering an equal area. Find the length of the square field in yards.

15. In a garden cocoanut trees are planted and there are as many rows as there are trees in each row. If there be 1369 trees on the whole, find the number of rows as well as the number of trees in each row.

16. The subscribers to a fund together contributed Rs. 46,875. If each of them contributed thrice as many rupees as there were subscribers, find the no. of subscribers and the contribution given by each.

17. How long will it take a person to walk round a square garden of area 3 acres, 121 sq. yds. at the rate of 3 miles an hour?

18. The cost of levelling a square field was Rs. 216 at 2 pies per sq. yd. Find the cost of fencing it at 8 as. a yd.

19. Find the value of 'x' when

(i) $\frac{5{\cdot}85}{x} = \frac{x}{23{\cdot}4}$.

and (ii) $\frac{x}{1{\cdot}3} = \frac{105{\cdot}3}{x}$.

20. Given that $A = P\left(1 + \frac{r}{100}\right)$, find '$r$' when $A = 1188{\cdot}1$.

21. From the corners of a square of perimeter 25 ins. four equal squares are cut off and the remaining part contains 36 sq. ins. Find the length of each square cut off.

22. The difference between the areas of two squares = 1600 sq. ft. The bigger square is 85 ft. long. Find the length of the smaller square.

23. *If in a rt.-∠d △ one ∠ = 60°, show that the sides are in the ratio 1 : 2 : √3. If, instead, one ∠ = 45°, show that the sides are in the ratio 1 : √2 : 1.*

A ladder inclined at 45° to the ground just reaches the top of a wall 12 ft. from the ground. If it slips and occupies a position inclined at 60° to the wall, calculate how far from the wall its other end lies.

24. Given the co-ordinates of a point to be (+ 4, + 5), calculate its distance from the origin.

25. A, B, C, D are 4 villages in the neighbourhood of a town T. A is 2 miles due North of T, B 4 miles due South, C 3 miles due West and D 2½ miles due East. If straight roads have to be constructed leading from each village to every other, find the length of each road.

26. Two vertical poles firmly fixed to the ground are 24 ft. apart. If their heights above the ground be 12 ft. and 20 ft. respectively, find the length of a wire connecting their tops.

27. Calculate to the nearest hundredth of a unit the perimeter of a quadrilateral whose vertices are the points (−10, 0), (0+20), (+ 20, + 15) and (+ 30, 0).

28. Draw a line AB representing 1 unit. Draw BC ⊥ to AB and equal to it. Join AC. Show that AC represents √2. Draw D ⊥ to AC and = 1 unit. Join AD. Show that AD represents √3. Draw DE ⊥ to AD and = 1 unit. Join AE. Show that AE = √4. In this way, represent graphically √6, √7 and √8.

29. If $\frac{v^2}{r} = \frac{g}{284 + 5280}$, find '$v$' to the nearest hundredth when 'r' = 3612·5 and $g = 32$.

30. If $v^2 = u^2 + 2fs$, find 'v' when $u = 16$, $f = 37$ and $s = 1000$.

31. The rule $\frac{W \times V^2}{1{\cdot}25 \times R}$ = Elevation of outer rail in inches is used in laying rails along a railway curve, where 'W' = the

width of gauge in ft. 'R' = the radius of the curve in ft, and 'V' = average velocity of trains in miles per hour. Find 'V' when 'W' = 3·28, 'R' = 1000 and Elevation = ·5".

32. The pressure of wind in lbs. (P) per sq. ft. against a surface ⊥ to its direction is given by $P = \frac{v^2}{200}$, where 'v' = its velocity in miles per hour. Find to the nearest tenth of a mile the velocity of wind when it exerts on the surface a pressure of 10·825 lbs. per sq. ft. If in the cyclone of 1918 a house had to bear a pressure of 24½ lbs. per sq. ft., find the velocity of the cyclonic wind.

33. A reservoir would supply a place without fail if it is stored with $\frac{1000}{\sqrt{F}}$ days' supply of water, where 'F' = $\frac{5}{6}$ of the average yearly rainfall in inches for 3 consecutive dry years. If near Cape Comorin the above average be 23", find how many days' supply of water should be stored in a proposed reservoir so as to supply the place without fail.

34. The space (s) feet traversed in (t) seconds by a body let fall from rest is given by $s = \frac{1}{2}gt^2$. Find the time in which a body would reach the ground if let fall from a height of 1000 ft. [Assume $g = 32·2$].

35. Of two proper fractions, one a square and the other its sq. root, explain why the latter is always greater than the former.

36. Taking ½" = 1 unit, draw lines to represent $\sqrt{5}$, $\sqrt{7}$, $\sqrt{17}$, $\sqrt{72}$, $2 \times \sqrt{5}$, $3 \times \sqrt{2}$ and $(\sqrt{5} - 1)$.

37. Find the value of $\sqrt[4]{18}$ correct to 2 places of decimals by finding the sq. root of $\sqrt{18}$.

38. Given that $\sqrt{2} = 1·414...$, deduce the value of $\sqrt{5000}$ to the nearest tenth.

39. The total surface area of a cube is 144 sq. ins. Find, to the nearest $\frac{1}{10}$, the length of its edge and that of the diagonal of a face.

40. When a cuboid is placed on a horizontal surface the corners P, Q, R, S in the top face are found to be vertically above

the corners A, B, C, D, in the opposite face. Explain how you would find (*a*) the diagonal of each face and (*b*) *the diagonal PC of the solid*. Hence show that (i) if the edge of a cube $= a''$, the diagonal of each face $= a \times \sqrt{2}''$ and the diagonal of the cube $= a \times \sqrt{3}''$ and (ii) if cuboid be l'' long, b'' broad and h'' high, its diagonal $= \sqrt{l^2 + b^2 + h^2}''$.

Give the numerical results when $a = 5$, $l = 5$, $b = 3$ and $h = 4$.

41. Find the values of the following correct to 3 places of decimals:—

$$\sqrt{\tfrac{1}{3}}; \frac{2\sqrt{2}}{3\sqrt{3}}; \qquad \frac{\sqrt{5}+1}{\sqrt{5}-1} \text{ and } \sqrt{16} + \sqrt{18}.$$

42. Construct a rt.-$\angle$d $\triangle$ with the sides containing the rt. $\angle$ equal to $(\sqrt{3}-1)''$ and $(\sqrt{3}+1)''$. Calculate the length of the hypotenuse and verify the result by measurement.

43. The adjacent sides of a rect. are in the ratio 1 : 2. Find their lengths when the diagonal $= 3' 5''$.

44. A ladder 15′ long is placed against a wall with one end resting on the ground at a distance of 9′ from the wall. Calculate the height of the other end of the ladder and verify your result from a diagram drawn to scale. Find also the inclination of the ladder to the horizontal.

45. A ladder 20′ long leans against a wall, making with it an $\angle$ of 30°. How far would the top descend if the foot of the ladder is pulled out 2′ farther from the wall?

46. A ladder 15′ long would reach to a window 12′ from the floor in one wall of a building. If the foot of the ladder be retained in the same position it would reach to a window 10′ high in a parallel wall. Find the distance between the walls.

47. A householder finds a certain number of beggars at his door. He gives to each man one-fourth as many pies as there are beggars, to each woman three-fourths as much as to each man, and to each child two-thirds as much as to each man. If he spends Rs. 2-8 as. and if the number of men, women, and children are as 5 : 4 : 3, how much does each child get?

CHAPTER XVII.

SYMMETRY IN FIGURES; CONSTRUCTIONS.

SECTION I.

Properties of symmetrical figures and their applications.

74. SYMMETRY.

Fold a sheet of paper along a st. line AB. Prick the double sheet with pins along a broken line drawn in it as in Fig. 128. If the paper be unfolded and laid flat, the portion bounded by the dotted lines formed would be like Fig. 129 and consist of two parts lying on either side of the crease-line AB. Since each part is exactly like the other the whole figure is said to be symmetrical about AB. The line AB is called its axis of symmetry. Also, two points, *e.g.*, P,P′ which would coincide when the figure is folded along AB are said to be symmetrical about AB. Similarly, two lines, *e.g.*, PS, P′S′ which would exactly coincide when the figure is folded along AB may be said to be *symmetrical* about it.

Fig. 128.

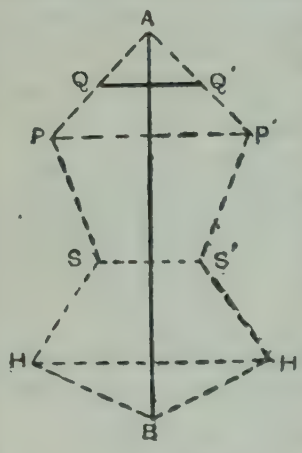


Fig. 129.

The following properties may be observed by folding :—

(*a*) *The axis of symmetry divides the whole figure into two congruent parts.*

(*b*) *The st. line joining any two symmetrical points is bisected at rt. ∠s by the axis of symmetry, e.g.*, AB bisects at rt. ∠s PP′. QQ′, SS,′ etc.

(*c*) *Any two lines which are symmetrical about a given axis are equal in length and are equally inclined to the axis, e.g.*, PS = P′S′, QS′ = Q′S, ∠ABH = ∠ABH′, etc.

(*d*) *Any point (k) in the axis of symmetry AB is equidistant from any two symmetrical points like P, P′ [as well as from any two symmetrical st. lines like SP, SP′. Why?]*

75. A few applications :—

(i) Draw an isos. △ ABC with AB, AC equal. Fold it so that AB falls along AC. It will be found that *the isos. △ is symmetrical about AD the line of fold.* Hence,

∠BAD = ∠CAD,

∠ABC = ∠ACB.

BD = DC

and ∠ADB = ∠ADC = a rt. ∠.

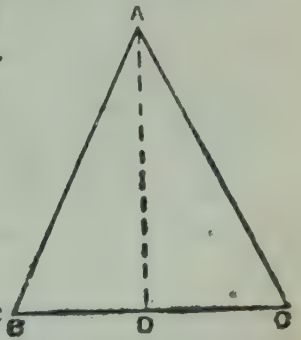


Fig. 130.

Note:—(1) In an isos. △ the bisector of the vertical ∠ bisects the base at rt. ∠s.

(2) All points equidistant from B and C must lie in the line AD, or AD produced, which bisects BC at rt. ∠s, *i.e.*, in the *perpendicular bisector* of BC.

(3) If in an isos. △ ABC the vertical ∠BAC = x°, ∠ACB = $\left(90 - \frac{x}{2}\right)^\circ$ = ∠ABC.

(ii) **Smmetry in a circle:—**

Describe a circle with centre C and with any radius. Fold it along any diameter AD. As the two parts on either side of AD exactly coincide, *the circle is symmetrical about the diameter.* The following properties may be observed:—

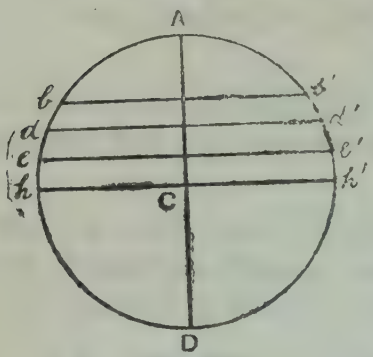


Fig. 131.

(*a*) The chords joining sets of symmetrical points on the Oce like b, b'; d, d', etc., are bisected at rt. ∠s by the axial diameter AD.

(*b*) If a circle be folded so that the two parts of any chord bb' drawn in it exactly coincide, the line of fold would be a diameter ⊥ to the chord.

(*c*) The chord joining symmetrical points on the Oce are equal, *e.g.*, $Ab = Ab'$; $db = d'b'$; $bh' = b'h$; etc.

(*d*) Similarly, the arcs Ab, Ab'; db, $d'b'$; etc., are also equal.

(iii) Draw two circles with unequal radii to cut each other. Let C, D be their centres and P, Q their points of intersection. Join CD, PQ. It may be seen by folding that the figure is symmetrical about CD and hence that the *common chord* PQ is bisected at rt. ∠s by CD. [Find the arcs and segments bisected by CD.]

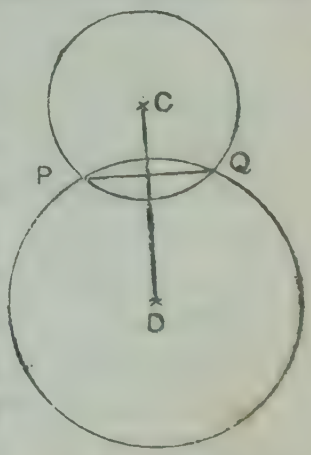


Fig. 132.

Draw some more circles to cut at P, Q. Where do their centres lie? Hence, if two or more circles intersect so as to have a common chord

PQ, their centres would lie in the *perpendicular bisector* of PQ.

(iv) Draw two equal circles with centres P and Q so as to cut at A, B. Join PQ, AB and let them intersect at O. It may be found that the figure is symmetrical about PQ and AB. Hence AP = AQ, BP = BQ, OA=OB, OP=OQ and ∠AOP=∠AOQ = a rt. ∠.

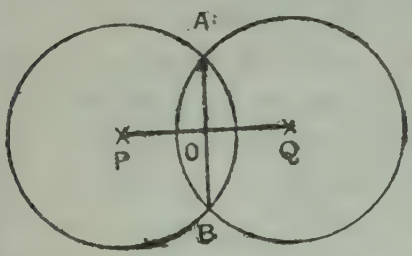


Fig. 133.

Also, if pairs of equal circles be drawn with centres P and Q, their points of intersection would be equidistant from P and Q and would be in AB or AB produced, *i.e.*, in the ⊥ bisector of PQ.

[The centre of a circle should lie in the ⊥ bisector of any of its chords.]

76. The undermentioned construction methods follow from the above:—

(*a*) *To bisect a st. line* PQ:—

With centres P and Q and with equal radii draw arcs to cut at A and B. Join AB to cut PQ at C. Then C will be the middle point of PQ. [Why?]

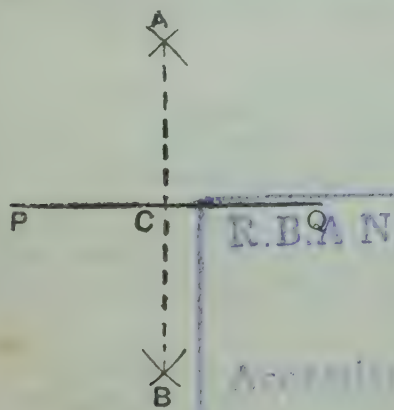


Fig. 134.

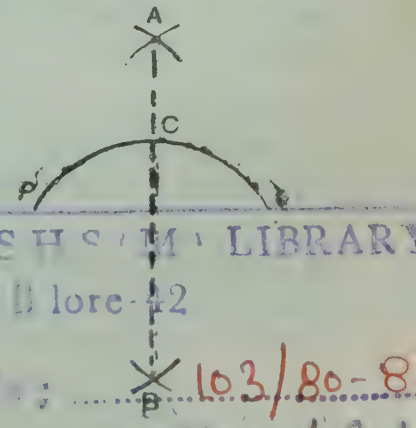


Fig. 135.

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The same construction may be used to obtain the ⊥ bisector of a st. line PQ as well as to find the middle point of an arc PQ of a circle.

(*b*) *To draw a* ⊥ *to* PQ *through a point* C *in it* :—

With centre C and with any radius draw arcs to cut PQ at A, B. (See Fig. 136). With centres A, B and with a convenient radius draw arcs to cut at D. Join CD. Then CD will be ⊥ to PQ. [Why ?]

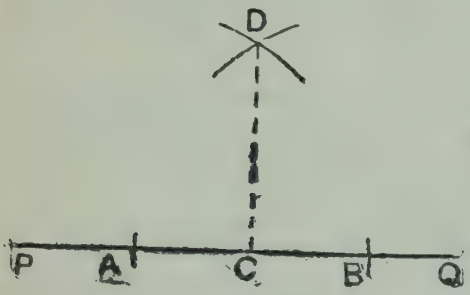


Fig. 136.

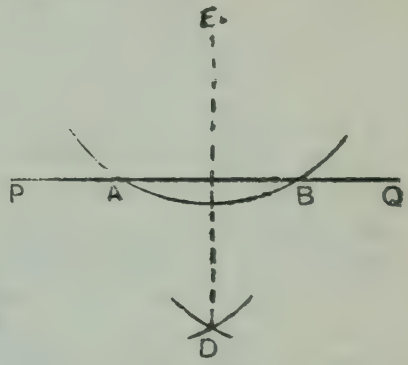


Fig. 137.

(*c*) *To draw a* ⊥ *to* PQ *from an external point* E :—

With centre E and with any convenient radius draw an arc of a circle to cut PQ at A, B. (See Fig. 137). With centres A and B and with any convenient radius draw arcs to cut at D. Join ED. Then ED will be ⊥ to PQ. [Why ?].

EXERCISES.

1. Mark two points A, B which are respectively 2 cm., and 4 cm. from a st. line PQ. Mark two points C, D so that the pairs of points (A, C) and (B, D) are symmetrical about PQ.

2. Construct a triangle ABC so that ∠ABC = ∠ACB. Find a line of symmetry in the triangle. If st. lines be drawn through B and C so as to make equal ∠s with BC and on the same side of it, where would they intersect ?

3. Construct a △ ABC anywhere on one side of a st. line PQ. Draw another △ on the other side of PQ so that PQ may be the axis of symmetry of the whole figure.

4. Construct an equilateral △ of side 3 cm. Find, by folding, lines of symmetry in it, Show that there are three lines of symmetry, that they bisect the ∠s of the △ and also bisect the sides at rt. ∠s, that they are *concurrent, i.e.*, meet in a point and that the point of concurrence is equidistant from the sides and corners of the △.

5. The accompanying figure represents a sector of a circle, *i.e., a part of the circle bounded by two radii and the arc between them.* The ∠ACB is called the ∠ of the sector and is said to be subtended at the centre by the chord AB. Find a line of symmetry in the sector. What properties do you deduce therefrom ?

Fig. 138.

[Note that unlike a circle a sector has only a single line of symmetry.]

6. Draw a segment of a circle and find a line of symmetry in it. What properties do you deduce therefrom ?

7. Show by folding that a rhombus is symmetrical about its diagonals and has thus two axes of symmetry. What properties do you deduce therefrom ?

8. Show that a rectangle has two axes of symmetry, *viz.*, the lines joining the middle points of opposite sides and that a square being a rectangular rhombus has four axes af symmetry.

9. Mark symmetrical points about a given st. line and examine the truth of the following :—"*If one pair of points is symmetrical with a second pair, the straight lines joining the pairs are either parallel to, or meet on, the axis.*"

10. Construct a quadrilateral ABCD with ∠B=60°, AB=3 cm, BC = 4 cm., CD=6 cm,. and DA=1 cm. Draw the figures which would be symmetrical about AB, and CD. Mention the symmetrical lines and points in each figure.

11. Draw an ∠BAC. Draw its bisector AH. (Refer to page 88 for the construction method). Show that the angular space BAC

is symmetrical about AH and that any point in AH is equidistant from AB, AC.

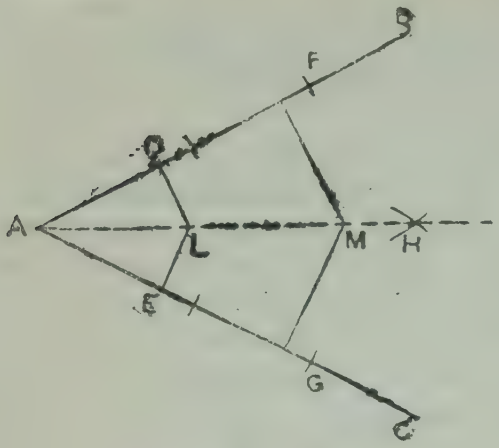


Fig. 139.

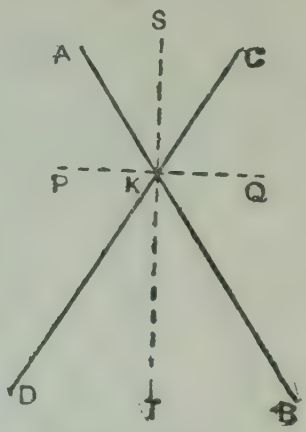


Fig. 140.

12. Draw two st. lines AB, CD to intersect at K. Draw PQ, ST to bisect the ∠s at K. Show that (1) each of the angular spaces AKD, CKB is symmetrical about PKQ; (ii) each of the angular spaces AKC, DKB is symmetrical about SKT, and (iii) hence any point which moves so as to be equidistant from AB, CD must move along PKQ or SKT.

(*Thus, if a point moves so as to be equidistant from two intersecting st. lines, its path or* **locus** *is one of the two bisectors of the ∠s formed.*)

13. Construct a △ ABC. Draw the ⊥ bisectors of the sides AB, AC and let them meet at S. Show that S lies in the ⊥ bisector of BC as well and that it is equidistant from the corners A, B, C of the △. [Hence a circle described with S as centre and SA as radius would pass through the corners of the △. Such a circle is called the **circumscribing circle** or **circum-circle** of the △ ABC. Its centre is called the **circum-centre** of the △ and its radius is known as the **circum-radius**.]

14. Draw an acute-∠d., a rt.-∠d. and an obtuse-∠d △. Find the circum-centre of each and measure the circum-radius. Repeat the above exercise with an equilateral, an isos. and a scalene △.

15. Mark three points P, Q, R so that P is 2" from Q and 1·6" from R and Q is 1" from R. Find the centre of a circle passing through P, Q and R.

16. The arc in Fig. 141 represents part of a circular bund around a lake. Explain how you would draw the whole ⊙ce which would represent the bund.

Fig. 141.

17. The chord of an arc =8 cm., the chord of half the arc = 5 cm. Draw the arc.

18. The sides of a triangular field are 5 chains, 4 chains and 5 chains respectively. There is a spring in the field equidistant from its corners. Mark its position in a diagram drawn to scale. Find how far it is from each corner of the field.

19. Construct a quadrilateral ABCD in which AB = AD = 4·5 cm., BC = CD and ∠BAD = 75°. Describe a circle so as to pass through B, A and D. What should be the lengths of BC and CD in order that the circle may pass through C? Measure the ∠BCD for that position of C.

20. Draw two st. lines AB, AC 2 cm. and 5 cm. in length, so as to intersect at an ∠ of 45°. Describe the circle of which AB, AC are chords. Measure its radius.

SECTION II.

Construction of Regular Polygons.

77. THE ANGLES OF A POLYGON.

Draw a convex polygon ABPQT, *i.e.*, a polygon with none of its interior ∠s reflex in nature. Produce its sides in order as in Fig. 142. Measure the exterior ∠s formed and find their sum.

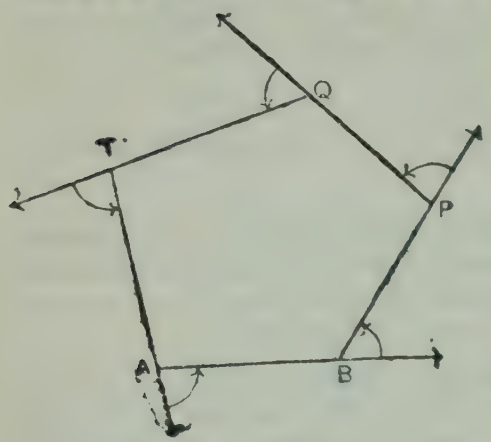


Fig. 142.

If a person starts from A in the direction AB and walks once round such a polygon turning at its corners as indicated by arrow-marks in the figure until he is again at A in the direction AB, it may be seen that he has turned *once* round himself and has hence turned through 4 rt. ∠s.

Hence **the sum of the exterior ∠s formed by producing the sides of a convex polygon in order=4 rt. ∠s.** Also, since the two adjacent ∠s at each corner are together equal to 2 rt. ∠s, **the sum of the interior ∠s of the polygon =** $(2n-4)$ rt. ∠s. where 'n' represents the no. of corners, ∠s. or sides.

Therefore, if a polygon be regular, *i.e.*, has all its ∠s equal as well as all its sides equal, each exterior $\angle = \frac{4}{n}$ rt. ∠ or $\frac{360}{n}$ degrees and *each interior* $\angle = \left(2 - \frac{4}{n}\right)$ *rt.* ∠ *or* $\left(180 - \frac{360}{n}\right)$ degrees. Thus, in a regular

ext. $\angle = \frac{360}{5}$ or 72 degrees and each int. $\angle = (180 - 72)$ or 108 degrees. [Why ?]

It may be seen that polygons which have reflex $\angle$s do not possess the above properties.

[*Ex.*—Find the ext. and int. $\angle$s of a regular hexagon, octagon and decagon.]

78. INSCRIBING A REGULAR POLYGON IN A GIVEN CIRCLE.

Method (i). In a circle of radius 2 cm. set off successive $\angle$s AIB, BIC, etc. around the centre equal to 45^0. [Refer to Fig. 143.] As the sum of the $\angle$s around the centre $= 360^0$, the no. of $\angle$s that could be so set off $= \frac{360}{45}$ or 8. Draw the chords AB, BC, etc. opposite to these $\angle$s and compare their lengths. By super-position it may be seen that the eight isos. $\triangle$s AIB, BIC, etc. are congruent and that the chords AB, BC, etc. are equal. Hence, if

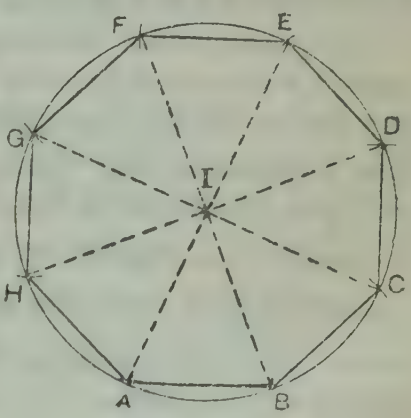


Fig. 143.

in a circle equal $\angle$s be drawn at the centre the chords opposite to them are also equal. Also, the $\angle$s ABC, BCD, etc. are equal (why ?) and bisected by BI, CI, etc. Thus a regular octagon ABCDEFGH is obtained and since its vertices lie on the Oce of a circle, it is said to be *inscribed* in the circle.

In a similar manner, a regular polygon of 'n' sides could be inscribed in a circle by setting off successive $\angle$s

around the centre each equal to $\frac{360^\circ}{n}$. [For what values of 'n' is the construction possible with a protractor?]

Method (ii). In a circle of radius 2 cm. set off a central $\angle AIB = 45^\circ$ and draw the chord AB. Step off successive chords BC, CD, DE, etc. in the circle each equal to AB. (See Fig. 143). Join their extremities to the centre C and compare the central $\angle$s opposite to them. It may be seen that the central $\angle$s are equal. Hence, **if equal chords be drawn in a circle the central $\angle$s opposite to them are also equal.** Also, after stepping off the chord GH equal to AB it will be found that the remaining sector AIH has its central $\angle$ equal to 45° (why?) and hence its chord AH = AB. Thus, eight chords equal to AB could be successively placed in the circle. Also, the eight isos. $\triangle$s AIB, BIC, etc. are congruent and hence together form a regular octagon inscribed in the circle.

Similarly, a regular polygon of 'n' sides would be inscribed in a circle if a central $\angle AIB$ equal $\frac{360^\circ}{n}$ be constructed and then successive chords equal to AB be drawn in the circle.

[NOTE :—(1) *The st. lines joining the centre of a circle to the angular points of an inscribed regular polygon of 'n' sides divide it into 'n' congruent isos. $\triangle$s and hence the area of the polygon = 'n' times the area of any of the $\triangle$s.*

(2) If AB, BC be two adjacent sides of such a polygon inscribed in a circle with centre I and if AB be produced to E, $\angle IBE = \angle AIB + \angle IAB$. [Why?] *Hence the exterior $\angle CBE$ of the polygon = the central $\angle AIB$ subtended by a side of the polygon* $= \frac{360^\circ}{n}$. [How?]

(3) Cut out regular polygons of 5, 6 and 8 sides inscribed in circles of different radii and find lines of symmetry in them. It may be seen by folding that *the polygons are symmetrical about the bisectors of their interior ∠s as well as about the ⊥ bisectors of their sides, that the lines of symmetry in each polygon are concurrent and that the point of concurrence is equidistant from the angular points of the polygon and hence forms the centre of its circum-circle.*]

79. CONSTRUCTING A REGULAR POLYGON ON A GIVEN BASE.

The following method is suggested by the note to Art. 78 as well as by Art. 75 :—

Suppose a regular polygon of 6 sides has to be constructed on a base AB. Produce AB to E and set off $\angle EBC = \frac{360^\circ}{n}$ or 60 degrees, since $n = 6$. Make BC = AB. Draw the ⊥ bisectors of AB, BC and let them meet at I. With I as centre and IA as radius describe a circle. It would pass through B and C. [Why?] Place successive chords CD, DF, etc. in the circle equal to AB. It may be seen that six chords could be so placed in the circle and that the hexagon bounded by them is regular. (How?)

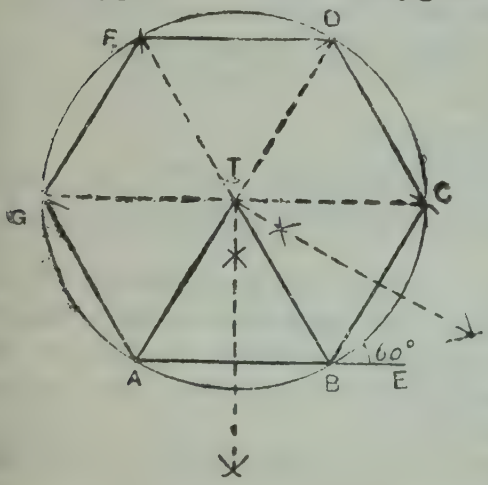


Fig. 144.

The same method may be adopted in constructing any regular polygon of 'n' sides in cases where an exterior angle equal to $\frac{360^0}{n}$ could be set off.

[Note that in the $\triangle$ AIB in Fig. 144 each $\angle = 60^0$. (Why?) and hence the $\triangle$ is equilateral. Thus, **in a regular hexagon the radius of the circum-circle = a side of the hexagon.** Hence (1) *a regular hexagon could be inscribed in a given circle by stepping off successive chords equal to the radius, and* (2) *it could be constructed on a given base AB by first drawing an equilateral* $\triangle ABI$ *and then placing successive chords each equal to AB in a circle drawn with I as centre and IB as radius.*]

Ex. Show that if in Fig. 144 the hexagon be folded along AF, BD the vertices G and F would coincide with the circum-centre I and the crease-lines would bisect IG and IC at rt. $\angle$s.

EXERCISES.

21. If each exterior $\angle$ of a regular polygon be 30°, find the number of its sides. Can there be a regular polygon with each exterior $\angle = 27^0$?

22. In a circle of radius 1·5" inscribe an equilateral $\triangle$, a square and a regular pentagon. Find the lengths of their sides.

23. Show how a regular dodecagon could be inscribed in a circle of given radius or constructed on a given base by using a ruler and compasses only.

24. Construct polygons with all the sides equal. Examine if they are necessarily equiangular.

25. Construct polygons with all the $\angle$s equal. Examine if they are necessarily equilateral.

26. Construct a regular pentagon, hexagon, octagon and decagon on bases 1·5" lang. Find the area of each.

27. Inscribe a regular pentagon, hexagon and octagon in circles of radius 5 cm. each and find their areas.

28. What is the size of an $\angle$ subtended at the centre of a circle by a chord equal to the radius? *Show that the following construction would give a* $\perp$ *to a given st. line AB through a point C in it* :—

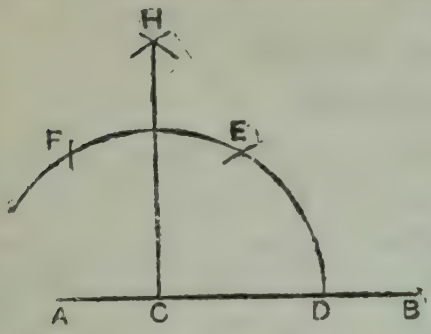


Fig. 145.

With centre C and with radius CD draw an arc cutting AB at D. Step of chords DE, EF equal to DC With centres E and F and with radius DC draw arcs cutting at H. Then CH is $\perp$ to AB.

29. Construct a regular hexagon on a base of 1". Produce its alternate sides so that a star-shaped figure is formed. Find the area of the figure.

30. Describe a circle with centre C and with radius 4 cm. and fold it along a diameter AB. Fold the circle again so that A and B coincide with C and the crease lines are $\perp$ to AB. Show that the six angular points of an inscribed regular hexagon could thereby be obtained.

31. Show how you would lay out a grass plot in the form of a regular hexagon so as to have an area of 756 sq. yds. What is the length of an edge?

32. A triangular fortress has its sides 10 chains, 8 chains and 6 chains respectively. It is surrounded by an outer wall which forms a regular octagon with each side $1\frac{1}{2}$ furlongs in length. Find the area lying between the outer wall and the fortress.

SECTION III.

Construction of Tangents.

80. ON TANGENTS.

(*a*) In Fig. 146 the st. line I cuts the circle at the points *a*, *a'*. Such a line is called *a secant* of the circle. Distinguish it from the chord *aa'* which forms a part of the secant intercepted by the circle. The st. lines II, III, IV, etc., represent the positions of a secant moving parallel to itself. Draw a st. line through the centre of the circle ⊥ to the secant. It may be seen that as the secant recedes from the centre of the circle the length of the intercepted chord gets less and less (*e.g.*, $ee' < dd' < cc'$) until at length it becomes quite negligible as at P, when the parallel APB does not cut the circle but only **touches** it. In that position the st. line APB is said to be a **tangent** to the circle and P is called **the point of contact** of the tangent with the circle. The following properties may be observed :—

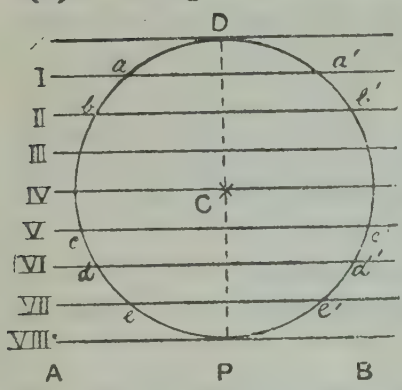


Fig. 146.

(1) *The distance of the tangent from the centre of the circle = the radius.*

(2) *The tangent to a circle and the diameter through the point of contact are ⊥ to each other.*

(3) *The tangent at the extremities of a diameter are parallel.*

Hence to draw a tangent to the circle through a point D on the Oce draw a diameter through the point (refer

to Fig. 146) and then a ⊥ to it through the same point. The perpendicular will be the tangent required.

[NOTE:—The centres of circles touching a given st. line at a given point would lie in a ⊥ st. line drawn through the same point.]

(*b*) Draw an ∠BAC and bisect it by AK. Then AK is the axis of symmetry of the angular space BAC. From any point K in AK draw KT, KL ⊥ to AB, AC respectively. Then KT = KL. (Why?) Draw a circle with centre K and with radius KT or KL. It will be found to touch AB, AC. Draw secants through A to cut the circle. These secants would lie in the angular space BAC. Also, T and L are symmetrical about AK. Hence AT = AL. [Note also that ATK and ALK are rt. ∠d △s, and hence $AT^2 = AL^2 = AK^2 - KL^2$.]

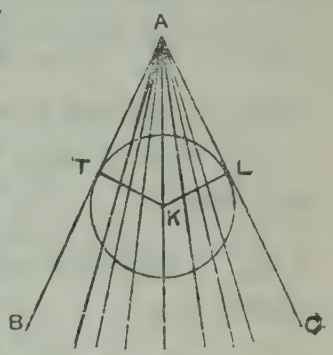


Fig. 147.

81. (1) Construct a △ABC with ∠B a rt. ∠. Find its circum-centre and draw its circum-circle. It may be seen that the centre of the circle is the middle point of AC. Mark other points D, E, F, etc., on the Oce. Measure the ∠s ADC, AEC, AFC, etc. Each such ∠ is called *an ∠ in a semi-circle, and would be found to be a rt ∠.*

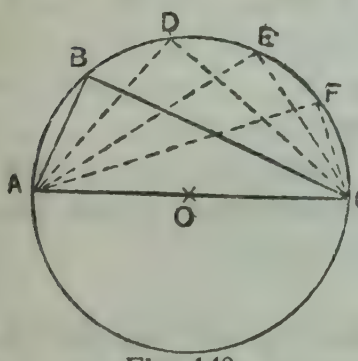


Fig. 148.

[Join EO and apply Ex. 48, page 93, or produce EC to H and apply the same Ex.]

(2) Construct rt. ∠d △s ABC, ADC, etc., on a common hypotenuse AC. Draw a semi-circle on AC as diameter. It may be seen that the semi-circle passes through B, D, etc.

(3) Place one of your set-squares on a flat surface and then rotate it over the surface keeping the middle point of its longest edge exactly over a point O. What is the path traced by the right-angular corner?

Hence, *if a point moves so that a given st. line* AB *subtends a rt.* ∠ *at it, its locus is the Oce of a circle drawn with* AB *as diameter.*

(4) The constructions given below follow from the above:—

(*a*) To construct a rt. ∠d △ ABC with ∠B a rt. ∠, AC = 5·5 cm. and AB = 2·5 cm.; draw a st. line AC equal to 5·5 cm. Bisect AC at D by the construction method. On AC as diameter draw a circle. In it place a chord AB equal to 2·5 cm. Join BC. Then ABC is the △ required [Why?]

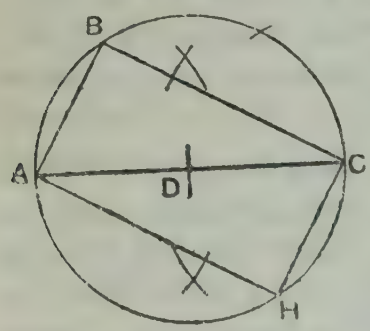


Fig. 149.

Deduce from the above how a rectangle ABCH could be constructed when the diagonal AC and the side AB are given.

[If the ⊥ bisector of AC cuts the Oce at P the △ APC would give a model of the 45° set-square (How?) If a chord CM be drawn equal to CD, the △ACM would give a model of the 60° set square. (How?)]

(*b*) *To draw a* ⊥ *to a st. line AB from an external point* P :—

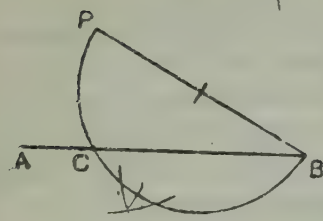


Fig. 150.

Join PB. Draw a semi-⊙ on PB as diameter cutting AB at C. Join PC. Then PC is ⊥ to AB [Why ?] Note that the construction is specially useful when the point C is very near A and it is not possible to produce BA beyond A.

(*c*) *To draw a tangent to a circle from an external point* P :—

Let O be the centre of the circle. Join OP. With OP as diameter describe a circle cutting the given circle at A, B. Join PA, PB. Then since PA, PB are ⊥ to the radii OA, OB (Why ?) they are tangents to the circle.

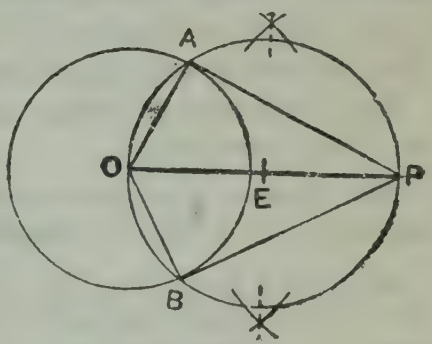


Fig. 151.

Note that *two, and only two, tangents could be drawn to a circle from an external point and that these tangents are equal.*

EXERCISES.

33. Draw two circles to touch each other at a point P. Show that the figure formed is symmetrical about the line joining the centres of the circles. Draw some more circles touching at the same point P. Where do their centres lie? Draw a tangent common to all these circles.

34. Construct any △ABC. Draw the paths of points moving so as to be equidistant from AB, AC and AB, BC respectively. Let the paths intersect at I. Show that I is equidistant from AB, BC, CA. Hence show that *the bisectors of the ∠s of a △ meet at points equidistant from the sides.*

Draw a circle with I as centre and with its distance from AB as radius. [This circle would touch the three sides of the △ and is

called the **in-circle** of the △. Its centre is known as the **in-centre**.]

35. Construct a △ABC and produce its sides AB, AC to D and E respectively. Bisect the ∠s DBC, ECB and let the bisectors meet at H. Show that H is equidistant from the three sides of the △ and that a circle could be drawn with H as centre so as to touch the three sides of the △. [This circle is known as the **escribed circle or ex-circle** of the △.] Show also that four circles could be drawn to touch the three sides of any △.

36. A st. line AB cuts two other st. lines PQ, RS. Find two points equidistant from the three st. lines.

37. Draw a st. line AB and a point P as in Fig. 49, page 91. Describe a circle with centre P so as to touch AB.

38. Draw a circle with radius 3 cm. so as to touch the arms of an ∠BAC equal to 72°.

39. Draw two || st. lines 3 cm. apart. Describe a circle to touch the two || s. What is its radius?

40. Draw two st. lines PQ, SR so as to intersect at an ∠ of 60°. Draw circles with rad. 1″ to touch the two st. lines. How many such circles could be drawn?

41. A point 'A' is marked 1·5″ from the centre of a circle of radius 1″. Show that two circles could be drawn with centre A to touch the other circle.

42. Three towns P, Q and R lie so that P is 5 miles north-east of Q and R is 15 miles due east of Q. S is another town equidistant from P, Q and R. Mark the position of S in a carefully drawn figure and find its actual distance from each of the other towns.

43. Construct a △ABC with ∠B=68°, AB=2″ and BC=3″. Mark a point 1·4″ from BC and equidistant from AB, AC. Measure its distance from B and C.

In the same figure mark a point 1·2″ from BC and 1″ from A. How far is it from the other sides and corners of the △?

44. Mark in a page of your note-book four points A, B, C, D each of which is 2·6″ from the two adjacent edges nearer it.

If 'X' be equidistant from A, B and C and 'Y' be equidistant from AC, CD, DA, find from a figure the distance between X and Y.

45. Inscribe a circle in a square, a regular hexagon and a regular octagon of side 1·2″. Find the radius of each circle.

46. Construct a △ whose sides are 2·4″, 3″ and 4″ long. Describe a circle touching the sides internally. Measure the length of its radius.

47. Rameswaram island is roughly triangular in shape with its sides 3 miles, 5 miles and 3½ miles in length. In it a choultry is built equidistant from each coast. Draw a diagram to scale showing the position of the choultry and find its distance from the sea.

48. Draw a circle with radius 2 cm. In it place a chord 1 cm. from the centre. Find the length of the chord.

Show that $r^2 = d^2 + l^2$ where '$2l$' represents the length of a chord of a circle, 'd' its distance from the centre and 'r' the radius of the circle. From the relation $r^2 = d^2 + l^2$, deduce the following :—

(*a*) *Equal chords in a circle are equidistant from the centre.*

(*b*) *Chords which are equidistant from the centre are equal in length.*

(*c*) *The diameter is the greatest chord that could be drawn in a circle.*

(*d*) *The distance of the tangent to a circle from its centre = its radius.*

49. A cow is tied by a rope 15′ long to a fixed pole 10′ from the straight bank of a stream. Find from a figure drawn to scale what length along the bank is accessible to the cow.

50. Construct a quadrilateral ABCD with ∠B = 50°, ∠D = 130°, CB = 2″, ∠BCD = 60° and CD = 1″. If the quadrilateral represents a garden drawn to a scale of 1 inch to 200 ft., find by measurement from a figure the distance of a well equidistant from the corners A, D, C and of a stake equidistant from BC, CD, DA. If a cow be tethered to the stake by means of a rope 16 yds. long, mark her position in the diagram when she is 100 ft. from CD.

51. Three fortresses A, B, C are connected by three straight roads AB, BC, CA. AB = 15 miles, BC = 10 miles and CA = 20 miles. A gun is so placed that with its maximum range it just covers the roads in full. Find where it is placed. If a gun won't carry far enough to cover the roads in full, what should be its minimum range so as to command at least one station in each road?

52. Construct, by using the ruler and compasses only, a rectangle ABCD with AB = 3 cm., BC = 5 cm.

E is a point in AB such that EB = 2 cm. F is a point in BC such that when the rectangle is folded along EF, B falls on AD. Find the position of F by a construction method. How far is it from A, B, C, D and E?

53. Mark two points A and B, 3·4 cm. apart. Draw a circle with rad. 2 cm. to pass through A and B.

54. Find the locus of the centres of circles of rad. 2 cm. which pass through a given point P. Also find the locus of the centres of circles of rad. 2 cm. which touch a given st line.

Hence draw a circle of rad. 2 cm. which would touch a given st. line AB and would pass through a point P as in Fig. 49, page 91.

55. Draw a circle with any point C as centre and with rad. 1'6". What is the locus of the centres of circles of radius 2" which would touch the circle with centre C?

Mark a point P 4" from C. Describe a circle of rad. 2" to pass through the point P and touch the circle with centre C.

56. In equal circles of rad. 2·5 cm., inscribe an equilateral△, a square, a regular pentagon, hexagon, and octagon respectively. Through their vertices draw tangents to the circles and produce them till they meet. What kind of figure is formed in each case?

57. Construct a rt. ∠d △ with the sides containing the rt. ∠ equal and with the hypotenuse 3". In it mark a point equidistant from the sides.

58. Construct a quadrilateral ABCD in which AC = 4·4 cm. ∠ADC = ∠ABC = 90°, AB = 3·6 cm., and CD = 2·5 cm. Measure its other sides and ∠s.

59. Plot the points $(+6, 0)$, $(-12, 0)$, $(-18, 0)$, $(0, +4)$. Describe a circle to pass through any three of these points and verify if it passes through the fourth point. Try the same construction with the points $(-18, 0)$, $(+6, 0)$, $(-12, 0)$, $(0, +6)$.

60. Travelling along a straight road, I see a bungalow due west of me at a place A. Travelling 3 furlongs farther along the road I find the bungalow lying in a direction $\perp$ to the road. Travelling 5 furlongs farther still I find the bungalow lying due north of me. Find from a figure drawn to scale how far the bungalow is from the road.

61. Two beat constables have to visit four hamlets P, Q, R, T from a station S every night. Straight foot-paths run from hamlet to hamlet. P is 3 miles due west of S, Q and T lie on opposite sides of PS and P and R lie on opposite sides of QS, PQ = ST = 1 mile. SR = $\frac{1}{2}$ a mile The paths PQ, QS are $\perp$ to each other as also the paths PT, TS and RQ, RS. Find from a figure drawn to scale the minimum distance the constables are expected to travel for completing their beat.

62. Construct a rect. ABCD in which AB = 2 cm., BC = 4·5 cm. Mark a point F in AD and E in BC so that FD = 1 cm. and $\angle DFE = 75°$. Mark points K and L in CD, DA so that when the rectangle is folded along EK, EL the st. lines EC, EB may fall along EF. Join EK, EL. Calculate the size of each $\angle$ in the figure and verify by measurement.

63. L, M, N are three light-houses along a straight strip of coast. LM = 13 miles and MN = 10 miles. Mines are spread near the coast so that it is dangerous for ships to sail up to where LM and MN subtend rt. $\angle$s. Represent in a figure drawn to scale the danger-area.

CHAPTER XVIII.

AREA OF THE CIRCLE, SECTOR AND SEGMENT.

82. RELATION BETWEEN THE O^{ce} OF A CIRCLE AND ITS DIAMETER.

The figures drawn below suggest that the $\odot^{ce}$ of a

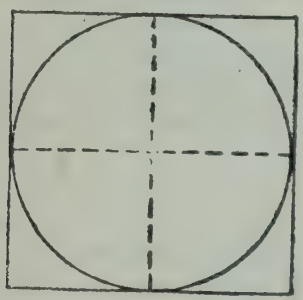


Fig. 152.

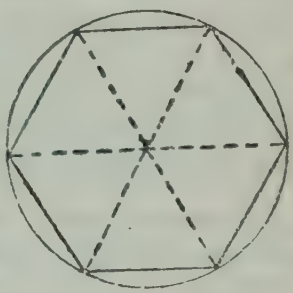


Fig. 153.

circle is less than four times its diameter but greater than three times the diameter. (How?)

Draw six circles with different radii. Measure the $\odot^{ce}$ of each by placing a string along it. Find the value of the ratio $\frac{\odot^{ce}}{\text{diameter}}$ in each case. The average of these values will be found to be 3·142 *nearly*. The above ratio is represented by the Greek letter 'π' (read as '*pi*') and is equivalent to a non-terminating decimal with 3 units in its integral part, *viz.*, 3·1415926.....

Unless otherwise stated, take $\pi = 3\frac{1}{7}$.

If C'' *be the* ⊙ce *of a circle of radius* R'' *and diameter* D'',

$$C = \pi \times D = 2\pi \times R;$$

$$D = \frac{C}{\pi} \text{ and } R = \frac{C}{2\pi}.$$

EXERCISES.

1. Explain how you would measure the Oce and diameter of a circular plate, a cylinder and a sphere.

2. The diameter of a rupee = 1·6″. Find the length of its circular rim.

3. In one complete revolution of a wheel it traces a distance of 11 ft. Find the diameter of the wheel.

4. A cycle wheel is 28″ in diameter. How many revolutions will it make in going ten furlongs?

5. Fill up the vacant columns below :—

	Oce.	Diam.	Rad.
(*a*)	1 chain.	( )	( );
(*b*)	( )	( )	1 chain;
(*c*)	( )	6/6″.	( ).

6. While measuring a curved line by rolling a half-anna coin over it, the coin went round 5 times. If the diameter of the coin be 1·75″, find the length of the curved line.

7. The outer Oce of a test-tube = 2·2″ and the thickness of the glass = ·051″. Find its internal diameter.

8. The Oces of two concentric circles are 8·4″ and 7·7″. Calculate the distance between the Oces.

9. The distance round the earth along the equator = 24904 miles. Supposing the earth to be a sphere, find its equatorial diameter.

10. The pulley in a well turns round a fixed rod 14 times during the time the bucket is raised from the water surface to a height of 22′. Find the diameter of the pulley.

11. An oil-mill in a village is worked as follows:—A pole MK is turned round and round the mill M by two buffaloes yoked at B and K. (See Fig. 154). Given that MK = 17·5′ and MB = 14·0′ calculate the distance each buffalo has to go in 85 complete turns round the mill. If the driver sits midway between B and K, show that he will be taken through half the distance gone by the two buffaloes together.

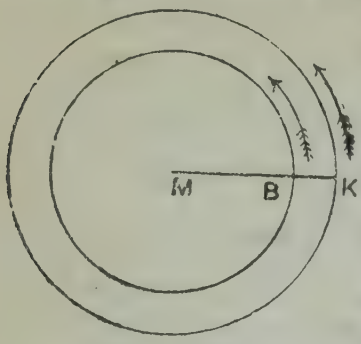


Fig. 154

12. In a school clock the minute and hour-hands are 3·15″ and 2·45″ in length. Find the distances travelled by their ends in 12 hours.

13. When a revolving wheel of diameter 25 ins. makes 50 revolutions, a point in its rim moves through 109 yds. 0 ft. 3 ins. Calculate the value of π from these data.

14. A circular ring of diameter 2·8′ is laid on a flat surface. A coin of diameter 1·6″ is also laid flat on the surface and rolled inside the edge of the circular ring. Find the distance traced by the centre of the coin in once completing the round.

15. A circular coin of diameter ·5″ rolls along the rim of a rupee of diameter 1·6″. Calculate the distance travelled by the centre of the coin when one turn round the rupee is completed.

16. PQRS is a rectangular piece of cardboard with PQ = 11″ and PS = 6″. From the position shown in Fig. 155 it is turned over by a lever applied at P, until SR, RQ and QP fall along a st. line XY in succession. Find the distance travelled by P and verify your result by measurement from a diagram drawn to scale. (Assume that $\pi = \frac{355}{113}$.)

Fig. 155.

83. THE AREA OF A CIRCLE.

Cut out a paper-circle of radius 1″. Fold it so as to get two ⊥ diameters. Again fold so as to trisect each rt. ∠ formed at the centre. The circle would then be divided in 12 equal sectors as in Fig. 156. Cut out

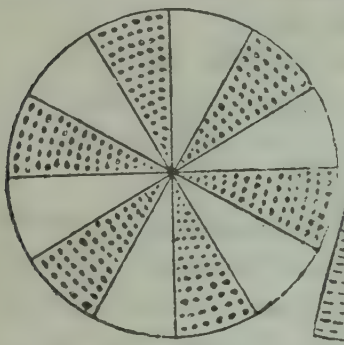


Fig. 156.

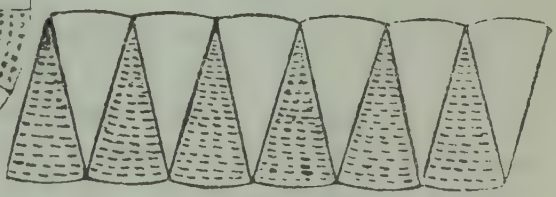


Fig. 157,

the sectors and place them alternately as in Fig. 157. They would almost form a parallelogram. If the number of sectors be sufficiently large, a rectangle will ultimately be formed with the semi-Oce as base and the radius as height.

Thus, the area of a circle

$=$ *the semi-Oce* $\times$ *the radius*

$= \pi \times R \times R$ *or* πR^2, *where*

'R' *represents the radius.*

[NOTE.—A circle could be cut into several equal sectors so that each of them ultimately resembles a △ and the sum of the bases of these sectors = the Oce of the circle. Hence the area of a circle = that of a △ whose base = the Oce of the circle, and altitude = the radius.]

Ex. 1. Draw two concentric circles with radii 2·8″ and 1·4″. Find the area of the *annulus* between their Oces.

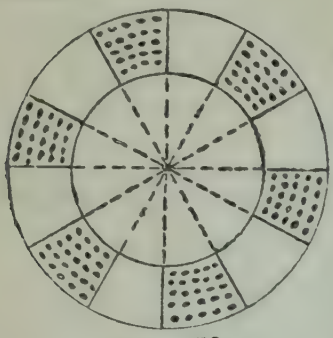


Fig. 158.

The area of the flat annulus
= the diff. between the areas of the two circles
= $\pi(2{\cdot}8^2 - 1{\cdot}4^2)$ sq. in.
= $\frac{22}{7}(2{\cdot}8 + 1{\cdot}4)(2{\cdot}8 - 1{\cdot}4)$ sq. in.
or 18·48 sq. in.

If 'R,' 'r' be the radii in inches of the larger and the smaller circles and D, *d* be their diameters in inches, *the area of the annulus* $= \pi(R^2 - r^2)$ or $\pi(R + r)(R - r)$ *sq. in.*

$$= \pi\left(\frac{D^2}{4} - \frac{d^2}{4}\right) \text{ or } \frac{\pi}{4}(D + d)(D - d) \text{ sq. in.}$$

[NOTE.—If several radii be drawn in the larger circle so as to divide it into several equal sectors, the annulus would *ultimately* be divided into several equal trapeziums. The sum of the parallel sides of all these trapeziums = that of the Oces of the two circles and the distance between them = the diff. between the radii. Hence the area of the annulus = $\frac{1}{2} \times (C + c) \times (R - r)$ where 'C,' '*c*' form the Oces and 'R' '*r*' the radii of the two circles.]

EXERCISES.

17. A paper circle of rad. 3·5″ is put in one scale-pan of a delicate balance. Three squares of side 3·5″ are cut from the same kind of paper and put in the other scale-pan. What fraction of a square of side 3·5″ should be cut from the same kind of paper and thrown in the lighter scale-pan in order that the scales may counterpoise?

18. Calculate the areas of circles of radii 2″, 3″, 5 cm., 6 cm., and 7·42 yds.

19. A boys' school is like the letter B and consists of a rectangular hall with two semi-circular rooms as shown in Fig. 159. If PQ = RS = 42', QR = ST = 18', find the area occupied by the ground-floor.

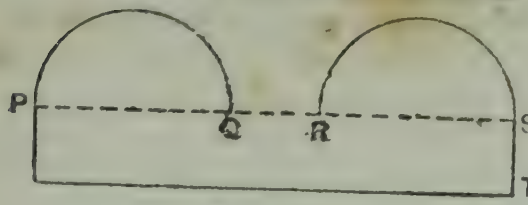


Fig. 159.

20. If 'A' represents the no. of sq. in. in the area of a circle of rad. r", diameter d" and Oce C", show that

$$A = \tfrac{1}{2} Cr = \pi \frac{d^2}{4} = \frac{C^2}{4\pi}; \; C = \frac{2A}{r} = 2\sqrt{\pi A}; \; r = \sqrt{\frac{A}{\pi}}$$

and $d = 2\sqrt{\frac{A}{\pi}}$.

Hence find the Oce and radius of a circle of area 154 sq. yds.

21. From a rectangular tin-plate 36" by 20" circular pieces of diameter 6" are cut in order to make lids for tin vessels. How many pieces could be cut and what area of plate would be left over?

22. In a rectangular grass farm 4 chains by 1·5 chains there is a tree in each corner. A cow is tied to each tree by means of a rope 70 links long. What fraction of the area of the farm is allowed for each cow to graze?

23. Assuming that the area of a circle equals that of a square whose side $= \frac{5}{8}\frac{2}{0}$ of the diameter of the circle, calculate the value of π. Show that $\frac{4}{1}\frac{8}{5}\frac{3}{0}$ gives a nearer value of π.

24. In a rt.-$\angle$d $\triangle$ABC semi-circles are described on the three sides as in Fig. 160. Show that the area of the shaded lunes = that of the rt.-$\angle$d $\triangle$.

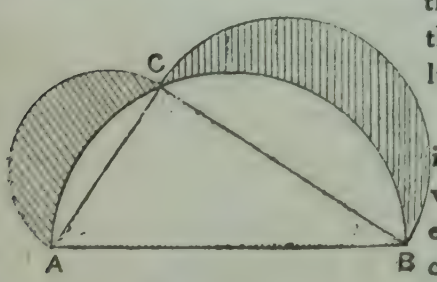


Fig. 160.

25. The circular race-track in Guindy *maidan* is 15 ft. wide and the ground which it encloses has a diameter of 10 chains 15 links. Find the area of the track.

26. A circular lake has a circular island in the centre. The area occupied by the sheet of water is 22 sq. miles. The distance between the shore of the lake and the edge of the island is 1 mile. Find the area of the island.

27. The minute and hour hands of a watch are 1·2" and 1" long respectively. What area is traced out by each hand in the course of 12 hours ?

28. A cast-iron pipe has an internal diameter of 10" and an external diameter of 11". Find the area of its cross-section.

84. AREA OF SECTORS AND SEGMENTS.

Cut out a paper circle and by folding or otherwise

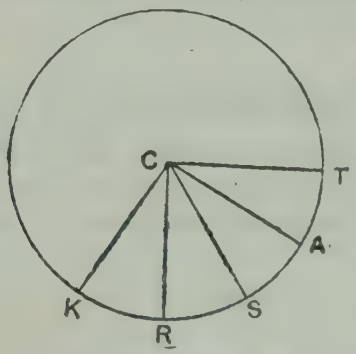


Fig. 161.

(refer to Art. 78) obtain a series of equal sectors as in Fig. 161. It may be seen that

$$\frac{\text{the arc KRSA}}{\text{the arc KRSAT}} = \frac{\text{the central } \angle KCA}{\text{the central } \angle KCT}$$

$$= \frac{\text{the sector KCA}}{\text{the sector KCT}}.$$

Hence it may be deduced that

$$\frac{\textit{the area of a sector of a circle}}{\textit{the area of the circle}} = \frac{\textit{the no. of degrees in its central } \angle}{360} \quad (\textit{How ?})$$

$$= \frac{\textit{the length of its arc}}{\textit{the length of the Oce.}}$$

Note:—If in a circle one arc is 'n' times another arc, the central $\angle$ opposite to the first arc is 'n' times the central $\angle$ opposite to the second arc and the same ralation exists between the corresponding sectors but *not between the chords.*

Ex. Find the area of a sector of a circle of rad. 2 cm. and central $\angle$ 30°.

The area of the sector

$$= \frac{30}{360} \times \text{the area of the circle.}$$

$$= \frac{1}{12} \times \frac{22}{7} \times 4 \text{ or } 1\frac{1}{12} \text{ sq. cm.}$$

The area of a sector of a circle may also be found as follows:—

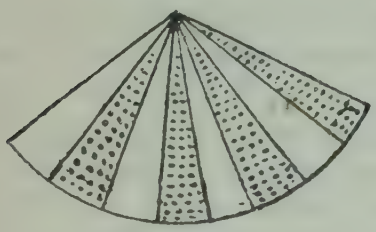


Fig. 162.

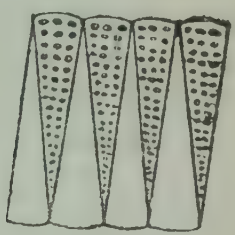


Fig. 163.

Fold the sector into many small but equal sectors. Cut out these sectors and place them alternately as in Fig. 163. Ultimately they will form a rectangle having *half the arc of the sector as base and its radius as altitude.*

Hence, the area of a sector of a circle = half the length of its arc × the radius.

Thus if the arc AB of a sector AOB measures 4 cm. and the rad. OA = 1·5 cm., the area of the sector $= \frac{1}{2} \times 1{\cdot}5 \times 4$ or 3 sq. cm. Also, the area of the *minor* segment of the circle bounded by the chord AB = the area of the sector AOB less that of the △ AOB.

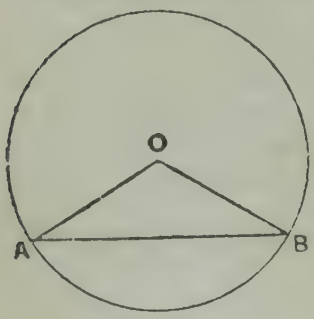


Fig. 164.

Hence, suggest how the area of the *major* seg ment bounded by the chord AB may be found.

EXERCISES.

29. The pendulum in our school clock in 3·64" long. During each oscillation it swings through an ∠ of 30°. Calculate (*a*) the distance traced by its moving end and (*b*) the area the pendulum describes during 24 hours.

30. Calculate the area of sectors of circles whose arcs are 4 cm., 6 cm., 5", 7", and 6·3" and whose radii are 2 cm,, 4 cm., 2", 4" and 6" respectively.

31. The sectors 1, 2, 3, 4, 5 in Fig. 165 represent the areas of the possessions of five Zemindars A, B, C, D and E respectively. If A's possessions contain an area of 'x' sq. miles, find the areas of the possessions of the other Zemindars.

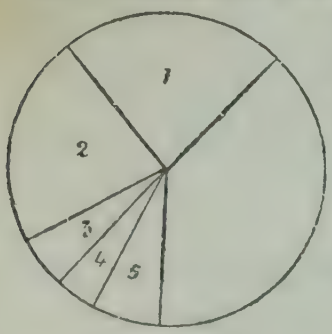


Fig. 165.

32. A circular curve in a road subtends an ∠ of 30° at the centre. the radius of the curve being 840 yds. Find the length of the curve.

33. Draw a sector of a circle whose chord = 1·5" and central ∠ = 108°, Find the area of the sector and of the segment cut off by the chord.

34. Two circles are described with centres 5 cm. apart and with rad. 3 cm, and 4 cm. Find the area common to them.

35. Two forts are situated at a diatance of 5 miles from each other. There are guns in each fort having a range of 3·5 miles. Find from a figure drawn to scale the area commanded by the guns of both the forts.

36. Two parallel chords 1·5″ and 1·2″ in length are drawn in a circle of radius 1·4″. Find what area in the circle lies between them when they lie (*a*) on the same side and (*b*) on opposite sides of the centre.

37. Two places A and B lie on the equator, A is 3113 miles west of B. If A is in 15° west longitude, find the longitude of B, assuming the equatorial diameter of the earth to be 7924 miles.

38. In making a semi-circular protractor a semi-circular plate of diameter 3·5″ is taken. After leaving a border of ·4″ all round the remaining portion is cut out. Find the area of the plate left.

[Note that the hollow portion is semi-circular.]

39. The arc AEB in Fig. 166 represents the circular arch of a bridge. Its *span* AB = 15′ and its *height* ED = 5′. Find the length of the arch.

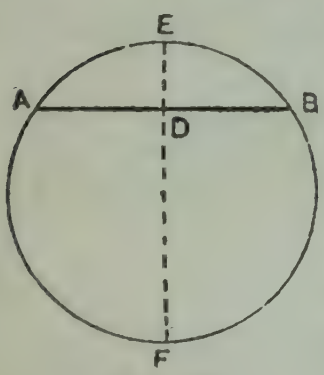


Fig. 166.

[Note that if ED be produced to meet the circle again at F, EDF will be a diameter of the circle and ED × DF = AD². [Refer to (Art. 72). Thus DF can be found and hence EF. The ⊙ can then be described with radius = ½ × EF.]

40. The chord of an arc of a circle = 2·4″. The chord of half the arc = 1·3″. Construct the circle and find the ratio between the minor segments formed by drawing the two chords. [Show that in Fig. 166, ED × EF = AE² and hence show how the ⊙ could be constructed when AB and AE are given.]

41. An arch like a minor arc of a circle 35′ in diameter is built over a channel 25′ wide. Find the height of the arch from a diagram drawn to scale.

42. Draw two concentric circles so as to form a flat annulus. Through a point C in the Oce of the smaller circle draw a tangent to it terminated by the Oce of the larger circle. If D be one extremity of the tangent in the outer Oce, show that the area of the annulus equals that of a circle of radius CD.

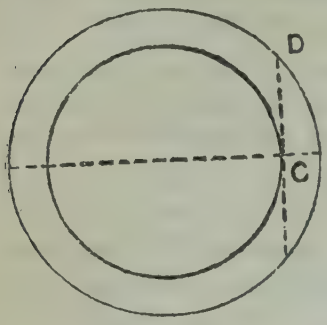


Fig. 167.

43. Three equal circles of rad. 3 cm. touch each other. Find the area lying amidst them. Explain how you would construct the circles.

44. The cross-section of a tunnel is like the major segment of a ⊙ whose chord is 11' and height 15'. Represent the cross-section by means of a diagram and find the greatest breadth of the tunnel.

45. Two friends leave a rest-house (R) in a circular island situated in a bay. One of them reaches the shore by the shortest straight path which is 4 furlongs in length. The other goes in a prependicular direction and reaches the shore after walking 5 furlongs. Find the area of the island.

46. Inscribe an equilateral △ in a circle of rad. 2″. Through the angular points of the △ draw tangents to the circle so as to form a △. What kind of △ is formed? In what ratio is the area between the two △s divided by the Oce of the circle?

CHAPTER XIX.

THE RIGHT PRISM, CYLINDER AND CONE.

85. THE RIGHT PRISM.

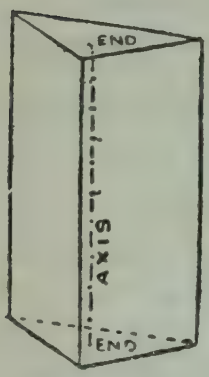


Fig. 168.

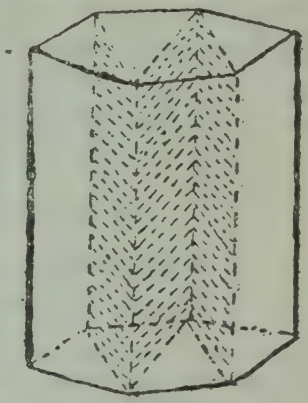


Fig. 169.

A prism is a solid bounded by plane faces as shown in Figs. 168 and 169. In every prism there are two faces which are parallel to each other. These faces are known as the *ends* of the prism and one of them is generally chosen as its base. The st. line joining the central points of the ends of the prism is known as its *axis*. The other faces of the prism are known as its *lateral* faces and are all parallel to the axis. If the ends of a prism are at rt. ∠s to the axis the lateral faces are rectangular and the prism is called a *right prism*. If otherwise, the prism is said to be *oblique* and its lateral faces are parallelograms.

Prisms are named after their ends and are called triangular, square, rectangular, hexagonal, &c., according as their ends are triangular, square, rectangular, hexagonal, &c.

It has been shown in Chapter XIII that the volume of a rectangular prism or cuboid = the area of the base × the vertical height.

The same relation holds good in the case of any right prism. For, just as a parallelogram, a triangle or any polygon may be cut and fitted into a rectangle (Refer to p. 137) a right prism having any such figure as its base may also be cut and fitted into a rectangular prism.

Hence, the volume of a Right Prism = the area of its base or end or a horizontal cross-section × the vertical height or length of the prism.

Draw the nets of right prisms with the bases a square, a triangle and a hexagon. It may be seen that the lateral faces of each prism are rectangles. (See Fig. 170).

Their total area = the height of the prism × the perimeter of its base. [How?]

Hence the total surface area of a right prism = (twice the area of its base) + (its height × its perimeter.)

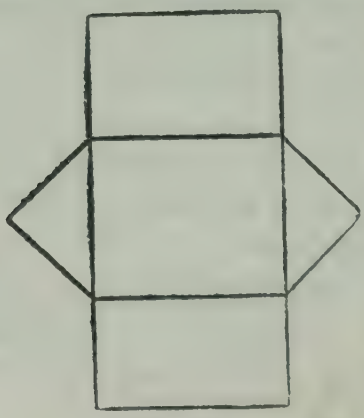


Fig. 170.

Ex. 1. The base of a prism is a rt. ∠d △ with the sides forming the rt. ∠ 3 cm. and 4 cm. long. If the height of the prism = 10 cm, find its total surface area and volume.

Since the base is a rt. ∠d. △ with sides 3 and 4 cm., its third side = 5 cm. [How ?]

∴ the perimeter of its base = 12 cm. ; and the area of its base = $\frac{1}{2} \times 3 \times 4$ or 6 sq. cm. ; and the total surface area of the three lateral rectangular faces

$= 3 \times 10 + 4 \times 10 + 5 \times 10$ or 120 sq. cm.

∴ the total surface area of the prism = $120 + 2 \times 6$ or 132 sq. cm.

and its volume = 6×10 or 60 c. cm.

EXERCISES.

1. Find the volumes and total surface areas of the right prisms given below :—

(*a*) Base is a square of side 1·5" and height = 2·5".

(*b*) Base is a trapezium ABCD as shown in Fig. 59, page 95 and height = 6".

(*c*) Horizontal cross-section is ·5" square and height = 3".

(*d*) Length of prism = 4' and each end is a △ with sides 12", 13" and 5" in length.

(*e*) Length = 8", each end is an equilateral △ of side 1·4".

(*f*) Length = 16", each end is a regular hexagon of side 1·55".

(*g*) Length = 8", each end is a △ with sides 6", 5" and 6".

(*h*) Base is a regular octagon of side 5" and height 1' 5".

2. Compare the volumes and the total surface areas of pairs of prisms of equal vertical heights but with the bases as follows:—

(*a*) The rectangle ABCD and the rt. ∠d △ ABC in Fig. 76, page 131.

(*b*) The parellelogram ABCD and the rect. BCHQ in Fig. 83, page 135.

(*c*) The △ABC and the △DFH in Fig. 85, page 137.

(*d*) The rect. PDHQ and the △ DAH in Fig.85, page 137.

3. A street well is in the form of a regular hexagon with a flat bottom, having each side equal to 12'. If the depth of the water in it is 12', find the quanity of water in the well.

4. A maize stack stands on a square base with gabled ends as in Fig. 171.

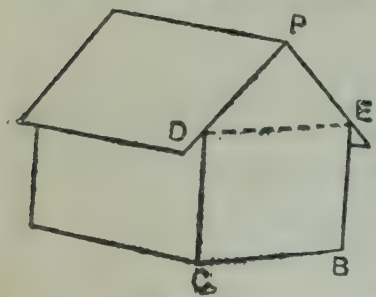


Fig. 171.

If CB = 16' = DE = twice the distance of P from DE and if the side faces of the stack are vertical, find the volume of the stack. [Neglect the overlapping at the eaves.]

5 A rectangular hall with gabled ends is used as a school dormitory. If the hall is 20' long and accommodates 10 boarders at 30 sq. ft. of floor-space and 400 c. ft. of air-space per pupil, find the area of each end of the hall.

6. The drainage canal in a city is 4 ft. wide at the top, 2 ft. wide at the bottom and 4 ft. deep, and its cross-section is a trapezium. If water flows through it in full at 2 miles an hour *on the average*, find the quantity discharged by it per minute.

7. The press for a tennis bat consists of two pieces, each in the form of a uniform hollow prism whose base consists of a trapezium ABCD with an inner trapezium EFHG cut out. (See Fig. 172). Find the volume of wood in the press given that the thickness of each piece is equal to ·5".

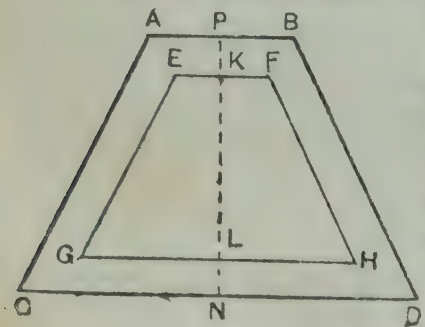


Fig. 172.

AB = 5·8"
CD = 11·4"
EF = 4", GH = 8·6"
KL = 8·9" and PK = LN = 1·1".

8. Assuming that the shaded portion in Fig. 79, page 132, roughly represents the cross-section of a rectangular prism with an inner hollow XZTY cut out in it, find the *total* surface area and volume of the solid given that PQ = 1'3", QR = 1'8", TY = TZ = 8", and XY = XZ = 10" and the height of the prism = 1".

9. A rectangular tin plate $\frac{a''}{2}$ by $\frac{a''}{4}$ lying on the ground is moved parallel to the ground to a height of h'. Find the volume of the solid traced.

10. A triangular steel plate is driven vertically to a depth of 4' so as to form a hollow in a ground of sticky clay. Find the capacity of the hollow when each edge of the plate is 1'6".

11. The accompanying figure shows the cross-section of a channel bed. If the *mean* speed of the current is 4 miles an hour, find how much of water is discharged per hour.

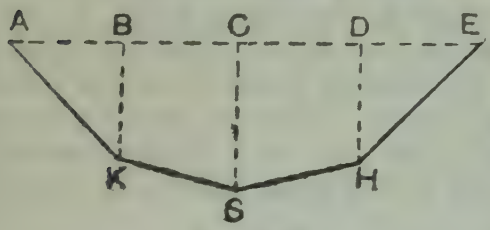


Fig. 173.

AB = BC = DE = KB = 4'
DH = 4½' and CS = 5'

12. Construct a hexagon ABCDEF in which AB = 2·5", AF = BC = 1·5", EF = CD = 1", ∠F = ∠B = ∠A = 185° = ∠C.

If the above hexagon is the ground-plan of a school-room drawn to a scale of 1" to 5 ft., find the surface area of the floor. If each wall is 10 ft. high, find how many students the room could accommodate, allowing 80 c. ft. of air space for each student.

13. ABCD is a rectangular plot of sloping ground in which AB = 2 chs., AD = 5 chs. and the edge AD is 5 ft higher than BC. Find how much of earth should be removed if a cutting be made so that the whole ground is reduced to the level of BC. [Assume that the soil when dug out increases by $\frac{1}{10}$ of its volume.]

14. A triangular iron prism has a cross-section of ·96 sq. in. Find the weight of the prism if its length = 6·6" and the specific gravity of iron = 7·5.

[*Specific gravity of a substance*

$$= \frac{\textit{the wt. of 1 c. ft. of the substance}}{\textit{the wt. of 1 c. ft. of water}}.$$

The wt. of 1 c. ft. of water = 1,000 oz.]

15. The ground beside a level road rises 1' vertical to every 2' measured along its slope. How much of earth must be removed

in order to leave a margin 10′ wide on a level with the road to a distance of 40′ ?

16. A strip of sand 10′ broad and 40′ long lies inclined to a vertical rock making 30° with the horizon. If the sand has to be removed to a depth of 16′ along the rock, find the quantity to be removed.

86. THE RIGHT CIRCULAR CYLINDER.

A right circular cylinder may be taken to be a right prism whose ends are circular. Since a circle could be cut into sectors which could be arranged so as to resemble a rectangle (refer to Art. 80) a cylinder could also be cut into pieces which may be fitted into a solid resembling a rectangular prism. Thus it may be seen that the volumes of a cylinder and of a prism would be equal when their bases are of equal area and their heights are also equal.

Hence, the volume of a cylinder of radius r″ *and height* h″ = *the area of its circular end* × *its height* = ‘ $\pi r^2 h$ ’ *cub. ins.*

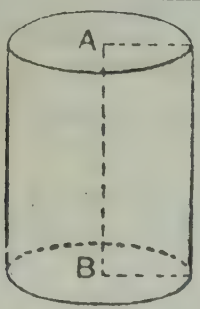


Fig. 174.

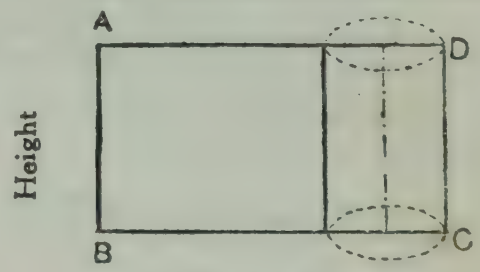


Fig. 175.

[A cylinder may also be treated as a solid generated by rotating a rectangular sheet of card-board swiftly round an edge kept in a fixed position. The fixed edge will then be the *axis* of the cylinder, *e. g.*, AB in Fig. 174.]

Roll the curved surface of a cylinder once completely over a flat surface so that its circular rim moves along a st. line BC. A rectangle will then be traced whose length equals the Oce of the cylinder and whose breadth equals the height of the cylinder.

∴ the area of the curved surface of a cylinder of diameter d'' or radius r'' and height h'' = $2\pi rh$ or πdh sq. ins.

∴ its total surface (including the areas of the ends) = $2\pi r + 2\pi rh$ or $2\pi r(h + r)$ sq. ins.

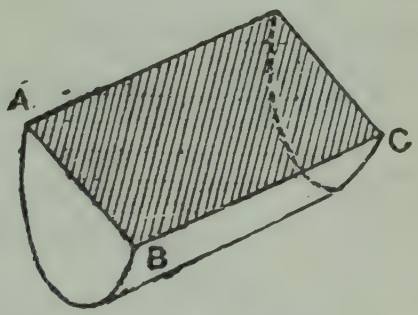


Fig. 176.

[If a cylinder be cut ⊥ to its axis, what will be the shape of the cross-section? If a cylinder is cut into two pieces along or parallel to its axis, explain how you would find the volume and surface area of the solids formed].

The solidity of a hollow cylinder of external diameter D'', internal diameter d'' and height h'' = the difference in volumes between two cylinders of diameters D'' and d'' and of height $h'' = \pi h\left(\frac{D^2}{4} - \frac{d^2}{4}\right)$

or $\frac{\pi}{4} h (D + d)(D - d)$ cub. ins.

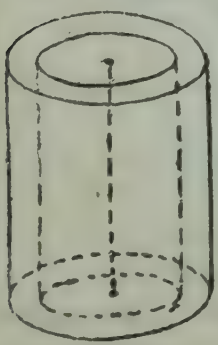


Fig 177.

= area of cross-section × vertical height = $\pi h(R^2 - r^2)$ or $\pi h(R + r) \times (R - r)$ c. ins. where R'' and r'' are the external and internal radii.

The total surface area of the hollow cylinder = the area of the internal and external curved surfaces + that of the two flat rims.

$$= \pi h\,(D + d) + 2 \times \frac{\pi}{4} \times (D^2 - d^2) \text{ sq. ins.}$$

$$= 2\pi h\,(R + r) + 2\pi\,(R + r)\,(R - r) \text{ sq. ins.}$$

Ex. Find the solid contents and surface area of a hollow cylinder 4 cm. long and of external radius 4 cm. and internal radius 3 cm.

The solid contents of the cylinder

$$= \frac{22}{7} \times 4 \times (4 + 3)\,(4 - 3) \text{ or } 88 \text{ c. cm.}$$

Its surface area $= 2 \times \frac{22}{7} \times 4 \times (4 + 3) + 2 \times \frac{22}{7} \times (4 + 3)\,(4 - 3)$ or 220 sq. cm.

EXERCISES

17. Calculate the total surface area and volume of each of the following cylinders :—

(*a*) Diameter 8″ and vertical height 10″
(*b*) ,, 10 cm. ,, 16 cm.
(*c*) ,, 15″ ,, 2′4″
(*d*) ,, 4·9′ ,, 6·16′
(*e*) ,, 5′ ,, 1′9″

18. A cylindrical ollock, a measure and a marcal are found to be 2·6″, 5″ and 10·3″ in diameter respectively. Find the depth of each, given that a measure holds 100 c. ins. and equals 8 ollocks or $\frac{1}{3}$ of a marcal.

19. A sheet of paper 8″ by 6″ is rolled along its length into a cylinder. Find the height and diameter of the cylinder.

20. A rectangular sheet of card-board 10″ by 5″ is rotated swiftly about one of the shorter edges. Find the volume of the solid generated. If the card-board be rotated about its longer edge, what will be the volume of the solid generated? (π = 3·1416).

21. A roller 3' long, has a Oce of 3'6". Find its weight if 1 c. ft. of its material weighs 175 lbs.

22. A cylindrical well 15' in diameter contains water to a depth of 10'. Find the volume of water in the well.

23. A petroleum tank is roughly in the form of a cylinder with diameter 3' 6" and height 12'. How many gallons could it held? (1 c. ft. contains 6·25 gallons).

24. A cylindrical tub open at one end has an internal diameter of 1' 9" and is 2' 4" high, If it is made of metal 1" thick and if 1 c, in. of the metal weighs ·2 lb., find the weight of the tub when empty.

25. The bore of a pipe in my house is 1" in diameter, Water flows through it (full bore at 44 ft. per minute). Find the quantity of water discharged by the pipe during the fifteen minutes of a bath.

26. Compare the lateral surface areas as well as the volumes of 3 cylindrical pillars of diameters 1', 1·5' and 2' and with vertical heights 14', 40' and 12'.

27. A cylindrical pail 7" in diameter and 1' 2" deep is used to draw water from a well. How many pails of water should be drawn to fill the tub in Ex. 24 above?

28. A cylindrical vessel 1'2" in diameter is partly filled with water. When the broken pieces of another vessel are submerged in it the water rises 2". Find the entire volume of the pieces thrown in,

29. Calculate the weight of a hollow brass cylinder with an external diameter of 1·4", internal diameter 1·26" and a height of 3·57". (1 c. in. of brass weighs ·32 lb.)

30. A cylindrical log of wood is 16' long and has a uniform thickness of 1' 6". From it is cut out a rectangular beam 16' by 1' by 10". How much of wood material would remain?

31. A cylindrical glass bottle contains ink to a depth of 10". The outer Oce of the bottle is 3·5" and the thickness of the glass is ·14. If five cylindrical bottles with an inner diameter of 2·1" are filled up to a height of ·2" with ink from the bigger bottle, how much will the level of the ink in the latter fall?

32. A boy who used to read with a candle shade said one night to his mother that he read from 7 P. M. to 10 P. M. If the candle he was using was 8″ long and 1″ in diameter, and could burn only for 7 hours, find what volume of candle he spent that night. (Neglect the conical portion at the top.)

33. Two cylindrical oil cans full of oil and with internal diameters 3″ hnd 4″ and heights 5·6″ and 6·3″ are emptied into a cylindrical vessel whose height is 2·625″. If the lather vessel is just full, find its Oce.

34. A circular grass-plot with a Oce of 14 chs. is surrounded by a circular path 7 yds, wide. If the path is to be metalled so as to raise its level by 7″. estimate the cost of metalling at 1 anna per c. ft.

35. An office is held in a rectangular hall with a semi-circular end as shown in Fig. 178. If 45 clerks are made to work in this hall, find how many c. feet of air space is allowed to each clerk, given that the hall has a flat roof, that the height of the hall is 18 ft. and that the records and furniture occupy one-fifth of the air space.

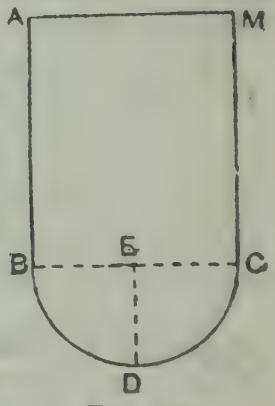


Fig. 178.

AB = MC = 41′
BC = 15′

36. Fig 179 is a rough sketch of a hay-stack consisting of a rectangular prism surmounted by a semi-cylinder.

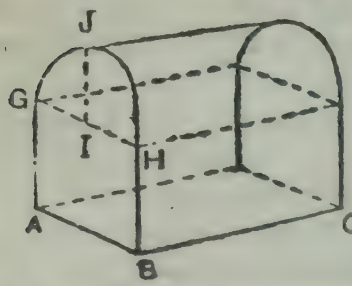


Fig. 179.

If AB = b', BC = c', AG = BH = h, find the quantity of hay in the stack. Also, give the numerical results when $b = 20$, $c = 30$ and $h = 15$.

37. The store-room in our Railway station contains a rectangular space 40 ft. long, 30 ft. wide and 10 ft. high, surmounted by a semi-circular roof which runs lengthwise. Find how much of air-space there is in the room.

38. A well with an internal diameter of 7 ft. and 30 ft. deep is lined with bricks to a thickness of 9 inches. Find the volume of the brickwork,

39. 'x' c. cm. of gold are melted and drawn into a cylindrical wire 'd' mm. thick. If the length of the wire drawn is 'l' cm., express 'l' in terms of 'd' and 'x'. Also, express 'd' in terms of 'l' and 'x'.

40. Two cylindrical measures are filled with oil. The diameter of one measure is d'' and its height is h''. The diameter of the other measure is e'' and its height is l''. A third cylindrical vessel x'' high can just hold the oil from the two measures. Find the diameter of the vessel.

41. In a cylindrical boiler with an internal diameter of d' and length l' there are n tubes each 2" in external diameter, traversing lengthwise along the boiler. Find the quantity of water the boiler can hold,

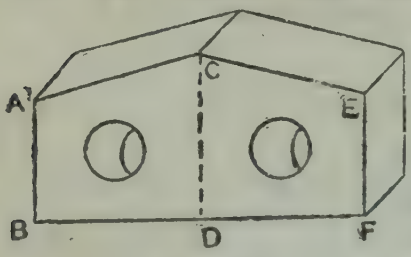
42. A length of glass tubing, having a, narrow circular bore, weighs 14·65 grams. A thread of mercury 10·5 cm. long is drawn into the tube. The tube now weighs 19·13 grams. If 1 cubic centimetre of mercury weighs 13·5 grams, find the diameter of the bore of the tube.

43. Water flows at the same rate through (1) a single pipe of diameter 8" and (2) a double pipe consisting of two separate tubes each 4" in diameter. Would both the pipes discharge equal quantities of water, when water flows through both (full bore) at the same speed?

If the pipes are kept open for 1 hour and if water flows in each pipe at the same speed of 2 miles an hour, which pipe would give the greatar quantity of water and by how much?

44. From a rectangular piece of wood 6" long, 3" wide and $2\frac{1}{2}$" thick is scooped out two cylindrical portions, each 2" in diameter and 1" thick for serving as ink-stands. Find the volume of wood scooped out and the weight of the ink-stand, the specific gravity of the wood being ·672

45. Two circular holes each 9″ in diameter are made in a steel plate which is in the form shown in Fig. 180. If the thickness of the plate is ·2′ and AB = 2′. EF = 2′6″ and CD = ¼ BF = 3′6″, calculate the volume and weight of the plate, the specific gravity of steel being 7·9.



Ffg. 180.

46. A cylindrical mud jar of mercury is 6·3″ high and 2·4″ in outside diameter. If the internal diameter is 2·1″ and the internal height is 6″, find the weight in ounces of the jar and also the total weight including contents, given that the specific gravity of mud is 2 and that of mercury is 13·6.

47. In making blacklead pencils 9″ long and ·4″ in diameter, rectangular blocks of wood 9″ × 2″ × 4″ are taken and five pencils are cut out from each block. If 2,000 pencils have been made in this way, find the total volume of wood used and the percentage of wasta e of wood.

48. In a thermometer of inner bore ·5 mm. in diameter there s 1·874 c. cm. of mercury. The bulb contains 1·848 c. cm. of mercury. To what height does the mercury stand above the bulb? If the volume of the mercury becomes 1·00018 times as arge for each degree (Centigrade) rise in temperature, find the height by which the mercury in the thermometer rises for a rise of 5° C. in temperature.

49. Find the weight of 100 ft. length of iron piping, external diameter 9 ins., and internal diameter 7·6 ins., if 1 c. in. of the iron weighs ·28 lb.

50. 21 grams of mercury are poured into a cylindrical glass tube and are found to occupy a height of 7 cm. Find the diameter of the tubs, it being given that 1 c. cm. of water weighs 1 gram and that the sp. gr. of mercury is 13·6.

51. The "Butter Ball" at Mahabalipuram is spherical in shape, being 20′ in diameter. Hard by is a circular pit, 7′ across and 5′ deep where the ball is said to have been moulded. If the volume of a sphere of radius $r'' = \frac{4}{3}\pi r^2$ c. ins., compare the volume of the ball with the capacity of the pit.

52. A hollow cylinder is of length l", inner diameter d", outer diameter D", and of volume V c. ins. Express 'l' in terms of V, D and d. Also, find 'l' when V = 3·52, D = ·5 and d = ·3

53. I want to buy 50 yards of a cylindrical brass wire which is ·23" thick, but I find it is sold only by the lb. Find the weight of wire I must buy, given that brass weighs ·32 lb. per c. in.

87. THE RIGHT CIRCULAR CONE.

A right circular cone is the solid generated by a right-angled triangle being rotated about one arm of the right angle kept fixed. This fixed arm is called the *axis* of the cone. In the rough sketch of a cone shown in the margin, V is the *vertex* or *apex* of the cone, AB is the diameter of the circular base, O its centre, VO is the *axis* of the cone and represents its *vertical height*. VA or VB is the *slant height* of the cone, the angle AVB is *the vertical* angle of the cone, the angle OVB is *semi-vertical* angle of the cone. Any section of a right circular cone parallel to the base is perpendicular to the axis and is a circle. A section of a right circular cone made along its axis is an isosceles triangle, *e.g.*, △AVB in the above figure. [Note that VA^2 or $VB^2 = VO^2 + OB^2$]. Take a hollow cylinder and a hollow cone with equal circular ends and heights. (A conical glass funnel closed at its apex and a cylindrical jar could be easily so adjusted).

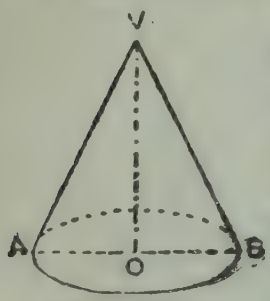


Fig. 181.

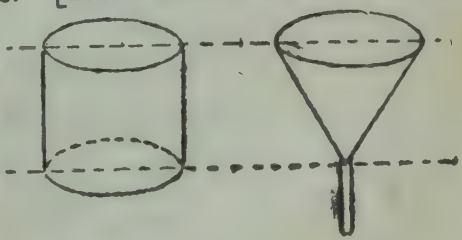


Fig. 182.

It could be seen that three conefuls of water would just fill the cylinder.

∴ the volume of a cone $= \frac{1}{3}$ *that of a cylinder of equal base and height* $= \frac{1}{3} \pi r^2 h'$ *cub. ins. where* r'' *is the radius of the base and* h'' *is the height of the cone.*

Roll the curved surface of a cone once round its apex.

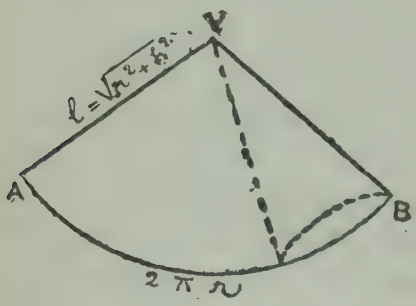


Fig. 183.

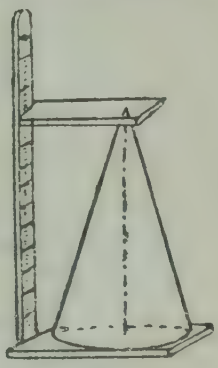


Fig. 184.

A sector will be formed whose arc is the circular edge of the cone and whose radius is the slant height of the cone.

∴ the area of the curved surface of a cone of slant-height l'', vertical height h'' and radius of base r''

$$= \tfrac{1}{2} \times 2\pi r \times l \text{ or } \pi r l \text{ sq. ins.}$$

$$= \pi r \sqrt{r^2+h^2} \text{ sq. ins.}$$

∴ the total surface area of a cone

$$= \pi r^2 + \pi r \sqrt{r^2+h^2}$$

$$\text{or } \pi r \{ r + \sqrt{r^2+h^2} \}.$$

Fig. 184 shows how the vertical height of a right circular cone could be measured.

Ex. Find the total surface area and volume of a cone whose circular bottom is of diameter ·7″ and whose vertical height is ·84″. Referring to Fig. 181 above, VO = ·84″ ; OB = ½ of ·7″ = ·35″.

$\therefore$ VB = $\sqrt{\cdot 84^2 + \cdot 35^2}$″
= $\sqrt{\cdot 7056 + \cdot 1225}$″
= $\sqrt{\cdot 8281}$″
or ·91″.

	·91
	·8281
	81
	181)181(

$\therefore$ the area of the curved surface of the cone = $\pi rl = \frac{22}{7} \times \cdot 35 \times \cdot 91$ sq. ins.
= 1·001 sq. ins.

The area of the circular face = $\pi r^2 = \frac{22}{7} \times \cdot 35 \times \cdot 35$ or ·385 sq. in.

$\therefore$ the total surface area of the cone
= 1·001 + ·385
or 1·386 sq. ins.

The volume of the cone = $\frac{1}{3} \pi r^2 h$
= $\frac{1}{3} \times \frac{22}{7} \times \cdot 35 \times \cdot 35 \times \cdot 84$ c. in.
= 22 × ·05 × ·35 × ·28 c. in.
= ·1078 c. in.

EXERCISES.

54. Find the volumes and the total surface areas of cones with the radii of their circular faces 1·5 cm., 6 cm., 4 ft., 4·2 yds. and with vertical heights 2 cm., 8 cm., 3 ft. and 6 yds. respectively.

55. The arc of a sector of a circle is 21 cm. and its radius is 14 cm. If the sector represents the development of the curved face of a brass cone, find the vertical height, volume and weight of the cone. (Assume that 1 c. in. of brass weighs ·32 lb.)

56. An oil-can is conical in shape. The diameter of its circular face is 3·5″ and its height = 9″. Find its cubical contents.

57. A rt.-$\angle$d $\triangle$ ABC with $\angle$ B a rt. $\angle$, AB = 12 cm. and BC = 5 cm., is rotated first about AB and then about BC. Compare the volumes of the cones generated.

58. An isos. $\triangle$ ABC with AB = AC = 5 cm. and BC = 8 cm., is rotated about BC kept fixed. Find the volume of the solid generated.

59. If 's' sq. ins. be the total surface area of a cone, 'V' c. ins. its volume, r'' the radius of its base and h'' its vertical height, show that $s = \pi r\,(r + \sqrt{h^2 + r^2})$ and $V = \frac{1}{3}\pi r^2 h$.

Express h (1) in terms of s and r and (2) in terms of V and r respectively.

Also give the numerical values of (1) V and s when $r = 2{\cdot}1$ and $h = 12{\cdot}5$; (2) r and s when $V = 404{\cdot}25$ and $h = 14$; (3) r and V when $s = 75\frac{3}{7}$ and $h = 4$ and (4) h and V when $s = 301\frac{5}{7}$ and $r = 6$.

60. A filter-paper is in the form of a semi-circle having a diameter of 10 cm. It is rolled into a conical filter. Find the vertical height of the filter and its capacity.

61. From a right circular cone of vertical height 4″ and diameter of base 2″ is cut off a small cone at the top of vertical height 2″ and diameter of base 1″. Calculate the volume of the solid remaining.

62. How many sq. ins. of the plate will there be in a conical tin funnel, the diameter of whose circular edge is $3\frac{1}{2}$″ and whose depth is $4\frac{1}{3}$″? (Neglect the overlapping portion, etc.)

63. The figure drawn represents a solid consisting of two right cones fitted with their bases together. If $AB = V_1O = OV_2 = 6{\cdot}4$ ft., calculate its volume and surface area.

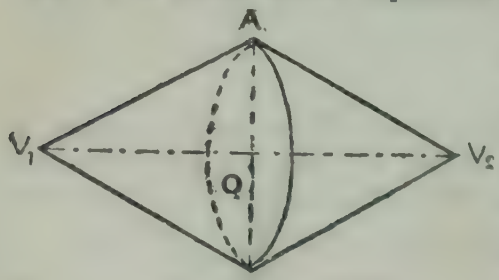


Fig. 185.

64. Find how many square yards of tent cloth there are in a conical tent of vertical height 11 ft. in which a man 5 ft. high may stand anywhere inside just within

a radius of 8 ft. from the centre without stooping. (Neglect seams, etc.)

65. The buoy of a ship consists of a cylindrical portion with two conical ends as shown in Fig. 186. The height of each conical end: that of the cylindrical part = 5 : 2 and the total length of the buoy is 8'. If the diameter of the cylindrical part = 10', find the total surface area and volume of the buoy.

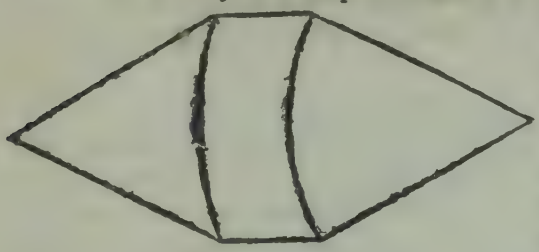


Fig. 186.

66. Find the volume and the total surface area of a cone whose vertical $\angle$ is a rt. $\angle$ and height 4".

67. I bought a new pencil 7·5" long and ·35" thick and mended it so as to have a conical end 1·2" long along its slant edge. If the length of the whole pencil remains the same, find how much of matter has been removed by mending.

68. Find the volume of a cone whose semi-vertical $\angle$ is 30° and whose bottom diameter is 2".

69. The interior of a tower consists of a cylindrical portion surmounted by a conical roof. If the diameter of the cylindrical portion is 14', the height of the cylindrical walls 40' and the total height of the tower 52', find the volume of air-space inside the tower.

70. A barn is in the form of a cylinder 20' in diameter and 25' high surmounted by a cone of vertical height 10'. Find the amount of grain that can be stored in it.

71. From a cylinder of wood two cones are scooped out as shown in Fig. 187. if the height of each cone= half that of the cylinder, express the quantity of wood material in the solid remaining in terms of d and h, the diameter and height in inches of the cylinder.

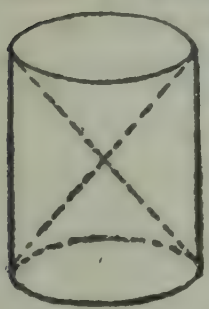


Fig. 187.

72. A solid brass cone, diameter of base 3·5 cm. and vertical height 6 cm. is placed in a cylindrical jar of diameter 10·5 cm. Water is then poured into the jar until the height of the

water column is 12 cm. If the cone be removed and replaced by a brass cylinder of the same base and height, how will the height of the water column be affected ?

73. Find the volume of the biggest cone that could be cut out of (1) a cubical block of edge 10" and (2) a rectangular block 12" by 8" by 7".

74. A circular sector whose radius is r" and area 'A' sq. ins. is folded into a conical cup. Find the cubical contents of the cup.

75. A conical heap of paddy which has a diameter of 14' at the bottom and a vertical height of 6' is measured by means of a cylindrical marcal which is 9·6" high and has an internal diameter of 10·3". If a marcalful of paddy measured consists of a cylindrical portion surmounted by a conical portion 2·2" high, how many marcals of paddy are there in the whole heap ?

76. A conical extinguisher whose vertical height is 4" is placed over a cylindrical candle, diameter 1". If a point of contact where the extinguisher touches the circular rim of the candle bisects the generating line of the cone, calculate the inside surface area of the extinguisher. (VA is the generating line of the cone.]

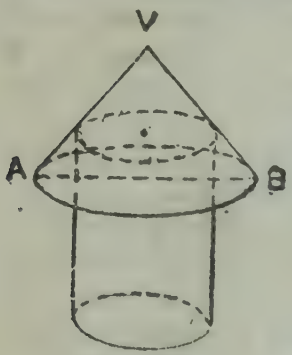


Fig. 188.

77. A Stephen's ink jar is in the shape of a cylinder 3'6" in diameter and 9·4" high surmounted by a cone of height 1·8". Find the volume of ink it will hold. The big bottle is priced Rs. 2-8. The ink in this is poured into small cylindrical bottles of diameter 1·2" and height 1·5" which cost one anna per 24 bottles. If 12 jars of ink were bought and each small bottle of ink be sold at one anna per bottle, how much is gained by the transaction ?

Fig. 189.

78. A measure is in the form of a cylinder of diameter 4" and height 8". But a tradesman by hammering at the bottom makes it assume the shape shown in Fig. 189 so that its bottom is now a cone of height ·5". If he sold at 4 as, 9 ps. per measure from a rectangular box 18" by 14" by 11" full of rice, find how much money he would have unjustly gained.

CHAPTER XX.

GRAPHS—(*continued*).

88. GRAPHICAL REPRESENTATION OF STATISTICS.

In Chap. XV, we have dealt with the graphical representation of variables connected by a linear relation, *e.g.*, $y = mx + c$. The values of either variable were obtained by calculation, with their help points were plotted and the graph was drawn as to pass through every plotted point. The same method of graphical representation may be followed in respect of quantities varying according to *any* mathematical law. But quantities like the growth of a plant, in different periods, the temperature of the atmosphere at different times, the yield of a crop in different years, etc., are not known to vary according to any law. The values to plot could be obtained only by experiment and observation or from a table of statistics. A clear general idea of the changes in their values could very often be gained by diagrammatic representation as shown in the examples below :—

Ex 1. The yield (in thousands of tons) of wheat in a province during the years 1911 to 1917 is given in the following :—

Year.	11	12	13	14	15	16	17
Yield of wheat (in thousands of tons).	465	620	525	335	435	678	447·5

The thick vertical lines in Fig. 190 represent the yield to a scale of $\frac{1}{8}$ to 100 thousand tons and the years are marked along the horizontal line at equal interspaces of $\frac{1}{4}''$. From the figure it may be seen at a glance during what periods the yield of wheat increased or decreased and when it was greatest or lowest.

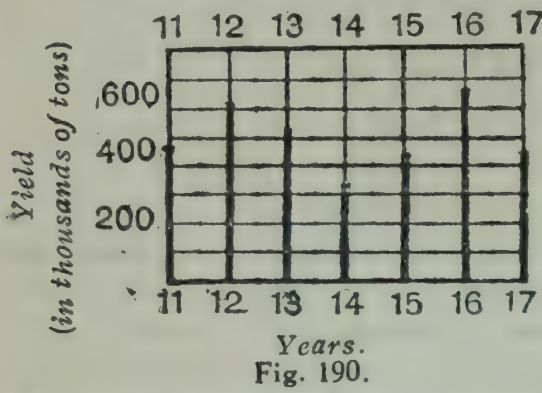


Fig. 190.

Ex. 2. The temperature (in degrees Fahrenheit) of the atmosphere in a certain place taken at intervals of 2 hours from 7 A.M one day to 7 A.M. the next day is found to vary as follows:—

7 A.M.	80°	9 P.M.	80°
9 A.M.	82·5°	11 P.M.	77·9°
11 A.M.	85°	1 A.M.	76°
1 P.M.	86·9°	3 A.M.	74·2°
3 P.M.	87·9°	5 A.M.	74·8°
5 P.M.	86°	7 A.M.	76°
7 P.M.	83°		

In Fig. 191 two ⊥ axes are chosen to represent the two variables, *viz.*, time and temperature. Along the horizontal axis time in hours is marked and along the other axis temperature in degrees. Points are plotted whose co-ordinates represent the temperature at particular moments and are then joined by a smooth curve.

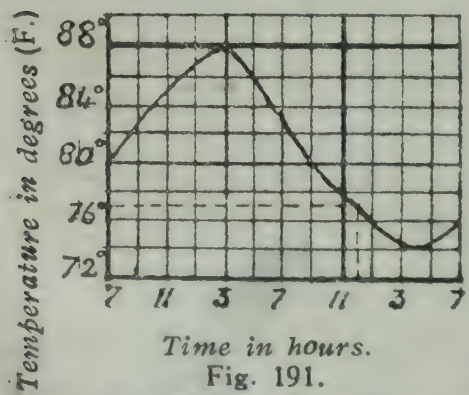


Fig. 191.

From the curve (or *graph* as it is otherwise called) we see that the temperature gradually rises from 7 A.M. to 3 P.M., that it falls from 3 P.M. to 5 A.M. the next day and that it again begins to rise. Also, we could read from it the *probable* temperature at any intermediate moment, *e.g.*, 77° at 12 midnight. Such a method of reading is said to be by Interpolation. A graph as above drawn is particularly useful in giving readings by interpolation.

In drawing the continuous curve it is presumed that if the temperatures at each and every moment between 7 A.M. the first day and 7 A.M. the next day were given, the series of points forming the curve would be plotted. Since different kinds of curves could be drawn between two plotted points *the readings given by interpolation are only approximate and what may be probable.* The accuracy of the readings depends upon the shortness of the interval between which two successive observations are recorded. If the temperature at 5 P.M. only were given after that at 11 A.M. the graph that would be obtained would give entirely different readings for the temperatures at moments in the interval. The consecutive points are joined by a smooth curve instead of by st. lines just to indicate that there are no abrupt changes in temperature as would be indicated by the deflections in a broken line.

[Note:—The points plotted may be joined by a line only if any meaning could be assigned to the intermediate points. Hence it would be meaningless to draw a graph through the points plotted in Ex. 1 above though in mercantile charts, etc., it is customary to join consecutive points and thus obtain a broken line.]

EXERCISES.

1. The curve in the marginal diagram represents the variation in the depth of water in a well as observed at intervals of one

hour from 1 A.M. to 12 noon on a certain day; read from the graph the depth of water in the well at 4 A.M., 5 A.M., 6 A.M., 8 A.M. and 9 A.M. Also, find approximately when the depth of the water was (i) 11 ft. (ii) below 8 ft. and (iii) above 10 ft. Also, find (i) when the depth was greatest and when least, (ii) during what periods it increased, (iii) during what periods it decreased and (iv) how much greater or less the depth was at 6 A.M. than at 2 A.M.

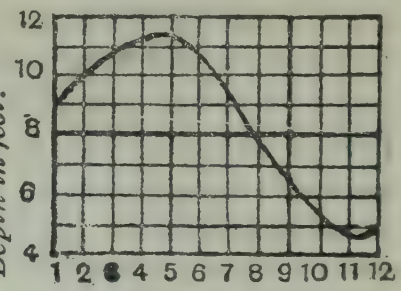


Fig. 192.

2. The height of a plant observed at intervals of 3 days from the 10th day after sowing the seed is given in the following table:—

Day of observation after sowing the seed.	10	13	16	19	22	25	28	31	34	37	40
Height of the plant in inches.	2	4·5	5·5	6	6·25	7	9	10·5	11	11·5	12

Illustrate graphically the variation in the height of the plant on the several days of observation and state from the graph the leading features of the variation.

3. The height of the barometer observed at certain moments in the course of a day varies as follows:—

Time observation.	12 Midnight.	2 a.m.	5 a.m.	9 a.m.	12 Noon
Height of the barometer in inches.	29·0	29·4	29·3	29·1	28·5

Represent the variation graphically. Read from the graph when the height may have been 29·2".

4. The length of a man's shadow measured at intervals of one hour from 7 a.m. to 4 p.m. in the course of a day has been 28 ft., 12 ft., 6 ft., 4 ft., 2 ft., ·3 ft., 2·1 ft., 6·4 ft., 8 ft. and 15 ft. respectively. Draw a graph showing the variation in the length of the shadow at different times. Read from the graph the length of the shadow at 11·30 a.m. and the time when the shadow was 7 ft.

5. The following table gives the total area and yield of sesamum in India during different years :—

Year.	1908-'09	'09-'10	'10-'11	'11-'12	'12-'13	'13-'14	'14-'15	'15-'16	'16-'17	'17-'18
Area (in million acres)	5	5·4	5·2	4·8	5	5·1	5·5	5·1	5	4·4
Yield (in thousand tons)	455	565	500	400	475	400	550	480	510	380

Represent in separate diagrams the variations in different years in the area and yield of sesamum.

6. The following table gives the average prices at Tinnevelly of some of the staple food-grains (in Imperial seers per Rupee) during 7 consecutive months :—

	Rice. (Second sort.)	Ragi.	Kumbu.
1917 November	6·6	11·1	9·6
,, December	6·6	12·0	9·1
1918 January	6·8	12·8	9·6
,, February	7·1	12·9	10·2
,, March	7·8	12·6	10·4
,, April	7·3	12·0	10·7
,, May	7·2	12·3	10·3

Represent the above variations diagrammatically.

7. The following table gives the times of sunrise (R) and sunset (S) in Madras in 1918 :—

		R (A.M.) hrs. mins.	S (P.M.) hrs. mins.
January	1	6—31	5—53
,,	31	6—36	6— 9
March	2	6—25	6—18
April	1	6— 6	6—21
May	1	5—48	6—24
,,	31	5—41	6—31
June	30	5—45	6—39
July	30	5—53	6—37
August	29	5—58	6—23
Sept.	28	5—59	6—25
Oct.	28	6—2	5—45
Nov.	27	6—13	5—39
Dec.	27	6—29	5—50

Represent in a diagram (1) the variation in the time of sunrise and (2) the variation in the time of sunset in the course of the year. Read from the graph during what part of the year (1) the day is longest and (2) the day is shortest.

Represent diagrammatically with the help of the same statistics the duration of night and day in the course of the year. Find from the diagram when the days and nights are of equal duration.

[Note :—If the abscissae of points represent dates they could be assigned a meaning only when they are integral. Hence the points plotted may be joined by a dotted line thereby indicating that the blanks represent points whose co-ordinates have no meaning. To distinguish the dotted line indicating the variation in sunrise from the corresponding line for sunset, the lines may be differently coloured. A like procedure may be adopted in similar exercises.]

8. The following table gives the rainfall in inches during the months of 1917 at Ootacamund :—

Serial order of month	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
Rainfall in inches.	0·59	3·42	1·91	1·94	3·24	5·25	1·94	7·5	11·13	5·27	9·34	·27

Represent the above variations in a chart.

9. The following table gives the temperature of water in degrees Centigrade and the maximum quantity of Epsom Salts expressed in grams which could be dissolved in 100 grams of water :—

Temperature of water in degrees (C).	0	20	40	60	80	100
Maximum quantity (in grams) of salt which 100 grams of the water could dissolve.	26·8	36·2	44·7	55·1	64·3	74

Draw a graph illustrating the variation in solubility. Read from the graph the maximum quantity of Epsom Salts which

water at 65°C. could dissolve and the temperature of water which could just dissolve 40 grams.

[NOTE.—*Since the values of the variables in the above tables can only be obtained by experiment and may be subject to errors it would be enough in such cases to draw a smooth curve so as to lie evenly amongst the points plotted without passing through each and every one of them.*]

10. The following table gives the same details about Potassium Alum and Potassium Nitrate as in the preceding exercise :—

Temperature in degrees C.	0	10	20	30	40	50	60
Potassium Alum in grams.	6	9	13	17	23	32	67
Potassium Nitrate in grams.	13	17	26	44	63	85	140·5

Illustrate graphically using the same scale and set of axes the solubility of Potassium Alum and Potassium Nitrate. Read from the graph the maximum quantity of each salt which could be dissolved in water at 35°C. and the temperature of water which could just dissolve 60 grams of either salt.

11. The height and the corresponding average weight of men are given in the following table :—

Height.	5'	5'-2"	5'-3"	5'-4"	5'-6"	5'-8"
Average weight in lbs.	110	120	130	140	145	156

Draw a graph exhibiting the variation and read from it the average weight of men 5'-1" high. Is the average weight proportional to the height? If the average weight of women be 100 lbs, 110 lbs., 120 lbs., 125 lbs., 140 lbs. and 150 lbs. respectively, whose heights are as mentioned in the above table, draw using the same set of axes a graph showing the relation between the height and average weight of women. Read from the graph

the probable difference in weight between men and women of height 5'-5" and the difference in height between men and women of weight 125 lbs.

12. The velocity of sound through air is found to vary with temperature as follows:—

Temperature in degrees Fahrenheit.	32	40	50	60	70	80	90
Velocity in ft. per sec.	1093	1102	1113	1124	1135	1146	1157

Read from the graph the rate of increase in the velocity for every degree rise in temperature.

13. The following table gives the distance in yards at which trainsshould be capable of being brought to rest on the level when moving at different speeds :—

Speed in miles per hour.	30	35	40	45	50	55	60
Stop in yards.	99	135	176	223	275	333	396

Represent the variations graphically.

14. The following table gives the distances traversed by a football after being kicked by a goal-keeper :—

Horizontal distance (in ft.) from the foot of the goal-keeper.	4	12	16	24	28	40	48	60	64	72	80
Corresponding height of the ball from the ground in feet.	3	11	15	22	25	30	27	18	15	6	0

Represent on squared paper the path of the ball and find from the graph at what distance from the goal-keeper a player 5'-2" high could just head the ball by rising 22' when the ball is about to fall.

15. From a set of experiments with pendulums the following data were derived :—

Length of pendulum in cm.	Period of vibration in seconds.
100	2
84	1·8
72	1·7
64	1·6
56	1·5
40	1·3

Represent the variation graphically. Find from the graph the length of the pendulum when the period of vibration is 1·4 seconds.

16. When the arms of a pair of dividers are widened the distance between the moving ends is found to vary as follows:—

Angle between the arms.	10°	20°	30°	40°	50°	60°	70°	80°	90°
Distance.	·7″	1·3″	2·05″	2·7″	3·45″	4″	4·5″	5·1″	5·5″

Represent the variation graphically. Find from the graph the ∠ between the arms of the dividers when their moving ends are 4·9″ apart.

17. In an Insurance Company the annual premiums for an assurance of Rs. 1,000 payable on the attainment of the 50th age, or at death if previous, are given in the table below:—

Age next birthday.	20	21	22	24	26	28	30
Annual premium.	Rs. as. 36 4	Rs. as. 37 8	Rs. as. 38 4	Rs. 42	Rs. as. 45 12	Rs. as. 50 2	Rs. as. 55 10

Illustrate the above graphically and estimate the premium when age next birthday is (1) 27 and (2) 29.

18. The cost per sq. ft. of wire-netting for meshes of varying dimensions is given in the following table:—

Mesh in sq. ins.	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Cost per sq. ft. in annas.	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$

Represent the variation graphically.

19. The following table gives the pressure exerted by winds of varying velocities on a surface ⊥ to their line of motion;—

Velocity in miles per hour.	Pressure in lbs. per sq. ft.	Velocity in miles per hour.	Pressure in lbs. per sq. ft.
0	0		
1	·005	30	4·50
2	·020	40	8·00
4	·080	50	12·50
8	·320	60	18·00
15	1·125	80	32·00
25	3·125	100	50·00

Draw a graph showing the relation between the velocity and pressure of wind. Read from the graph the velocity of wind exerting a pressure of 10 lbs. per sq. ft. and the pressure exerted when the velocity of wind is 10 miles per hour.

20. When trains of average weights pass along bridges in a metre-gauge line, the pressure exerted on the main girders is as follows:—

Distance in ft. bet. two consecutive piers.	10	20	40	55	85	130	185	230	285
Pressure in tons.	20	37½	50	75	100	125	150	175	200

Represent the variations graphically. Find from the graph the pressure in tons when the span is 200 ft.

21. The resistance (R) in lbs. per ton to the motion of a train moving at (V) miles per hour is found to vary as follows:—

Velocity (V) in miles per hour.	10	15	20	25	30	35	40	45
Resistance (R) in lbs. per ton.	6·9	8·0	9·6	11·6	14·1	17·0	20·4	24·1

Represent the variations graphically.

22. When a gas is subjected to different pressures its volume is found to vary as follows :—

Vol. (v) in c. cm.	220	200	191	180	145	125	110
Pressure (P) in mm. of mercury.	455	499·9	524	555·5	690	799·9	909

Draw a curve showing the variations in pressure and volume.

Read from the graph the volume of the gas when P = 150 and the pressure when v = 500.

89. MISCELLANEOUS CASES OF GRAPHICAL SOLUTION.

The examples worked in Art. 69 involve quantities varying at a uniform rate. It has been found that there exists a linear relation between them and that their graphs are straight lines. Hence it has been possible to solve the examples directly by means of graphs even without framing equations from the conditions given. (Refer to pages 225 and 226). A few examples of such graphical solution are added below. A clear understanding of the gradient of the graph would help much in rendering the solution easier.

Ex. 3. S and P are two places 72 miles apart. A motor-car leaves S for P at 7 A.M., travels at a uniform rate of 20 miles per hour till 8 A. M., then stops for 30 minutes and again travels uniformly at 16 miles per hour until it reaches P, stopping for 15 minutes after 10. Another motor-car leaves P for S at 7-15 A. M. and travels along the same road at a constant rate of 16 miles per hour. Find graphically (1) when each car will reach its destination and (2) when and where the cars meet.

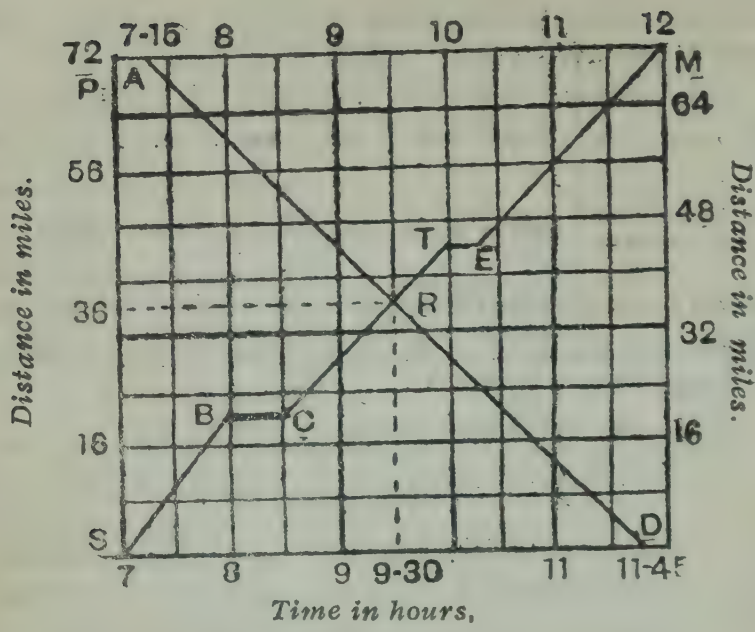


Fig. 193.

In Fig. 193 time in hours is marked horizontally and distance in miles vertically. The broken line SBCTEM represents the variation in the distance traversed by the first car at the several moments after it left S. The gradients of the lines SB, CT and EM indicate its speed. The horizontal lines BC and TE indicate that while the time changed the distance did not vary. From the graph it is seen that the first car would be 72 miles from S at 12 noon. The st. line AD indicates the variation in the distance traversed by the second car at the several moments after it left P. Its gradient indicates a movement of 16 miles per hour towards S, *i.e.*, away from P. The graph shows that the motor reached S at 11-45 A.M. Also, the co-ordinates of the point of intersection (R) of the graphs indicate that the cars passed each other at 9-30 A.M. at a place 35 miles from S.

[If a third car left S for P at 7 A.M. on the same day and along the same road and travelled uniformly at 15 miles per hour, find graphically when and where in its journey it would have the first car by its side.]

Ex. 4. By partly walking at 2 miles an hour and travelling at 5 miles an hour, I complete a journey in the same time as by travelling uniformly at 3 miles an hour. Find graphically the ratio between the distances traversed at either speed.

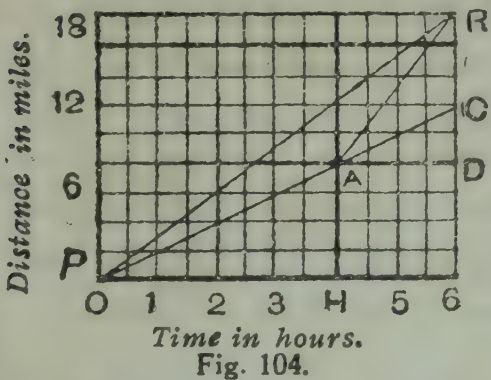


Fig. 104.

In Fig. 194 the lines PR and PC are drawn so that their gradients indicate a speed of 3 miles and 2 miles an hour respectively. Through any point R in the graph draw a line AR whose gradient indicates a speed of 5 miles per hour and produce it so as to meet PC at A. Then the broken line PAR indicates that if I travel the distance AH at 2 miles an hour and the distance DR at 5 miles an hour I would reach a place in the same time as another who leaves P and travels along the same road uniformly at 3 miles per hour. (See the line PR.)

∴ the ratio between the distances travelled at 2 and 5 miles per hour = AH : RD = 8 : 10 or 4 : 5.

[NOTE. By marking the cost in rupees along the horizontal axis and the quantity in measures along the vertical axis the above method may be adopted for finding the ratio in which rice selling at 2 measures per Re may be mixed with rice selling at 5 measures per Re. so that the mixture may cost 3 measures per Re.]

Ex. 5. A sluice in a canal if opened would let off water and fill an empty tank in 12 hours and another sluice would fill it in 24 hours. If both the sluices are kept open simultaneously and the rates of flow in them are uniform, find graphically in what time the tank would be full.

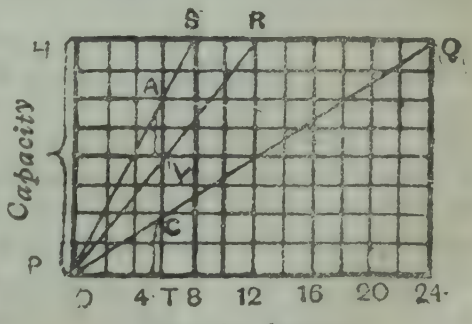


Fig. 195.

In Fig. 195 the line PH is arbitrarily chosen to represent the capacity of the tank. Then the line PQ indicates the variation in the quantity filled in solely by the second sluice and the ratio $\frac{TC}{PT}$ gives the rate of flow. Similarly the line PR indicates the variation in the quantity filled in solely by the first sluice and the ratio $\frac{TV}{PT}$ gives the rate of flow. Hence when both the sluices are simultaneously opened, the rate of flow would be given by $\frac{TV}{PT} + \frac{TC}{PT}$. Hence mark a point A vertically above V so that VA $=$TC, *i. e.*, $\frac{TA}{PT} = \frac{TV+TC}{PT}$. Join PA and produce it to meet the horizontal through H at S. Then the line PS indicates the variation in the capacity filled by both the sluices and from it it is seen that the two sluices would fill the tank in 8 hours.

EXERCISES.

23. A cyclist leaves Madras at 6 A.M. for Chingleput 45 miles away and rides at 10 miles an hour. He is followed by a motorist who leaves Madras at 8 A.M. and rides at 20 miles an hour. Find when and where the motorist overtakes the cyclist.

24. Water falls into an empty tub from a pipe which discharges at 2 gallons per minute. After 5 minutes, the tap is turned further on and the pipe then discharges at the rate of 4 gallons per minute. If when the tub contains 24 gallons it be emptied at a constant rate of half a gallon per minute, find from a graph when it would be completely emptied.

25 A thief steals a cycle at 3 P.M. on a certain day and rides on it at 12 miles an hour. Ten minutes later, the owner follows him on another cycle along the same road at 10 miles an hour but stops 5 minutes at the end of each mile to make inquiries. After cycling for half an hour the thief finds a puncture in the tube, spends 20 minutes in trying to close the puncture but fails. So he walks at 3 miles an hour leading the cycle himself. Find graphically when and where the owner would overtake the thief.

26. A motor-bus runs as follows:—

Station.	Distance in miles from starting point.	Arrives h. m.	Leaves h. m.
T	0	——	8
Q	15	9-15	9-20
R	25	10	10-2
S	37	11	11-5
X	50	12-5	——

A pleasure car leaves T at 8-30 and reaches X at 11. Assuming that the cars travelled at a uniform rate while they were in motion, find graphically when and where the pleasure car overtook the bus.

27. Two travellers A and B walk towards each other from two inns 12 miles apart. A walks at the rate of 3 miles an hour but halts for half an hour at the end of every two miles, while B travels at the rate of $2\frac{1}{2}$ miles an hour taking an hour's rest at the end of the fifth mile. Find graphically when and where they meet.

28. The following extracts taken from an S. I. R. Time-table relate to two trains, one up and one down:—

From Madras Beach Miles.	Station.	No. 223 S. H. M.	No. 118 P. H. M.
,,	Beach ...	8-40	9-50
1.	Fort ...	8-46	9-45
2.	Park ...	8—52	9—40
3.	Egmore *A.* ...	8-57	*D.* 9—35
	D. ...	9—0	*A.* 9—32
5.	Chetput ...	9—8	9—26
6.	Kodambakum.	9—20	9—19
7.	Mambalam ...	9—27	9—14
8.	Saidapet ...	9—32	9—9

Read downwards. Read upwards.

Assuming that the speed of the trains was uniform when in motion, find graphically when and where they passed each other and read from the graph the distance between them at 9-25.

29. No. 1 Ceylon Boat Mail is timed as follows:—

Distances from Madras Beach in Miles.			H. M.
$2\frac{3}{4}$	Egmore ...		20—15
$10\frac{3}{4}$	St. Thomas Mount.	*A.*	20—39
		D.	20-41
25	Guduvancheri ...		21—22
$37\frac{1}{2}$	Chingleput ...	*A.*	21-56
		D.	22-4
$42\frac{1}{2}$	Ottivakkam ...		22—16

No. 236 Shuttle Passenger is timed as follows :—

Distances from Dhanushkodi

in Miles.			H. M.
444½	Pallavaram	...	20— 5
446¼	Minambakkam	... A.	20—12
		... D.	20—14
448	St. Thomas Mount.	A.	20—21
		D.	20—24
449½	Guindy	... A.	20—31
		... D.	20—40
450¾	Saidapet	...	20—46

Draw graphs showing the distance of each train from Madras Beach at any part of the journey.

Find therefrom how far apart the trains are at 20—26 and where the Passenger waited for the Boat Mail to pass.

30. A bath could be filled by a pipe A in 2 hrs. and by another pipe B in 5 hrs. If both of them are opened simultaneously into the empty bath, find graphically in what time it could be filled.

31. Find graphically the ratio in which oil costing 2½ as. an ollock may be mixed with oil costing 4 as. an ollock so that the resulting mixture may cost 3½ as an ollock.

32. Find graphically at what time between 3 and 4 o'clock the minute and the hour hands of a clock are (i) coincident, (ii) ⊥ to each other, (iii) 10 minute-spaces apart and (iv) 30 minute-spaces apart.

[NOTE :— During the time the moving end of the minute-hand traverses sixty minute-spaces from the point marked XII to the same point again, the moving end of the hour-hand traverses 5 minute-spaces from the point marked III to that marked IV.]

(33) Work graphically the exercises (42), (45), (46), (49), (52), (53), (57) and (67) in Chapter XIV.

CHAPTER XXI.

MISCELLANEOUS EXAMPLES.

(1) The twelfth of December is the Indian Durbar Day. In 1915 the Durbar Day was a Sunday. What day of the week will it be this year?

(2) Show that the following statement is incorrect:—

'*The G.C.F of two numbers is contained 772 times in one of them and 954 times in the other.*

(3) The areas of two adjacent walls of a rectangular lecture hall are respectively 527 sq. ft. and 476 sq. ft. If the walls are of equal height, find the dimensions of the hall.

(4) When 150, 220 and 325 are each divided by 'x' the remainders are equal. Show that 'x' must be a common factor of (325—150), (220—150) and (325—220). Hence find its maximum value.

NOTE.—If $z = am + b$, $y = an + b$ and $z = al + b$ and if 'm,' 'n,' and 'l' are prime to one another, 'a' is the G. C. F. of $(x-y)$, $(y-z)$ and $(z-x)$.

(5) Show that if $P = m \times x$, $Q = n \times x$, $R = l \times x$ and if l, m and n are prime to one another, the L.C.M. of P and $Q = \dfrac{P \times Q}{\text{their H.C.F., } x}$ and that of P, Q and R $= \dfrac{P \times Q \times R}{x^2}$. Verify the above by means of numerical examples.

(6) Find the difference between $3\frac{1}{7}$ and $\frac{355}{113}$. The area of a circle is known to be $\pi \times$ the square of its radius. I calculate the area taking $\pi = 3\frac{1}{7}$. If you calculate it taking $\pi = \frac{355}{113}$, by how much will our answers differ if the radius is $2\frac{4}{5}$"?

(7) Given that $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{2}{f}$, find '$f$' when $v = \frac{1}{10}$, $u = \frac{1}{10}$

(8) The fraction $\dfrac{p}{q}$ of a certain ore is lost in roasting, the fraction $\dfrac{l}{m}$ of the remainder is lost in smelting. If the

rest be pure metal, find what quantity of pure metal will be yielded by 't' tons of ore and what quantity of ore will be required to produce 't' tons of pure metal ?

(9) A square tank 270 ft. wide has a square mantapam 30 ft. long in the centre. If the remaining portion of the tank be covered with water, find the area of the water surface.

(10) Given that $(29)^{\frac{1}{2}} = 5{\cdot}385$ and $8^{\frac{1}{2}} = 2{\cdot}828$, find the value of $(232)^{\frac{1}{2}}$ and $(3{\cdot}625)^{\frac{1}{2}}$.

(11) Assuming that $a^m \times a^n = a^{m+n}$ and that $a^m \div a^n = a^{m-n}$ for all values of 'm' and 'n,' find the values of $10^{1{\cdot}9031}$ and $10^{{\cdot}5051}$, given that $10^{1{\cdot}2041} = 16$ and $10^{{\cdot}6990} = 5$.

(12) A ship starts from —15° Longitude, sails + 20° in Longitude, then —26°, then + 40° and finally — 20°. In what Longitude is the ship after the sailings ?

(13) A rectangular block of iron is 5·55″ long, 4·54″ wide and 2·5″ thick. Calculate the weight of the block, given that 1 cubic inch of iron weighs 0·26 lb.

(14) Supply the figures marked '×' in the following multiplication :—

$$
\begin{array}{cccccc}
 & & 7 & 4 & \times & \\
 & & \times & \times & 2 & \\
\hline
 & \times & \times & \times & & \\
 & \times & \times & \times & \times & \\
 & & \times & \times & 9 & \times \\
\hline
\times & \times & \times & \times & * & 8
\end{array}
$$

(15) A fruiterer brought a basket of mangoes I bought half of what he had and he gave me an additional mango free. My brother went next and bought half of what the fruiterer then had. He too got an additional mango free. There were only 8 mangoes still left in the basket. How many mangoes were there in the basket at first.

(16) The jurisdiction of a village head-man is limited to a circle of radius 3 miles. The occupied houses form a circle with the same centre and with radius 4 furlongs. Surrounding them is a circular ring of green fields to a distance of 1 mile. The fields are fed by a circular lake, diameter 1 mile, whose edge is half a mile from the nearest

occupied house. The rest are waste lands. Represent the above in a diagram drawn to scale.

(17) A bottle when full of water weighs 680 grains and when filled with another liquid, 580 grains. When the bottle is half filled with the liquid, its weight = 380 grs. Find the weight of the bottle when empty. Also, what is the ratio between equal quantities of the liquid and of water?

(18) A contractor undertook to build a school in 3 months. He employed 41 men, 20 women and 5 boys. At the end of 1 month he found only $\frac{1}{4}$ of the work completed. How many more men should he employ so that he may keep up to his agreement? [Four men work as much in a day as 5 women, and 4 boys work as much in a day as 3 women.]

(19) If $\frac{a}{b} = 1{\cdot}414$, find the values of $\frac{a+b}{b}$, $\frac{a-b}{b}$, $\frac{a}{a+b}$, $\frac{a}{a-b}$, $\frac{b}{a}$, and $\frac{a-b}{a+b}$.

(20) The weight of a Roman coin is 60 grains and it contains $\frac{2}{3}\frac{9}{0}$ths of pure silver. Find the value of 1000 of these coins when the value of 480 grains of standard silver is 5*s*. 5*d*. (the value of the base metal being neglected).

(21) Fill up the missing columns below:—

A bankrupt's assets.	His liabilities.	The dividend he is able to pay on his debts.
(*a*) Rs. 150,000	Rs. 240,000	( )
(*b*) Rs. 8,000	( )	5 as. in the Rupee.
(*c*) ( )	Rs. 2,500	14 as. in the Re.
(*d*) £2,280	£6,080	( )
(*e*) ( )	£1700	14*s*. 8*d*. in the £

(22) Taking $\pi = 3\frac{1}{7}$, show that $\frac{1}{\pi} = \frac{1}{4} + \frac{1}{1\,6} + \frac{1}{1\,1}$ of $\frac{1}{1\,6}$. Use this to divide 576·576 by π.

(23) Show that $\frac{1}{1\,7\,6\,0} = \frac{1}{2\,0\,0\,0}$ of $(1 + \frac{1}{8} + \frac{1}{1\,1}$ of $\frac{1}{8})$ and use the equality to convert 528528·88 yards into miles.

(24) Show that $37\% = \frac{1}{8} + \frac{1}{80} + \frac{1}{800}$ and use this to find the values of 37% of Rs. 72,900 and of Rs. 720,927.

(25) Show that $\frac{1}{2240} = \frac{1}{8000}(1 - \frac{1}{10} - \frac{1}{140})$ and use this to convert 876545 lbs. into tons.

(26) Find the ratio between the areas of (1) two squares whose lengths are 10″ and 1′ and (2) of two rectangles whose lengths are as 3 : 2 and widths are as 2 : 5.

(27) The height of a pole is 18 ft. above the ground. A monkey gets up the pole. For every three feet it ascends in a second, it slips down two feet in the next second. Find in what time it could reach the top of the pole. Draw a graph representing its movement.

(28) The sides of a rt. ∠d △ are $(3x + 4)$ cm. $(4x + 10\frac{2}{3})$ cm. and $(5x + 11\frac{1}{3})$ cm. respectively. If 'x' be greater than 'o', find its value and hence calculate the number of cm. in the length of the hypotenuse.

(29) Four straight lines intersect at a point. The successive ∠s formed around it are $(4x + 5)°$, $(5x + 6)°$, $(7x - 8)°$, and $(2x - 3)°$. Find the value of 'x' and draw four such ∠s around a point A.

(30) A c. cm. of silver weighs 10·5 grams and a c. cm. of brass weighs 8·4 grams. A waist belt is made of an inner brass plate covered by an outer silver plate and weighs 160 grams. If the ratio of silver to brass in the belt = 5 : 3 by volume, find the weight of each ingredient in the belt.

(31) (*a*) Two roads running from north to south are cut by two other roads running north-west and south-east respectively. Bounded by these roads lies a flower-garden ABCD. I start from the north-east corner A, walk 280 yds. due south, reach the south-east corner B, turn north-west, walk 125 yards and reach the south-west corner C. Draw a diagram to scale showing the position of the garden amidst the roads. (Represent 100 yds. by 1 inch). Mark in it the position of a bower midway between the north-east and south-west corners. Find the distance of the bower from the other corners and from the north-to-south roads.

(*b*) At first a gentleman bought the land amid the roads for Rs. 4,375. Find to the nearest rupee, how much he paid per acre.

(*c*) Soon after buying, he dug 5 wells each costing Rs. 425, levelled the ground at a cost of Rs 190, put wire-fencing round the land at a cost of Re. 1 per yd. and for want of additional funds sold the whole plot for Rs. 8,000. How much did he gain or lose per cent. ? [Answer to the nearest tenth.]

(*d*) The new buyer at once took in two partners who contributed Rs. 3,000 and Rs. 1,000 for a proportionate share of the land. The three partners next contributed equally towards a working capital of Rs. 9,000, the interest on which met the annual expenditure on gardening and left a further balance of Rs. 340. The flower-show at the end of the year fetched a sum of Rs. 1,700. If the annual profits were divided among the partners in proportion to the capitals they had invested, find how much each received.

(*e*) Some time after, they leased the garden to a Horticultural Society for a period of four years on the understanding that at the beginning of each year the Society should pay them Rs. 1,500. Payment was made regularly for the first two years and then postponed. Assuming money to fetch 5% compound interest, find how much the partners could demand from the Society at the end of the period of lease.

(32) The following method is adopted by a person to find out a number selected by you. He requests you to perform in succession the following operations :—

(1) Multiply the no. by 5; (2) add 2 to the product; (3) multiply the sum by 4; (4) add 12 to the product, and (5) multiply the sum by 5. After you give the final result of these operations he divides it by 100, deducts 1 from the quotient and reads off your number. Explain with reference to any number 'x' how the above process is accurate.

(33) A room is 'l' feet long, 'w' feet wide and 'h' feet high. A spider in one corner of the ceiling wants to *creep* to a fly in the opposite corner on the floor. Discuss the shortest path it can take when $l = 18$, $w = 12$ and $h = 10$. If the spider spins a straight thread from the corner on the ceiling to the opposite corner on the floor, find the length of the thread.

(34) Bishop Hatto had stored a large quantity of corn in his granary. If 1,000,000 rats went there and each thousand

daily spoiled '015 per cent. of what was left unspoiled after the previous day, what percentage of the total quantity of corn would be left unspoiled at the end of a week? (Assume that the rats commit no havoc on the Sabbath Day which happened to be the seventh day after their coming.)

(35) Construct a rt.-$\angle$d $\triangle$ ABC with the hypotenuse BC 2" long and AB greater than AC. On BC and on the same side of it as A describe a square BCDE. Join AE, AD. Draw DL $\perp$ to AC and EF $\perp$ to AB. Show that EF = AD and DL = AC and the square BCDE = $\triangle$ ABC + $\triangle$ AED + $\triangle$ AEB + $\triangle$ ADC = $\frac{1}{2}$ ($BC^2 + AB^2 + AC^2$).

Hence deduce that *the square on the hypotenuse of a rt.-$\angle$d $\triangle$ = the sum of the squares on the other two sides.*

(36) Draw any $\triangle$ ABC. Bisect BC at D and join AD. Compare the areas of the $\triangle$s ABD, ACD.

[The line AD joining a vertex A to the middle point of the opposite side BC divides the $\triangle$ into two equal parts and is called a **median** of the triangle.]

(37) Construct $\triangle$s of different sizes and shapes. Draw the three medians in each $\triangle$. In what ratio is each median divided at the point of its intersection with another median?

(38) Show that the area of a $\triangle$ ABC could be found graphically as follows:—Along BC set off a segment BP=2". Join AP and draw CE $\parallel$ to PA to meet BA at E. Draw EF $\perp$ to BC. Then the no. of inches in EF will be the no. of sq. inches in the area of the $\triangle$ ABC.

(39) Explain how you would use the method suggested in Fig. 65, page 106 for (1) *bisecting a st. line;* (2) *for trisecting it, and* (3) *for dividing it into any number of equal parts.*

(40) In Fig. 65, page 106, show that the $\triangle$s having $\angle$A as the vertical angle and the $\parallel$s of the type SV, PR, QB, etc. as bases are equiangular. Compare the values of the ratios between the sides forming equal $\angle$s in the above series of equiangular $\triangle$s.

[The $\triangle$s are said to be *similar* and *the ratios between corresponding sides are equal.*]

(41) A triangular park is enclosed by three st. fences 1000 yds., 1500 yds., and 1200 yds. long. Draw a plan of the park to a scale of 1 inch to 100 yds. [*The* $\triangle$ *in the plan is similar to the* $\triangle$ *forming the park.*]

(42) The sides of a $\triangle$ are 5", 6" and 7·8". The corresponding sides of a similar $\triangle$ are m", n" and 5·2". Calculate the values of 'm' and 'n'. Construct the second $\triangle$ after drawing the first. Compare the areas of the two $\triangle$s.

(43) If in Fig. 65, page 106, AR, AB, RP, BQ represent $x+5$, $x+9$, $x+3$ and $x+6$ respectively, calculate the value of x.

(44) Show that if an isos. $\triangle$ ABC has its equal sides each x" long and the third side y" long, its area $= \frac{1}{4}y\sqrt{4x^2-y^2}$ sq. inches.

(45) If each side of an equilateral $\triangle = a$", show that its area $= \frac{a^2\sqrt{3}}{4}$ sq. inches.

(46) If 'a', 'b', 'c' respectively represent the lengths in inches of the sides opposite to the $\angle$s A, B, C in a $\triangle$ ABC and if 'p' be the length in inches of the $\perp$ AP dropped from A on BC, show that $BP = \frac{c^2+a^2-b^2}{2a}$ and

$p = \frac{1}{2a}\sqrt{(a+b+c)(a+b-c)(b+c-a)(a+c-b)}$. If 'A' be the area in sq. inches of the $\triangle$ ABC and $s = \frac{a+b+c}{2}$, show that $A = \sqrt{s(s-a)(s-b)(s-c)}$. Use the above formula to calculate the area of a $\triangle$ ABC when

(i) $a = b = c = 2$".

(ii) $a = b = 2$", $c = 1$".

(iii) $a = 2$", $b = 3$", $c = 4$".

(47) Find the area of an isos. $\triangle$ having (1) $a = b = 2$" and $c = 3$" and (2) of an equilateral $\triangle$ with $a = b = c = 4$".

(48) Find the area of a $\triangle$ whose three sides are (1) each 3" and (2) 10 cm., 12 cm. and 6 cm.

(49) Calculate the area of a quadrilateral ABCD with AB = 8", BC = 24", DC = 26", DA = 6" and DB = 10".

(50) Two boys A and B had some biscuits each. A said to B: "If you give me 5, we would have an equal no. of biscuits each." B replied: "Give me 5, so that I may have thrice as many as you." How many biscuits had each?

(51) Two asses were carrying firewood to a market. 'Lazy fool' said the one to the other, 'If you give me 1 stone I should carry twice as much as you; if I give you 1 stone we would have equal burdens.' What weight of firewood did each carry? [1 stone = 14 lbs.]

(52) From the castor-crop report of 1918-19 we learn that the area under cultivation was 387,500 acres as compared with 531,700 acres during the previous year. Owing to the failure of the south-west monsoon it was expeeted that the yield per acre would only be 91% of what it was in the previous year. Owing to the removal of restrictions in buying and selling it was estimated that the price would go up from Rs. 12-8 as. per bag during the previous year to Rs. 40. By how much per % would the value of the castor-crop have increased during the year under reference?

(53) It is found that 20 men and 10 boys could remove the goods from 10 waggons to the store-shed in 50 minutes. Twenty waggons full of goods have to be so cleared in 1 hour and 20 minutes. How many more men should be put in if 2 men work as much as 5 boys?

(54) A contractor employs a certain no. of coolies on a piece of work and finishes $\frac{5}{8}$ths of it in 20 days. 50 more coolies are then employed and the remaining work is thus finished in 8 days. How many coolies were employed at first? Find the cost of the whole work at 6 as. per cooly per day.

(55) A merchant invested Rs. 50,000 in purchasing and storing castor seeds at Rs. 12-8 as per bag of 176 lbs. He sold them all in three different markets at Rs. 16 8 as. per bag in one market, at Rs. 17-8 as. per bag in another and at Rs. 420 per ton in a third market. If the quantities sold in the three markets were as 3 : 4 : 9 and if the expenditure in the transaction amounted to $33\frac{1}{3}$ % of the sale receipts, find the amount gained by the merchant in the transaction.

(56) In a Form of 40 boys, the average age was 15 years. One boy aged 16 leaves the school and another joins. The average age of the class is then 15·1. Find the age of the new boy.

(57) A person bought 256 parahs of unslaked lime at P at the rate of 8 as. per parah. On slaking it, every parah gave 3 cubic feet of lime but of this $\frac{1}{16}$ was unserviceable. The remainder had to be carried to a place 10 miles distant where it was required and the cost of carriage was at 3 as. per 80 c. ft. per mile. At what rate per c. ft. must he sell it there in order to gain Rs. 35-2 as. on his outlay?

(58) If $(-2, +8)$ and $(+6, -8)$ are points in the graph of $\frac{x}{a} + \frac{y}{b} = 1$, obtain the values of '$a$' and '$b$'. Check your result by drawing the graph.

(59) ABC is a triangular piece of land with a well (W) in the border BC. Show how you would draw a line of demarcation so as to divide the land into two equal parts with the well common to both of them.

(60) A boy measures the length and breadth of a rectangular room with what he supposes to be a yard measure and from his measurements calculates the area of the room to be 180 sq. yds. But "*his yard measure*" is found to be half an inch short of a true yard. By how many sq. inches is he wrong in his calculation of the area of the room?

(61) A cyclist leaves P for Q at 2 P.M. and travels at 10 miles an hour. A motorist leaves P an hour and a half later and travels along the same road at 20 miles an hour. Find graphically when and where the motorist would overtake the cyclist. Verify your results otherwise.

(62) A train 88 yards long runs at 12 miles an hour. Another train 110 yards long runs at 28 miles per hour in the opposite direction. If the trains move on pairs of parallel rails, in how many minutes will they pass each other? Answer the above question when both the trains move in the same direction.

(63) A plot of ground is triangular in shape and contains 2 acres. One side is $16\frac{1}{2}$ chains. How long is a fence which is constructed from the opposite corner $\perp$ to this side?

If this fence divides the sides in the ratio 1 : 2, find the areas of the two parts into which the plot is divided by the fence.

(64) A could do a piece of work in 5 days and B in 10 days. In what time will they be able to do it together?

(65) A reservoir is filled by two pipes in 30 minutes and 40 minutes respectively and emptied by another in an hour. If all the three pipes be opened together, what portion of the cistern will be filled in 6 minutes?

(66) A rectangle ABCD has AB = 6 cm. and BC = 4 cm. If two points E, L be marked in AD such that AE = 1 cm. and LD = 2 cm. and EB, LC be joined, compare the areas of the three parts into which the rectangle is divided by EB, LC.

(67) If in the rect. in the last question a point P be taken in AB produced such that BP = 4 cm. and if a transversal PQR cuts BC at Q and AD at R, show, by measurement or otherwise, that $\frac{BQ}{AR} = \frac{PB}{PA}$ If the area of the quadrilateral ABQR is $\frac{7}{16}$ of the rect. ABCD, calculate the lengths of BQ and AR.

(68) A contractor undertook to clear away avenue trees which fell in a storm. He was to clear them in 10 days or pay a penalty fee of Rs. 8 extra for each day delayed out of the sum of Rs. 120 due to him for the work. He was paid only Rs. 88 at the end. For how many days did he pay the penalty?

(69) In a rectangular garden 'l' ft. by 'b' ft. there is a pathway 'w' ft. wide running all round inside. A pathway of the same width runs from the middle of each side to that of the opposite side. The garden is thus divided into 4 equal beds. Find the total area of the pathway and of each bed.

(70) A triangle LMN lies within the angular space of a rt. $\angle$ XOY. If the vertices L, M and N be respectively 2 cm, 5 cm. and 3 cm. from OY and 3 cm., 8 cm., and 5 cm. from OX, fix the positions of L, M and N in a diagram and calculate the area of the triangle LMN.

(71) An embankment is 20' wide at the top and 35' wide at the bottom. If a breadthwise vertical cross-section of it is 387·5 sq. ft. in area, find its height.

(72) If 'M' be the middle diameter, 'D' the diameter at the end and 'L' the length of the cask (all expressed in feet). the capacity in Imperial gallons of a moderately curved cask $= \cdot 0009442\ L \{ (2M^2 + D^2) - \frac{2}{3}(M - D)^2 \}$.

Hence calculate the capacity of a cask when M = 3·5, D = 2·5 and L = 4.

(73) Given that 1 c. ft. of water contains $6\frac{1}{4}$ gallons and 1″ = 2·54 cm., express correct to 4 decimal places a litre as the decimal of a gallon.

(74) (*a*). The number and value of suits instituted in the Civil Courts of British India during 1915 were 22,26,000 and Rs. 47,33,17,300 as against 18,04,000 and Rs. 36,74,53,000 respectively in 1914. Find the increase per cent. in the number and value of suits in 1915.

(*b*) Bengal and Madras have $\frac{3}{10}$ and $\frac{1}{3}$ of the total population of British India and are respectively responsible for $\frac{1}{3}$ and $\frac{2}{5}$ of the total number of suits The United Provinces having $\frac{1}{8}$ of the total population had 2,13,000 suits. Assuming the total population to be 342 millions, find the rates of suits per thousand of the population in each Province in 1915 and hence decide which of the three Provinces is the most litigous one.

(75) Whenever a trader buys paddy he uses a marcal which is $\frac{1}{16}$ too much of a true marcal and whenever he sells he measures with a marcal which is $\frac{1}{16}$ too short. Find the gain per cent. he actually makes if as aforesaid he buys paddy and sells at a gain of 4% on what he actually paid for each true marcal of rice that he bought.

(76) If a stick having weights '*w*' lbs. and '*m*' lbs. attached to its ends A, B respectively should rest horizontally over a fulcrum F, the point C in it resting on the fulcrum is obtained from the law AC × '*w*' = BC × '*m*.' Using a device like the above a merchant was weighing and selling articles. I placed some sugar in the scale-pan at A and the weights placed in the scale-pan at B to equipoise showed $\frac{5}{8}$ viss. Removing the weights, I next placed the sugar in the scale-pan at B and the weights placed in the scale-pan at A to equipoise showed $\frac{3}{4}$ viss. Find the ratio of AC to BC.

(77) The weight of a rupee = 1 tola. Find how many rupees could be coined from 5000 silver bars, each 27″ × $2\frac{1}{4}$″ × $\frac{1}{2}$″ given that 1 c. ft. of silver weighs 653·8 lbs. and 1 lb. = 38·889 tolas.

(78) Given that a gallon of water contains 277·274 c. ins., a c. in. of water weighs 252·458 grains and 1 lb.=7000 grains, find which of the following is the more accurate statement:—

"*A gallon of water weighs 10 lbs.*
A c. yd. of water weighs 15 cwt."

(79) Given that 1 litre of water weighs 2·205 lbs. and one metre = 39·3701 inches, find to the nearest tenth the weight in oz. of 1 c. ft. of water.

(80) Describe the major segment of a circle with radius 2 cm. on a chord BC 3 cm. long. Mark points A, D, E, F on the arc of the segment. Measure and compare the ∠s BAC, BDC, BEC, BFC, etc. Also, mark "O" the centre of the circle and compare the ∠BOC with each of the above ∠s in the segment.

N.B.—*Angles in the same segment of a circle are equal. Also, an angle subtended by a chord at the centre of a circle is double the ∠ subtended by it at any point in the Oce.*]

(81) If the chord of a segment should subtend an ∠ of 60° at the Oce, calculate the size of the ∠ it should subtend at the centre of the circle.

(82) Mark several points where a st. line AB subtends an angle of 60°. Verify if they lie on the arc of a circle of which AB is a chord.

(83) Construct a △ ABC in which BC=2 cm., the ⊥ AD on BC = 3 cm. and the angle BAC 30°.

(84) Construct an isos. △ ABC with its vertical ∠=45°, and base = 1·5 cm.

(85) P is 20 miles nearer T than Q. Tiles are sold at Rs. 'r' per 100 at P and at Rs. 'l' per 1000 at Q The cost of carrying 1000 tiles is 8 annas per mile. A person at T finds it as cheap to get the tiles from Q as from P. Express the relation between 'l' and 'r' in the form of an equation.

(86) In what ratio should ghee costing Rs. 3 per measure be mixed with oil costing 14 as. per measure so that the mixture may be sold at Rs. 2 7 as. 10 ps. per measure without gain or loss?

(87) In what ratio should two kinds of coffee seed costing Rs. 2·4 as. a viss and Re. 1-14 as. a viss be mixed in order to gain 25% by selling the mixture at Rs. 2·9 as. a viss?

(88) Find the ratio (1) between 'x' and 'y' if $2x - 3y = 6y - 9x$, (2) between 'x,' 'y' and 'z' if $20x - 12y = z$ and $2y - 2z + x = 0$.

(89) If $v = u + ft$. and $s = ut + \frac{1}{2} ft^2$, find the value of '$s$' when $v = 30, f = 10$ and $u = 5$.

(90) A railroad crosses a small channel at an angle of 30°. A railway station is 200 yards from the crossing and the station-master's quarters are built by the side of the channel and at a distance of 100 yds. from the station. Find from a diagram how far the crossing is from the station-master's quarters.

(91) The formula for discharge through a small hole in a thin plate is given by $Q = 5A\sqrt{h}$, where Q = the quantity of discharge in c. ft. per second, A = the area of the hole in sq. ft. and 'h' = the height of the water-level above the hole in feet. A leaky pot of butter-milk has a circular orifice at the bottom, ·05″ in diameter. Find how much of butter-milk would be spilt in the course of an hour, assuming that there was kept all along the period butter-milk to a height of 1′ —9″ in the pot above the orifice.

(92) Draw a circle with radius 2 cm. Mark 4 points A, B, C, D on its Oce and join AB, BC, CD, DA. Find the sum of the ∠s ABC, ADC, as well as that of the ∠s BAD, BCD and compare. Repeat the above exercise with many quadrilaterals inscribed in circles. What result do you get? Examine if the same result could be obtained in the case of quadrilaterals whose vertices do not all lie on the Oce of a circle. [A quadrilateral ABCD whose vertices lie on the circumference of a circle is called *a cyclic* quadrilateral and *its opposite angles are always together equal to two rt. ∠s.*]

(93) Inscribe in a circle of radius 1·8″ a quadrilateral ABCD. Through its vertices draw tangents to the circle so as to form another quadrilateral. Find the sum of pairs of opposite sides of this quadrilateral and compare.

(94) In a circle of radius 2″, place a chord AB 1·5″ in length. Through B, draw a tangent to the circle and measure the angles formed on either side of AB. Compare them with the angles subtended by the chord AB at any point in the major and minor arcs of the circle divided by AB.

Repeat this exercise with different chords and in different circles. What result do you get?

Hence deduce the following method of constructing a segment containing an angle = $\alpha°$ on a given base AB:—Set off ∠ ABC = $\alpha°$. Draw BH ⊥ to BC. Draw the ⊥ bisector of AB to meet BH at H. With H as centre and HB or HA

as radius describe a circle. The segment which is not on the same side of AB as the $\angle ABC$ is the segment required,

(95) Construct a $\triangle ABC$ with vertical $\angle ABC = 40°$, base $AC = 2{\cdot}4$ cm. and the altitude $BD = 3$ cm.

(96) P, Q, R are three light-houses along a convex stretch of coast PQR such that $PQ = 16$ miles, $QR = 18$ miles and $\angle PQR = 120°$. A ship S is off the coast and it is warned that there are rocks and shoals near the coast up to where PQ and QR subtend $\angle$s of 30°. Draw a diagram to scale showing the *danger area* through which the ship should not sail.

(97) Draw a circle of radius 5 cm. with centre C. Draw a central $\angle ACB$ of 35°. Show how you could then draw a central $\angle$ of 122·5° without using the protractor.

(98) The discharge of water per second passing over a tank *kalingula* is given by the formula $Q = 3{\cdot}1 \times l \times h\sqrt{h}$, where Q = the quantity in c ft per second, l = the length in feet of the *kalingula* and h = height of water in feet above the top of the *kalingula* wall. What length of *kalingula* should a tank have in order to carry off a rainfall of 1″ an hour over an area of 1 sq. mile, assuming that the tank is full, that 60% of the rainfall reaches the tank and that the height of water passing over the *kalingula* is 4 ft.?

(99) Two years ago my father was 14 times as old as my sister. Two years hence, he will be 26 years older than she. Find their present ages.

(100) An old man was asked his age by his grandson. He replied "Twenty years ago when you were born I was 20 years older than your father. Twenty years before you were born I was twice as old as he." Find the present ages of the father and the grand-father.

(101) An alloy made of copper and zinc in the ratio of 5:1 by volume weighs 525 lbs. per c. ft. Another alloy made of copper and zinc in the ratio of 1 : 4 by volume weighs 460 lbs. per c. ft. Calculate the weight of a c. ft. of copper and of zinc.

(102) A motor-bus left P for Q 25 miles away at its customary speed, stopped for half an hour on the way owing to an accident, travelled the remaining distance at half its usual rate and thus completed the journey in two hours. Next morning it left Q for P at its usual speed. Its tube

burst when it went to the place of accident on the previous day and was replaced in 15 minutes. Travelling the remaining distance at $\frac{4}{5}$ of its customary speed, it completed the journey in $1\frac{3}{4}$ hours. Find the customary speed of the motor-bus and the distance of the place of accident from P.

(103) One bottle contains a liquid, 25% of which is strong sulphuric acid, and the rest water. Another bottle contains a liquid, 40% of which is strong sulphuric acid, and the rest water. How much of liquid must be taken from each bottle in order to obtain a third mixture containing $3\frac{1}{2}$ oz. of strong sulphuric acid and $7\frac{1}{2}$ oz. of water?

(104) To find the breadth of a river without crossing it, show that the following method may be adopted:—"Observe a tree (T) on the opposite bank. Mark a point (A) just opposite T on this bank. Walk straight along the bank as far as B and continue walking as far as C until BC = TB. Then move away from the river along CD ∥ to TA until D, B, and T are found to lie in the same st. line. Measure CD. The length of CD gives the breadth of the water.

(105) A skew bridge 770 yds. long makes an angle of 70° with the direction of a river. Find from a diagram drawn to scale the breadth of the river.

(106) Wishing to know the height of a tree, I fix a pole $3\frac{1}{2}$ yds. long at a distance of 10 yards from the foot of the tree. Proceeding 2 yds. farther, I see the top of the pole in a line with that of the tree. If my eye is $1\frac{1}{2}$ yds. above the ground-level, find the height of the tree.

(107) Standing 18 ft. from a vertical pole I look at its top H. The angle between the horizontal line EP through (E) my eye directed in the same side as H and the straight line EH = 60°. If my eye (E) be 5 feet from the ground, find from a diagram the height of the pole. [The angle HEP above described *is called the angle of elevation* of the top H above me.]

(108) From T, the top of a lighthouse, its keeper looks at a ship (S). The ∠ between the horizontal TH, through T, directed in the same side as (S) and the st. line TS = 30°. If the ship be 600 ft. from the foot of the lighthouse, find the height of T above the ship.

[*The ∠ HTS is called the angle of depression* of the ship below the observer.]

(109) Construct a $\triangle$ ABC having AB = 2 cm, $\angle$ACB = 40° and AC + CB = 5·4 cm. Construct the above $\triangle$ when AC − CB = 1 cm.

(110) Construct a quadrilateral ABCD of any chosen dimensions. Bisect the sides at E, F, G, H. Show that EFGH is a $\parallel$ m whose area = half that of the quadrilateral ABCD. Deduce a method of cutting out any quadrilateral into pieces which may be fitted into a $\parallel$ m.

(111) In a ⊙ of radius 3·4 cm. mark a point E 1 cm. from the centre. Through E draw several chords of the circle. Find by measurement the area of the rectangle contained by the segments of each chord. Compare the areas so obtained. What result do you get?

(112) If the arc given in Fig 141, page 255 represents a part of a broken bangle, explain how a new bangle of the same size could be constructed. Find its diameter

(113) From the formula

$$T = \frac{v}{608 \cdot 3}\sqrt{\frac{w}{d}} - \cdot 14d.$$

calculate 'w' when T = 10·5, v = 1687, and d = 6. Also calculate 'v' when T = 6·9, w = 50 and d = 5.

Does your ignorance of the meaning of the formula hinder you from finding out the values?

(114) Construct a $\triangle$ ABC with the following measurements :—A = 30° ; a = 4·7 cm. ; b = 7·5 cm. Show that there are two solutions. Find, by measurement, the two values of 'C' and also of B.

(115) Given that 1 c. cm. of water weighs 1 gram and 1 c. cm. of sulphuric acid weighs 1·85 grams, find how many c. cm. of water should be added to 110 grams of the acid so that 1 c cm. of the diluted acid may weigh 1·4 grams on *the average*.

(116) An elastic rubber string hanging vertically from a fixed point is 10″ long in its unstretched position. When a weight of 'x' oz. is attached to its end, its length increases to y″ and the relation between 'x' and 'y' is given by the formula

$$(y - 10) = \tfrac{2}{5}x.$$

Draw a graph showing the relation between 'x' and 'y' for varying values. If the string would break if a weight of

2 lbs. be attached to it, find from the graph the length of the string at the breaking point.

(117) A yard length of iron pipe weighs 242 lbs. when the diameter of the bore is 4" and the thickness of the metal is 1·5". Find the weight of a c. ft. of iron. If 1 c. ft. of water weighs $62\frac{1}{2}$ lbs., find the ratio of the weight of a c. ft. of iron to that of 1 c. ft. of water.

(118) A cyclist can bike 'x' miles an hour in still weather, 'm' more miles per hour with a certain wind, 'n' miles less per hour against a certain wind. In what time could he go a distance of 20 miles with the wind and return against it?

(119) Thambu Chetty Street is 1500 yds long. A student starts from one end of the street to the opposite end, walks at 100 yds. per minute but stops for 10 minutes in a house half way up the street. Another student goes from the same end of the street and in the same direction but starts 10 minutes later and rides in a rickshaw moving at 150 yds. a minute. Find graphically when and where the latter would overtake the former.

(120) The gabled ends of a granary are 18 ft. wide, the height at the ridge of the roof being 15 ft. and that at the eaves 10 ft. If the granary is 40 ft. in length, find its capacity.

(121) The catalogue price of an article is 40% greater than the real cost of it to the firm. The customer is allowed 10% discount on the catalogue price. If the firm thereby gained Rs. 39 by selling a cycle, find the catalogue price of the cycle.

(122) A and B are two bathing ghats along one bank of a river, where its course is straight, F is the foot of a tree in the opposite bank. The $\angle$s ABF, BAF are found to be 60° and 45° respectively. If AB = 200 yds., find from a diagram the breadth of the river. If the $\angle$ of elevation of the top of the tree is found to be 30° at B, calculate the height of the tree.

(123) The population of Madras is 'p' and its area is 'm' sq. miles. 'Triplicane' has an average 'q' inhabitants per square mile and the remaining portion of Madras has 'r' inhabitants per sq. mile. Find the areas of Triplicane and of the remaining portion of Madras in terms of 'm,' 'p,' 'q' and 'r.'

(124) The total area of the surface of the earth = 193,100,041 sq. miles, of which the ocean surface covers 141,499,999 sq. miles. If the Pacific Ocean covers $\frac{36}{87}$ of the ocean surface of the earth, find correct to two significant figures what °/₀ of the total surface of the earth is covered by the Pacific Ocean.

(125) Given that the area of a || m. = 4·2 sq. inches, its altitude = 1·4″ and one side = 1·7″. Construct the || m. Measure its other sides and ∠ s.

(126) A man sells all his property for Rs. 5000 and invests the sale-proceeds in a bank which pays 6°/₀ interest per annum, interest payable yearly. If he draws Rs. 1200 at the end of each year for his expenses, find in what time he would be ruined.

(127) Two rectangular prisms have their lengths in the ratio 2 : 3, their widths in the ratio 3 : 4 and their heights in the ratio 3 : 2. Compare their cubical contents.

(128) In two different maps drawn with different scales, a railway line is represented by a length of 2″ in the one and by 5″ in the other. Assuming that the areas of similar figures are as the squares of corresponding sides, calculate the area of a Province in the first map when it is represented by 5 sq. inches in the second map.

(129) A milk seller pays $3\frac{1}{2}$ as. per measure for the milk he buys and mixes it with water in a ratio 5 : 1. He also defrauds by using, while selling, a false ollock which is $\frac{1}{16}$ short of a true ollock. If he sells the milk at the rate of half an anna per ollock, find his gain per cent. in the transaction.

(130) A merchant reduces the price of an article by 10°/₀. By what per cent. should the no of articles sold increase so that there may be a rise of 5°/₀ in the gross receipts.

(131) Daily I attend school from my village 3 miles distant by going in a bandy. Starting at 9-10 A.M., I usually go there at 9-50 A.M. One day I start only at 9-30 A.M. In order to be in the school percisely at 10 A.M., by what °/₀ must the usual speed of the bulls be increased ?

Another day I start so early as 8-45 A.M. By what °/₀ may the usual speed of the bulls be slackened so that I may reach the school precisely at 10 A.M.

(132) Construct a $\triangle$ whose $\angle$s are in the ratio 2 : 3 : 4 and one of whose sides is 3 cm. in length. Find the ratio between the sides of the $\triangle$.

(133) On January 1st of a certain year I lend Rs. 350 to A at 8 per cent. per annum simple interest and on March 1st of the same year I lend an equal sum to B at 12 °/₀ per annum simple interest. Draw graphs showing the amounts at simple interest on each of the loans. I get back the amounts on the day when they are equal. Read from the graphs when I should receive the loans?

(134) I lend Rs. 250 to A and Rs. 500 to B on the same day at 9 °/₀ per annum. Show graphically that the difference between the simple interest on them is always constant.

(135) A sum of money amounts at simple interest to Rs. 180 in 3 years and to Rs. 200 in 5 years. Draw a graph from which may be read of the amounts at the end of any no. of years. Also, read from the graph the sum lent, the rate per cent. of interest and the interest on the sum at the end of 2 years.

(136) I borrowed a sum of money from a money-lender. At the same time his son-in-law borrowed an equal sum from him. Three years hence I had to repay the money with 6 °/₀ comp. int., while his son-in-law repaid it with 6 °/₀ simp. interest. I found I was charged Rs. 31 more than he. How much was lent to each of us at first?

(137) A merchant sells firewood at a gain of 1 anna in the Re. on the cost price he shows in his accounts, while the price marked there is 12°/₀ more than the actual cost. In weighing he so arranges that $\frac{9}{10}$ of a ton is sold as 1 ton. Find his gain °/₀.

(138) Show that the area of a $\triangle$ = half the perimeter × the radius of the inscribed circle.

(139) Show that the area of a regular hexagon described on a side of $R'' = \frac{3R^2}{2} \times \sqrt{3}$ sq. ins. Hence calculate the areas of the regular hexagons whose sides are 10 cm. 2″, 3 ft. respectively. Deduce the length of the circum-radius of a regular hexagon containing 50 sq. cm.

(140) s'' = the length of a side of a square inscribed in a circle of radius r'', show that the area of the square =

$2r^2$ sq. ins. Hence calculate the length of the side of a square inscribed in a ⊙ of rad. 2′ and the radius of the circum-⊙ of a square of side 52 cm.

(141) The following approximate rule is used for finding the length 'S' of a circular arc:—$S = \frac{1}{3}(8l - L)$ where 'l' is the length of the chord of half the arc and 'L' is the length of the chord of the whole arc. Use the above rule to find the length of the arc of a minor segment of a circle whose base = 10″ and height = 3″.

(142) The area of a segment of a circle smaller than a semi-circle $= \frac{b^3}{2c} + \frac{2}{3}cb$ where 'b' is the height of the segment and 'c' is the length of its chord. Find the area of a minor segment of a circle whose height = 2 cm. and base = 5 cm. Also, find the length of the chord of a minor segment of a ⊙ when its area = 5 sq. cm. and its height = $\frac{1}{3}$ of its chord.

(143) If 'A,' 'a' be the areas of the ends of the frustum of a cone and 'b' its vertical height, its volume $= \frac{b}{3}(A + a + \sqrt{Aa})$. Hence find the capacity of a bucket whose circular ends are 1 ft. and 8″ in internal diameter and vertical height = 11″. Also calculate the weight of water it could hold when 1 c. ft. of water weighs 1000 oz.

(144) In building a hostel 30 men and 15 women are employed at Rs. 15 per day. Each man is paid 2 as. more per day than each woman. Find the daily wages of a man and of a woman respectively.

(145) An æroplane ascends from Fort St. George and goes to St. Thomas Mount 8 miles away. The flash of a gun fired from the Fort is noticed in the æroplane and the report is heard 40 seconds later when the æroplane is vertically above St. Thomas Mount. Calculate the velocity of sound in feet per second.

(146) The amount collected for celebrating a school-day is Rs. 100. In the previous year, the expenditure for the celebration was as follows:—(1) Refreshments Rs. 30-12 as. (2) Music Rs. 10-4 as. (3) Magic performances Rs. 15-8 as. (4) Flowers, fruits, &c., Rs. 20. (5) Lighting charges Rs 9-8 as. and (6) Miscellaneous Rs. 11-4 as. If this year's collection is to be spent upon each item of expenditure in

the same proportion, how much should be spent under each item?

(147) Construct a quadrilateral with one side = 5 cm. and with the diagonals 6 cm. and 8 cm. in length at rt. ∠s to each other. Show how the quadrilateral may be cut into pieces which may be fitted into a rectangle.

(148) Construct a rhombus whose area is equal to 5·1 sq. ins, and one diagonal = 3·4". Measure its ∠s and sides.

(149) Explain how the following construction would give lengths equal to ·01" to ·09". Draw an ∠BAC=80°. Make AB = ·1" and set off equal lengths AD, DE, EF, FG, GH, HK, KL, LM, MN, NO along AC. Join BO and draw ‖ s to AB through D, E, F, G, H, K, L, M, N, O, and terminated by BO. The parts of these ‖ s lying between AO and BO give the required lengths. [Refer to the *diagonal scale* in a rectangular protractor.]

(150) In Form VI of a High School 10 pupils have taken up Group C Mathematics. During the short term the percentage of marks scored by them is as follows:—2 pupils 90% each, 2 pupils 70% each and 4 pupils 60% each. The rest got an equal percentage of marks and the class average was 63 per cent. Find the percentage of marks scored by the last two boys.

(151) In a pentagon like ABPQT in fig. 142, pages 256, a point D is taken in AB produced such that BD = AB, DE is drawn ‖ to BP to meet AP produced in E. EG is drawn ‖ to PQ to meet AQ produced in G. GL is drawn ‖ to QT to meet AT produced in L. Compare the lengths of the sides of the pentagon ADEGL with those of the corresponding side of ABPQT. Also compare the areas between the two pentagons. [Note that △ABP : △ADE = △ADH : △ABC in fig 85, page 137 = 1 : 4.]

Hence deduce a method by which pentagons may be constructed whose sides will be half, one-third of, thrice and five times the corresponding sides of the pentagon ABPQT.

(152) Draw a pentagon of exactly the same size as ABPQT in fig. 143, page 256. What measurements will you take? Mark a point M within it and join it to the vertices of the pentagon. Mark a point N in MA so that MA : AN = 2 : 3. Through N, draw ND ‖ to AB to meet MB at D. Draw DF ‖ to BP to meet MP at F., FG ‖ to PQ to meet MQ at G, GK ‖ to QT to meet MT at K and join

KN. Show that the pentagon ABPQT is thus developed with its lines in the scale 3 : 2.

(153) Draw the *polygons* in figures 89, 90, 92 in page 139 with their lines in the scale $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{4}$, $\frac{6}{5}$.

(154) In an army consisting of the French, Britons and Belgians, the number of Frenchmen : that of the Belgians $= p : k$ and the number of Britons : that of the French $= q : t$. When 'x' more Britons joined the army, the total strength of the Britons was 'r' $\%$ of the whole army. What was the original strength of the army? Verify your result when $p = 50$, $k = 41$, $q = 2$, $t = 10$; $x = 606{,}800$ and $r = 85$.

(155) Show that the common solution of $ax + by + c = 0$ and $lx + my + p = 0$, is given by

$$\frac{x}{bp - cm} = \frac{y}{cl - ap} = \frac{1}{am - bl}.$$

Hence solve (i) $4x + 2y + 3 = 0$;
$5x + 6y + 4 = 0$.
(ii) $3x - 2y + 5 = 0$;
$4y - 3x - 20 = 0$.
and (iii) $5x - 7y - 5 = 0$;
$3x - 2y - 14 = 0$.

(156) A boy was asked to find the L. C. M. of 9225, 15170 and another number. He took down 925 instead of 9225 and found out the L. C. M. corect. Find the least value of the third number. Also assign 3 more values to it.

(157)

District.	No. of letters, parcels, &c., delivered by post-offices in a year.	Average No. of letters, &c. delivered per head of the population in the year.
Coimbatore ...	$31{\cdot}74 \times 10^6$	15
Trichinopoly ...	$26{\cdot}45 \times 10^6$	25
Tanjore ...	$58{\cdot}667 \times 10^6$	35

Find the average number of letters, &c., delivered per head of the population in all the three districts put together.

(158) The following table is said to give the average cost, receipts and expenditure per mile of railway in three different countries :—

	Cost.	Receipts.	Expenditure.
United Kingdom ...	£43,600	£3,680	£1,910
India ...	£9,500	£1,050	£525
United States ...	£12,500	£1,290	£900

Calculate the yearly rate °/₀ of net profit from each Railway and find which of them is the most profitable one.

(159) Construct a $\triangle ABC$ in which the base $BC = 2{\cdot}5$ cm., the median AD corresponding to $BC = 4$ cm. and $\angle ABC = 60°$. [The use of the protractor is not allowed.]

(160) The distance around a circular tank-bund is 2000 yards. A person starts from a place in the bund and bikes round and round along the tank-bund at 6 miles an hour. An overseer starts from the same place and walks once round the tank in the same direction at 3 miles an hour. Find graphically how often, when and where the former will overtake the latter.

(161) In a 200 yds. race, A gives B a start of 20 yds. and gives 'C' a start of 40 yds. If A beats B by 10 seconds and C by 5 seconds, find graphically the relative position of the runners when A reaches the winning post a minute after starting. Supposing the rates of running to be uniform, find from the graph the speed of each in yds. per minute.

(162) The outer edges of the parapet wall of a well form a regular hexagon of side 4 ft., while the inner edge is circular with a diameter 5 ft. If the well is 26 ft. deep, find the quantity of masonry work used for its walls.

(163) By joining the diagonals of a regular pentagon of side 5·2 cm., show that another regular pentagon is formed inside it. Compare the areas of the two pentagons.

(164) It is known that, if sound travels at 'v' feet per second when the temperature of the atmosphere is 't' Fahrenheit, 'v' and 't' are connected by the equation $v = m(t - 32) + c$ where 'm' and 'c' are constants. If the velocity of sound be 1126 ft. per second in 60° F. and 1114 ft. per second in 50° F., find from a graph or otherwise the values of the constants 'm' and 'c.'

(165) A metal nut for an engine has to be 2·5″ long whose cross-section is a regular hexagon of edge 3″ with an inner circular bore of diameter 3·5″. Find the volume of metal used for its construction, allowing $1\frac{1}{8}$ c. in. for the thread of the screw. Also, find its weight assuming that 1 c. in. of the metal weighs ·26 lb.

(166) A hexagonal table each side of which is 2·8 ft. has its central portion covered with a circular marble cloth of radius 2·1 ft. Find the area uncovered. If a circular marble cloth is big enough to be just within the table and touch its edges, what area could it cover? If it just covers the whole of the area, how much of it will lie over the edges of the table?

(167) With the same total perimeter 12 ins., draw a square, a *regular* pentagon, hexagon, octagon, and dodecagon. Show that as the number of sides increases the area increases.

[Hence *with a given length of string the greatest area will be surrounded by it when it is in the form of a* ⊙].

(168) A square floor is paved with two kinds of marble. The centre is a big square of white marble slabs costing Re. 1-8 as. per sq. ft. and the rest of the floor is paved with figured slabs at Re. 1-4 as. per sq. ft. If the cost of paving the centre of white marble be Rs. 121-8 as. and that of paving the figured border be Rs. 398-12 as., find the width of the latter.

(169) I had to go from S to M a distance of 10 miles to catch a train leaving M at 10-45 A.M. I left S at 8-30 A.M. and travelled in a jutka going at 7 miles an hour. Half an hour hence I found that I left my money bag at S and hence returned to S in the same jutka at the same rate. It took me 10 minutes to secure the purse and I at once rode to the station M on a cycle. If I biked at a uniform rate and reached the station 5 minutes before time, find from a graph my rate of cycling.

(170) Assuming that for all values of 'x' the value of $(1+x)P = 1 + Px$ (approximately), when 'x' is very small compared with '1,' find the approximate values of $(1{\cdot}000)^5$; $\sqrt{1{\cdot}0001}$; $\frac{1}{1{\cdot}0005}$; $(\cdot 9999)^{\frac{1}{2}}$ and $\sqrt[3]{\cdot 999}$.

(171) P, Q are two consecutive furlong stones in a straight road leading to a tower T. A person in the tower observes a bandy proceeding towards the tower and notes the $\angle$s of depression to be 30° and 35° when the bandy is at P and Q respectively. Find the vertical height of the tower from a diagram drawn to scale.

(172) If $x = \cdot 005$ and $y = \cdot 003$, calculate the error per cent. in assuming that $\frac{1+x}{1+y} = 1 + x - y$, and $(1+x)(1+y) = 1 + x + y$.

(173) Two globes are made representing the earth but with different diameters 10″ and 12″ respectively. Assuming that the surface areas of spheres are as the squares of their radii, calculate by what area a country would be represented on the smaller globe if it is represented by 12 sq. inches on the larger one.

(174) Show with reasons which of the following statements are obviously wrong:—

(*a*) $\sqrt{0{\cdot}3716} = \cdot 2104$.

(*b*) The area of an equil. $\triangle$ of side 5 ft. = 10·825 sq. ft.

(*c*) The area of a $\odot$ of diameter 6·6″ = 80·8 sq. inches.

(*d*) $425{\cdot}5 \times 400{\cdot}4 = 160{,}000$.

(*e*) The area in acres of a trapezium, the parallel sides of which are 62·87 chains and 38·94 chains with the distance between them 12·65 chains = ·1665 acres.

(*f*) The total cost in Indian money of 25 copies of Potter's Geometry at 3*s*. 6*d*. (net) per copy and of 17 copies of Eastwood's Arithmetic at 4*s*. 6*d*. a copy = Rs. 1110 at 9 as. per ordinary shilling and at 12 as. per shilling (net).

(175) A bankrupt's assets are really 25 °/₀ more than what he shows them to be. He owes A twice as much as he owes B and owes C twice as much as he owes A. If his nominal assets allowed him to pay a dividend of only 10 as. per rupee

of his debts and if after such payment he really had a balance of Rs. 5250, find how much he owed A, B and C respectively.

(176) B, C and D have respectively borrowed Rs. 2500, Rs. 5500, and Rs. 2000 from A. If B, C and D become bankrupts all on a sudden such that B pays A at the rate of 8 as. in the Re., C at the rate of 10 as. in the Re. and D, 12 as. in the Re., find how much per Re. A realises for the money he lent.

(177) At each stroke, an air-pump exhausts from a bell-jar 'α' °/₀ of the air which it contained at the beginning of the stroke. If at the end of 12 and 14 strokes the quantity of air in the bell-jar was reduced to $\frac{1}{14}$ and $\frac{1}{15}$ of the original quantity, find the rate at which the air is being exhausted.

(178) A party of pilgrims set out for a temple 4 miles distant and walked at the rate of 2 miles per hour. After going 1 mile, one of them has to return to the starting place to fetch a money bag. At what rate must he now walk so as to reach the temple along with the rest? Verify your result graphically.

(179) Before a certain war, biscuits cost 3 as. per lb. to manufacture and were sold in packets at 5 as. per packet. During the war the cost of manufacture rose by 25 °/₀ and the packets were sold at 6 as. per packet and each packet contained only 7 oz. instead of $\frac{1}{2}$ a lb. as before. Find the percentage of profit to the manufacturer both before and during the war.

(180) A certain train is 2 hrs. late when it performs a journey at an average speed of 18 miles per hour and is $2\frac{1}{2}$ hrs. late when it travels at 6 miles an hour. Find the length of the journey.

(181) Periapet is roughly in the form of a quadrilateral ABCD. From A to B it is 10 chains south-west, the $\angle ABC = 95°$, BC = 2 chains, AD = 6 chains and CD = 12 chs. Draw a plan of Periapet to any convenient scale and in it mark the position of the Bazaar Road which passes through the middle of CD and meets AB 1 chain from A. Find from the plan the length of the road and the area of Periapet.

(182) My milkman who generally gives unadulterated milk had to serve one morning an additional customer half a measure of milk. His cow as usual gave him only 3 measures that morning. So he added enough of water to suit the demand. That morning he charged his customers at the

same rate as on other days, *viz.* at $3\frac{1}{2}$ measures a rupee. Find his special gain that morning. If he did not charge for the water he added, how much should he demand per measure from his customers that morning?

(183) According to the valuation by an assistant examiner the highest mark scored in a subject was 35 and the lowest 10. The chief-examiner altered the marks so that the highest mark was 45 and the lowest 5. If he did this according to a linear law, draw a graph from which could be read the marks given to the same candidates by both the examiners. Read from the graph the marks awarded to a candidate who scored 32 before the alteration. Check your result otherwise.

(184) The population of a town was 24875 in 1901 and 35666 in 1911. Find the percentage of increase in the population during the period of 10 years. If the rate of change in the population was uniform, find the population in 1881.

(185) A's movable property : B's = 3 : 4. Each gets Rs. 20000 in addition. Then A's movable property : B's = 13 : 16. How much was A's movable property worth at first?

(186) The expression $P(x^2 + y^2 + z^2) + Q(xy + xz + yz)$ = 30 when $x = y = z = 1$, and = 35, when $x = 2$, $y = 1$, and $z = 0$. Find the numbers represented by P and Q.

(187) In an S. S. L. C. public examination 'x' candidates appeared for Elementary Mathematics in the Presidency. The Presidency average in the subject was 'n' per cent. 'm' of the candidates that appeared had taken up Algebra and Geometry or Practical Mathematics as their special subject and they scored on the average 'y' per cent. in Elementary Mathematics. Calculate the average marks scored by the candidates who did not specialise in Mathematics. If the latter average be 24 %, $n = 30$, $x = 6300$, and $y = 66$, find what per cent. of the total number of candidates had specialised in Mathematics.

(188) A boy's register-number in an examination is remembered as follows:—The middle digit is 'o,' the digit 'y' occurs on either side of 'o' and the digit 'x' occurs on the extreme right and left. The number formed by the two digits in the extreme left is $\frac{5}{6}$ times the number formed by the two extreme digits in the right. Find the register number of the boy, if $x = y + 1$.

(189) The sum required to pay a dividend of 10% on the capital of a company was Rs. 40,000 at the end of the 12th year of its existence. If the capital employed each year was 20 % greater than that during the previous year, calculate the capital employed by the company during the 8th and 10th year of its existence.

(190) A circular well has a ⊙ce of 41·8 ft. when measured outside the parapet wall. Its inner diameter is 10·5 ft. Find the area on which the wall rests.

(191) Compare the cost of fencing a circular plot of land, diameter 1 chain, with that of fencing another plot, diameter 80 ft.

If both the plots yield at the same rate per acre, compare the total produce of the first plot with that of the second.

(192) A rectangular hall 46 ft. long, 25 ft. wide and 24 ft. high is divided into four equal rooms by cross-wise walls, 1 ft. 6 ins. thick. Find the cubical contents of each hall.

If a new floor 1 ft. 6 ins. thick be put in between the old floor and the ceiling and thus the hall be divided into 8 equal rooms, find the cubical contents of each room.

Find a general expression for the above results taking l' to be the length, b' to be the breadth, h' to be the height and t' to be the thickness of the cross-wise walls as well as of the new floor.

(193) A circular sheet of water whose diameter is 'x' ft. is surrounded by a path 'y' ft. wide. If the area of the path be $\frac{1}{4}$ of that of the water surface, find the relation between 'x' and 'y.'

(194) The gross income of a certain person was Rs. 4 more in 1901 than in 1900, but as he paid income-tax at the rate of 4 pies in the Re. in 1900 and at 5 pies in the Re. in 1901 his net income in the later year was Rs. $6\frac{1}{2}$ less than that in the first year. Find his gross income each year.

(195) A cyclist starts on a 16-mile journey; but after an hour, his bicycle broke down and he had to walk the rest of his journey which he did in $1\frac{1}{2}$ hours after the accident. Had the accident happened 12 minutes earlier he would have reached his destination 40 minutes earlier. Find his rates of cycling and walking.

(196) The price of a box of mathematical instruments is 12 as. more at one firm than at another. By buying at the

former firm a tradesman finds that with Rs. 100 he could get only $\frac{5}{8}$ths of the number of boxes that he could at the latter for the same amount. Find the price of a box in each firm.

(197) A owes B Rs. 2,000 and owes C Rs. 3,000. His assets amount to Rs. 3,625, out of which he pays B at the rate of 14 as. in the Re. What rate of dividend could C be paid?

(198) I borrowed a sum of money at 'r' °/₀ simple interest and forthwith invested it in buying house sites in a town. The price of the sites rose each year by 'r' °/₀ of what it was in the year before. I found that if I should sell the sites and clear my debts at the end of the 2nd year I could gain Rs. 1,000; or if I should do so at the end of the third year, I could gain Rs. 3,100. Calculate the sum I borrowed and the rate of interest I paid.

(199) A railway train, after travelling for 3 hrs., meets with an accident which delays it one hour; after this it proceeds at $\frac{4}{5}$ of its former rate and arrives at the terminus $2\frac{1}{4}$ hours behind time. Had the accident happened 50 miles farther on, the train would have arrived half an hour earlier. Find the length of the line and the original speed of the train.

(200) The expenses of a hostel are partly constant and partly dependent on the number of boarders. The total expenses in a month when there were 54 boarders amounted to Rs. 599 and in another month when there were 48 boarders to Rs. 538. Find, graphically or otherwise, the expenses for a month when there were 50 boarders.

(201) A mirasdar's income was as follows in one year:—

	Rs.
From rice-fields	100,000
„ gardens	55,000
„ trade	40,000
„ other sources ...	5,000

Next year, for want of a timely rain, the income from gardens decreased by 85 °/₀ that from rice-fields by 40 °/₀, that from trade increased and the income from the other sources fell by 15 °/₀. If the total fall in income was only $26\frac{1}{4}$ °/₀, by what °/₀ did the income from trade increase?

(202) In a palace kitchen 'x' viss of genuine ghee was supplied for the preparation of sweets to the royal family. A cook came in, melted the ghee and took away 'y' °/₀ of the ghee, replacing it by cocoanut oil. Another cook

next came in, took away 'y' $\%$ of the mixture and replaced it by oil. If one more cook did likewise before the sweets were prepared, find the ratio of genuine ghee to oil in the adulterated ghee used in the preparation.

(203) A carpenter is given a fragment of a wheel which is in the form of a circular arc whose height $= 10''$ and span $= 2'\ 6''$. Explain how he may find the radius of the wheel he has to make.

(204) A kite is in the form of an isos. $\triangle$ with equal sides of length, $1{\cdot}2''$ inclined at $30°$ with a semi-circular end bulging out of its base. Draw a plan of the kite and find its area.

(205) An agent to a minor's estate writes false accounts by entering every item of income received by 10 $\%$ less and every item of expenditure by 10 $\%$ more. During a certain year the actual expenditure in the estate was 30 $\%$ of its real income. If the income according to the accounts was Rs. 18,000, find how much the agent has embezzled. Calculate the actual estate income and expenditure. What $\%$ of the actual income was embezzled by the agent and what $\%$ of it are the estate savings for the year.

(206) A farmer bought a cart-load of hay containing 80 bundles, distributed it into 100 bundles and sold each of them at 4 $\%$ more than what he paid for a bundle of the kind that he bought. Find his gain $\%$. If thereby he gained Rs. 6, find how much he paid for the cart-load of hay.

(207) A merchant states his profits as a percentage on his cost price and another states it a percentage on his sale price. A stove costs each of them Rs. 10. For how much should each of them sell it in order that he may say that he gained 25 $\%$ profit?

(208) Two thieves P and Q steal some jewels and hide them under-ground in a spot which is at a distance of 100 footsteps of P's from a mango tree and 250 footsteps of Q's from a cliff. Some time hence they come back to the spot, but see no trace of the tree. If they know that the cliff is N. N. E. of the tree at a distance of 150 foot-steps of P's from it, draw a plan showing how they may discover the treasure.

(209) A school-boy records that the ends of the dividers he was using were $7''$ apart when the $\angle$ between the arms was $135°$. Find the length of each arm. What would be the

distance between the ends when the arms are inclined at 45°, 90°, and 60° respectively?

(210) Mark four points A, B, C, D so as to lie on the Occ of a circle so that CD = 2 cm., D B = 4 cm. ∠ ACD = 45° and ∠CAD = 110°. Draw the quadrilateral ABCD and find its other sides, and ∠s.

(211) Wellington Cinema showed its exhibitions on the 20th December 1918. The rates of the tickets for admission were in the ratio 8 : 4 : 2 : 1 and the amounts collected were in the ratio 4 : 9 : 8 : 6. If the total collection be Rs. 1350, find an expression for the number that attended the exhibition. If the total number that took the last class ticket be 1200, find the no. that attended the show.

(212) When the export duty on an article was increased by 'x' %, the export decreased by 'y' %. Find the percentage of decrease in the revenue from the export duty on the article. Give the numerical result when $x = 10$ and $y = 20$.

(213) Messrs. P. Orr & Sons advertised to sell three kinds of rolled gold watch-chains. The Company allowed 10% discount for cash. The prices advertised were in the ratio 9 : 8 : 7. A merchant ordered for 120 chains consisting of the 1st, 2nd and 3rd kinds in the ratio 3 : 4 : 5. The first sort he sold at an advance of 10 %, and the third sort at an advance of 15% over the nominal value. The second sort he sold at the face-value. If his profits amounted to Rs. 173½, find the catalogue price of each sort.

(214) A rustic wanted to catch a train and walked at 3 miles per hour. An hour after he left his village he was informed that the station was 4 miles away and that he should catch the train within an hour and a half. He ran back to his village at 5½ miles an hour to take his purse and returned in a jutka. What should be the minimum speed of the jutka in order that he may catch the train? Verify your result graphically.

(215) If an alloy be made of 'x' grams of gold and of 'y' grams of silver so that the mixture weighs 15 grams per c. cm., compare the ratio by volume between the gold and the silver mixed, assuming that the specific gravity of gold = 19 and that of silver is 10·5.

(216) Standing 18 feet from our school building I find the angle of elevation of its top to be 60°. If my eye is 5 ft.

from the ground level, find the height of the building above ground level.

(217) The speeds with which two cyclists could ride against a certain wind are as 2 : 3 and their corresponding speeds against the wind are as 8 : 9. Find the ratio of their speeds in still weather, other circumstances not being taken into account.

(218) A boy cuts out from a sheet of card-board a 60° and a 45° set square with the longest edge of each = 6". Find how many sq. ins. of card-board there are in each of the set squares.

(219) A street hawker has two kinds of damaged silk which cost him Re. 1 and Rs. 2 per yard respectively. He sells four yards of the first kind for every three yards of the latter but the sale price per yard of the first kind : that of the second kind = 2 : 3. If he gains $27\frac{1}{2}$ % on the whole, find the sale price per yard of each kind of silk.

(220) A merchant buys mangoes wholesale at Rs 28 per thousand. He has also to pay Rs. 2 per thousand for railway and other charges. If 10% of the mangoes are damaged in transit and are hence unsaleable, find at what price he must sell the rest so as to gain $7\frac{1}{2}$% on his outlay.

(221) A number of hexagonal pencils 20 cm. long with each edge 5 cm. are cut out of a block of wood 60 cm. × 10 cm. × 5 cm. Find the number of pencils that can be made. Show by drawing a net work of the bases of the pencils that there is a very little wastage of material in making a large number of such pencils.

(222) I have a square plot of land in a Municipal street. The Municipality opens two lanes within it along two adjacent sides. The lanes are of a uniform width of 5 yds. and cover 675 sq. yds. of ground. Find the area of the plot left.

(223) A proposed railway runs right through a rectangular garden cutting the longer sides at an ∠ of 60° and taking up $2\frac{2}{3}$ acres or $\frac{2}{9}$ of its entire area. If the sides of the garden are in the ratio of 6 to 5, find the breadth of the proposed railway.

(224) The S. I. R. line has a circular curve along the slope of the Thenmalai Hills. A, B, C are three consecutive level crossings along the curve, the distance being 4 furlongs along the line from A to B, 2 furlongs from B to C. If a planter's

bungalow is equidistant from the three crossings, fix its position in a diagram drawn to scale.

(225) A paper-weight is in the form of a solid cone of base 6" in diameter and slant height 5". If a c. in. of the material weighs 0·3 lb., find its weight and surface area.

(226) The increase in the number of males in a district was 12% one year and the corresponding increase in the no. of females 20%. If there was a total increase of 15·5% in the population, find the ratio between the no. of males and females in the district at the beginning and at the end of the year.

(227) Find the values of the following, using short methods:—

(i) $\dfrac{(15\frac{3}{4}-4\frac{2}{7})}{(5\frac{1}{4}-1\frac{3}{7})} \times \dfrac{(5\frac{1}{4}+2\frac{1}{6})}{(2\frac{1}{6}-1\frac{1}{4})} \times \dfrac{(5\frac{1}{4}-2\frac{1}{6})}{(2\frac{1}{6}+1\frac{1}{4})}$.

(ii) The value of x when

$$\cdot00157 \times \cdot00732 = x \times \cdot0183.$$

(iii) $\dfrac{7{\cdot}75 \times (6{\cdot}34)^2 - 6{\cdot}25 \times 7{\cdot}75}{4{\cdot}65 \times 6{\cdot}14 + 4{\cdot}65 \times 3{\cdot}86}$.

(iv) $\dfrac{(1+\frac{1}{1+\frac{1}{3}})(1+\frac{1}{1+\frac{1}{3}})-(1-\frac{1}{1+\frac{1}{3}})(1-\frac{1}{1+\frac{1}{3}})}{(1+\frac{1}{1+\frac{1}{3}})+(1-\frac{1}{1+\frac{1}{3}})}$.

(228) If in a $\triangle$ ABC, D be the middle point of BC and AD $= m$, it is known that the lengths of the sides a, b and c are connected by the relation $2m^2 + \dfrac{a^2}{2} = b^2 + c^2$.

Using the above formula, construct a triangle in which $b = 3"$, $c = 4"$ and $m = 3{\cdot}5"$.

(229) A road rises 1 in 10 for the first 10 miles, 1 in 20 for the next 2 miles and 1 in 5 for the next 15 miles. If it is to be lowered in level so as to have a gradient of only 1 in 20 all through, find the quantity of earth contained in the cutting to be made. [Assume that '1' in '10' means a rise of 1 ft. vertical for 10 ft. measured along the road.]

(230) During the Third Mysore War a sepoy buried his treasure in four different places in his garden. He left a parchment stating that they lay in a circle in four straight

paths drawn from a well in the directions North, South, East and West, that his father's tombstone was South-West of one of the places and that two of the rest were equidistant from the well and the tombstone. If he was suddenly killed in battle, explain how his son may discover each treasure with the parchment left.

(231) The distance to which we could see from hill-tops is given by the formula $d = 0{\cdot}511\sqrt{h}$ where 'd' = the distance in miles and 'h' = the height in feet of the hill. Calculate the height of a hill if from its top we are able to see just as far as a tank 10 miles away.

If the height of a rock-fort is 700 ft., find the maximum distance that we could ordinarily see from its top.

(232) If the speed of water in a river is 'b' yards per second at the bottom and 's' yards per second at the surface, 'b' and 's' are connected by the formula $b = (s - \sqrt{s}) - (\sqrt{s} - 1)$. Find '$b$' when $s = 20$, 30 and $50\frac{1}{2}$ respectively.

(233) A train leaves (M) at 19-15 and reaches (S), 5 miles away at 19-33 without stopping at any of the intermediate stations. Another train leaves (S) towards (M) at 18-46, reaches (H) one mile away; at 18 49, reaches (K) one mile further away at 19-2, reaches (C) one mile further away at 19-10 and M, two miles away at 19-22. Find from a graph when and where the trains pass each other.

(234) Daily I attend school one mile away from my house by walking the distance in 20 minutes. At a certain spot I used to meet the postal runner running in the opposite direction at 4 miles an hour. One day I was late and I hence met the runner a furlong nearer my house. If I walked as usual, find graphically by how many minutes I would reach the school late on that day. [Choose the usual meeting spot and time as the co-ordinates of the origin.]

(235) Use the formula

$$P = \frac{900C}{D}\left(1 + \frac{25}{D^2}\right),$$

to find the value of C when P = 12,000 and D = 50.

(236) 1500 coolies are employed to construct a railway line within a year. When 5 months passed, half the work was finished but the monsoon commenced very severely and

for one month the work was stopped because of its severity. If the monsoon washed away $\frac{1}{3}$ of the lines constructed, calculate how many more coolies should be employed so that the line may be constructed before the close of the year. [Assume that other conditions are unaltered.]

(237) If a point P be marked at a distance of 'x' inches from the centre of a circle of radius 'r' inches, find an expression for the length of the tangent to the circle drawn from P.

If secants be drawn from P to cut the Oce at Q and R, find the areas of the rectangles contained by segments of the type PQ, PR when $x = 2{\cdot}3$ and $r = 1{\cdot}5$.

(238) Walking along a straight road I turn to the left at a place A along a straight foot-path and reach a fountain F 250 yards away. I then walk in a direction $\perp$ to AF and reach the original road 400 yds. from A. Find from a diagram how far the fountain is from the road.

(239) Ice is manufactured at 4 pies a pound and sold for 7 pies a pound in the factory and at 10 pies a pound in the branch shops. If 55 % of the quantity manufactured be kept in the factory and the rest sent to the branch shops and if the quantities lost in melting be 5 % in the factory and 10% in the branch shops, find the percentage of profit to the manufacturing firm.

(240) A farmer has a rectangular garden containing 2 acres in area. He digs in it a circular well of diameter 15′ and depth 20′ and the soil removed is spread uniformly over the garden. Find by how much the level of the garden would be raised. Assume that the soil increases in volume by $\frac{1}{12}$th when dug out.]

(241) A railway station (S) is due East of a village P and due West of another village R with the railway line running from North to South. A train moving at 20 miles an hour is observed North-East of P at 6-20 and South-West of Q at 6-40. Find the distance of each village from the station if the train did not stop but passed by it at 6-28. Also find at what times the train would be in places where the line joining the villages subtends rt. $\angle$s at them.

(242) A raw compounder was directed to prepare a mixture of three ingredients in the ratio $a : b : c$ by weight. He mixed them by mistake in the ratio $a : b : c$ by volume. If the specific gravities of the substances were in the ratio 1 : 2 : 3,

compare the weights of equal volumes of the mixture he had to prepare and of what he actually prepared. Also, calculate the volume of each ingredient that he should add to 100 grams of his mixture in order that he may get the mixture he was directed to prepare, taking $a : b : c = 2 : 3 : 5$.

(243) Our town is situated on the left bank of a river which flows North-East and is half a mile from it. Its tributary which flows due North empties itself 1 mile away from our town. An anicut is built across the river one mile further away. Mark in a figure the position of a lake fed by the river which is equidistant from our town, the anicut and the point of confluence of the tributary with the main-river. Calculate how far apart they are from each other. If a temple is equidistant from the river and its tributary and is due South-East of our town, find how far it is from our town, the breadth of the river being 3 furlongs.

(244) Construct a rhombus and a set of $\|^{ms}$. with the diagonals 2·4″ and 3·5″. Show that the area of the rhombus is greater than any of the other parallelograms.

(245) I have a rectangular garden ABCD in which AB = 150 ft. and AD = 80 ft. Two survey stones E, F have to be placed in AB, CD so that EB = 40 ft. and EBCF = $\frac{1}{4}$ of the garden. Explain how this may be done.

(246) How much $\%_o$ above cost must a tradesman mark his goods so that he may allow $10\%_o$ discount and still gain $10\%_o$ on his outlay? What will be the marked price of a watch which cost the tradesman Rs. 90? What sum of money will he gain in selling it?

(247) T, S and P are three villages along a road, S being at the 10th mile-stone from T and P at the 17th mile-stone. Two gentlemen engage a spring-cart for Rs. 2-8 as. to travel from T to P. They take in three more travellers from T to S. If the fare is to be distributed in proportion to the distance travelled, how much should each passenger pay?

(248) Two solid cones are such that in one the slant height = the diameter of the base and in the other the vertical height = the radius of the base. If, when the bases are placed over a rectangle, 1′ by 5″, the circular rims pass through its angular points, calculate the total surface area and volume of each cone.

(249) Find the weight of iron required to make 10,000 iron nuts each of which is ·4″ thick and has a cross-section ·9″ square with an inner circle of diametr ·35″ cut out. [Specific gravity of iron = 7·6.]

(250) The basins of the Nile and the Mississippi are each 1·3 million square miles in area. The average daily rainfall is about ·01 ft. in the Nile basin and ·008 ft. in the Mississippi basin. The Nile discharges on the average 10,000 million c. ft. of water per day into the sea and the Mississippi 50,000 million c. ft. of water. What fraction of the rain which falls in each basin is discharged into the sea?

(251) A rectangular granite block 1′ thick 1′ wide and 12′ long is worked into a cylindrical pillar of diameter 10·5″ and length 11·5″. Compare the weights of the original block and of the cylindrical pillar.

(252) A manufacturing town with a population of 40,000 has to be supplied with water at 25 gallons per inhabitant per day. If water flowing (full bore) at 4 miles an hour along a cylindrical pipe for 6 hours is sufficient for the town for each day, calculate the diameter of the pipe. [1 c. ft. = 6¼ gallons.]

(253) The world's production of sugar has been 15,994,711 tons in 1918; 15,451,504 tons in 1917 and 16,440,664 tons in 1916. Cuba alone produced 3,350,000 tons in 1918; 3,023,720 tons in 1917 and 3,007,915 tons in 1916. Express as a percentage Cuba's share in the total sugar output of the world for each year. (Answer to the nearest unit.) Also, find in what year the percentage was the greatest.

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CHAPTER XXII.

DISCOUNT AND EXCHANGE.

90. DISCOUNT.

The word *discount* means deduction made. This deduction is made in two ways, *viz.*, (1) without any reference to the element of time and (2) with due reference to time also.

The first kind of problems arises when a tradesman allows a customer, who pays cash, a reduction of 10% (say) from his bill or account. This reduction is made with a view to promote business or to encourage cash payment. Such reduction is known as trade discount, and is calculated just like simple interest with the difference that the element of time does not enter in our calculation.

Thus a customer buying goods to the value of Rs. 1000 from a merchant whose principle is to allow his cash customers 10% discount off his bill, would be allowed to deduct from his bill a sum of $\frac{10}{100}$ of Rs. 1000 or Rs. 100 and he would therefore pay in cash Rs. 900 only. This deduction of Rs. 100 from the face-value of the bill is the discount allowed for cash payment. Care should be taken to remember that the discount is expressed as a fraction (generally a percentage) of the face-value of the bill. When, as it generally happens, the commodity bought passes through several middle hands, and every one of them

allows some trade discount, the respective face-values of the several bills ought to be taken into account in the calculation of the deductions allowed at each successive stage.

EXERCISES.

1. A dealer buys a car from the manufacturer for £900. (*a*) If he sells it at a gain of 40°/₀, what does he receive for it? (*b*) At what price must he mark it in his shop so that he may be able to allow a purchaser a reduction of 20°/₀ for cash and yet, make for himself a profit of 40°/₀?

2. A cycle manufacturer posts in his lists prices which would give him a profit of 60°/₀ on the cost of manufacture and then allows for 'cash down' a discount of 30°/₀ off the list price. What profit °/₀ does he actually make? Find the actual profit made on a cycle whose list price is Rs. 350. If the manufacturer actually gain Rs. 75 in a particular case, determine (*a*) the cost of manufacture of the cycle, and (*b*) its list price.

3. If Rs. y is the cash payment for a sum of Rs. x after discount at r °/₀ has been deducted, express y in terms of r and x.

Exhibit this relation graphically for the particular case when $r = 12\frac{1}{2}$. Read off the cash payments for bills of Rs. 37-8-0 and Rs. 9-0-0. Also determine what amounts can be settled by cash payments of Rs. 13-4-0 and Rs. 30-0-0.

4. The real cost of an article is 80°/₀ of the price at which it is marked for sale; a discount of $12\frac{1}{2}$°/₀ is allowed for cash payment. Determine the °/₀ of gain on cash sales.

5. A tradesman allows a discount of 6 pies in the rupee off his customer's bill. He then makes a profit of 20°/₀. What would he have gained if no discount were allowed? If a customer paid Rs. 750 for goods, find the prices at which they were marked and they were made.

6. The wholesale price of watches is 20°/₀ lower than the retail price, and one watch is given free for every two dozen bought at a time. What °/₀ profit does the retailer get?

7. An Indian manufacturer makes an article the catalogue price of which is Rs. 200. An American merchant buys it at a discount of $33\frac{1}{3}$ °/₀ from the catalogue price and has to pay an import duty of 40 °/₀ on the net cost of the article to him. He desires to make a profit of 25 °/₀ on his outlay. What should he charge for the article in America if 1 Re. = 1*s.* 4*d.* and 1 dollar = 4*s.* $1\frac{1}{4}$*d.*?

8. A retail bookseller is supplied by a wholesale firm which allows 30 °/₀ discount on Mathematics and Science books and 50 °/₀ discount on Novels and History books. Another wholesale firm offers a uniform reduction of 40 °/₀. If the bookseller stocks the two kinds of books whose catalogue prices are in the ratio of 3 : 2, determine which offer will be more advantageous and by how much?

9. The expenses attending the production of a book (the retail price of which is Rs. 5-10-0) is Re. 1-12-6. The publisher allows the bookseller 25 °/₀ on the retail price and gives 13 copies to the dozen. 3,900 copies are printed and sold. The author is to have $62\frac{1}{2}$ °/₀ of the profits. How much will he receive?

10. To make an article takes 25 hrs. of a workman's time paid at the rate of Re. 0-8-0 per hour. The material, 15 °/₀ of which is wasted in the working, costs 10 as. a pound. The final weight of the article is 20 lbs. At what price must it be sold to gain $12\frac{1}{2}$ °/₀ on the total cost after allowing a discount of 25 °/₀ on the published price?

11. A merchant marks his goods 25 °/₀ above the cost price; but for cash sales, he allows a discount of 10 °/₀ of the bill. Find his profit per cent. What is the cost price of an article bought for Rs. 86-10-0?

12. A tradesman marks certain goods 35 °/₀ above cost price, he allows cash customers a discount of 10°/₀ and credit customers a discount of 5 °/₀. $62\frac{1}{2}$ °/₀ of his sales are for cash and the rest on credit. When the whole stock has been sold and paid for, find his profit °/₀. (Correct to two places of decimals.)

13 A publisher sells 2,600 copies of a book to booksellers at a discount of 30 °/₀ on the marked price which is 7*s.* 6*d.* and also sold 13 copies at the price of 12. If the booksellers allow a

reduction of 25 °/₀ to their customers, what °/₀ do they make and how much does the publisher receive ?

14. A merchant bought 60 measures of castor oil at Re. 1-2-0 a measure and purified it at a cost of 2 as. per measure. During the process 5 °/₀ of the oil was lost, and the rest was bottled at a cost of Re. 1 per dozen bottles. If the market price per bottle was 12 as. and a discount of 12½ °/₀ was given for wholesale dealers, find the merchant's gain, if he sold one-fourth of the quantity in retail and the rest wholesale (1 measure = 4 bottles).

15. A bookseller is allowed by a publisher 20 °/₀ discount on the nominal price of a book and 5 °/₀ on this last amount for cash. For every dozen books, he gets an extra copy for nothing. The bookseller allows his cash customers 4 as. in the rupee off the nominal price. Find the bookseller's profit on cash sales.

16. 2,000 copies of a book are issued at Rs. 4-8-0 per copy: the cost of printing is 12 as. per copy, of binding 4 as., and of carriage and advertising 2 as.: the publisher disposes of them to the retail book-seller, charging 25 copies as 24, and 30 °/₀ less than the selling price, and upon the whole receipts takes 10 °/₀ commission for himself. Find how much is gained by the author, the publisher and the bookseller, respectively, on this edition.

17. A dealer bought a job lot of goods at 40 °/₀ off list prices and sold them to the trade at 30 °/₀ off. What rate per cent. did he make on cost ?

91. The second kind of problems where the deduction made involves also the element of time arises when payments of money are made much earlier than when they become legally due.

Suppose Rama promises to pay Krishna Rs. 1020 at the end of 9 months but that, after 3 months have passed, he proposes to pay Krishna in settlement of the debt by a cash payment at once. What would be a fair arrangement if money be worth 4°/₀ per annum ? Here Rama proposes to pay off his debt 6 months before it

is really due and hence he is in fairness entitled to some reduction. Krishna is entitled now to get from Rama only that sum which would amount to Rs. 1020 at the end of the remaining 6 months at 4% per annum. This sum is easily deduced to be $\frac{100}{102} \times 1020$ or Rs. 1,000. Hence a sum of Rs. 1020 due at the end of 6 months is really *now equivalent* to a sum of Rs. 1000 at present. The deduction of Rs. 20 (Rs. 1,020—Rs. 1,000) is known as the *discount on* Rs. 1020. The actual present cash equivalent of Rs. 1000 is said to be the *present worth* or present value of the debt. The nominal value of Rs. 1020 is the *face-value* of the debt.

The discount of Rs. 20, as calculated above, is called the *theoretical, or mathematical or true discount.*

A careful analysis of the above procedure will show that all problems on true discount are cases of inverse problems on simple interest where we are asked to calculate the interest when we are given the amount, time and rate %. Such problems therefore are easily worked if we remember carefully—

(1) The sum due = true present value + true discount.

(2) The true discount = the interest on the true present value for the remaining time.

The above statements are unaltered if we reckon compound interest or simple interest.

EXERCISES.

1. Determine the true present value and the true discount.
   (*a*) On Rs. 5,722-8-0 due at the end of 3 years at 6%.
   (*b*) On Rs. 4.810 due in $4\frac{1}{2}$ yrs. at $7\frac{1}{2}$ %.
   (*c*) On £108-6-8 due at the end of $2\frac{1}{2}$ years at $3\frac{1}{3}$ %.

2. The true discount on a certain sum due $2\frac{1}{2}$ years hence at 3% per annum S. I. is Rs. 354-8-0. What is the sum?

3. Find the true discount on a bill of Rs. 2,417-12-0 due 146 days hence, at $4\frac{1}{2}$% interest. Show that this discount is equal to the simple interest on the present worth of the bill, at the same rate for the same time.

4. Allowing $6\frac{1}{4}$% per annum S.I., determine what sum of money will now discharge a debt of Rs. 18,071-4-0, which falls due 11 months hence.

5. A dealer bought a horse for Rs. 1,200 and sold it the same day for Rs. 1,325-12-0. allowing the buyer 7 months' credit. If money be worth 9% per annum, determine his gain %.

6. A debt of Rs. 10,150 with C. I. at 3% is paid off by two equal annual payments. Find the amount of each payment.

7. If Rs. 315-0-0 is the true discount on Rs. 3,657-8-0, find when the latter sum is due, interest being at the rate of 6% per annum.

8. If the present value of a bill of Rs. 1,91,011-9-0 due 5 months hence is Rs. 18,750, what is the present value of a bill of Rs 19,382-13-0 due 9 months hence, at the same rate of interest?

92. BANKER'S OR COMMERCIAL DISCOUNT.

In commerce, when a man buys goods, say from a wholesale dealer, he frequently does not pay 'cash down' for them, but gives what is called a Bill of Exchange which is simply an agreement to pay for the goods after a certain specified time. The following is a form of a Bill of Exchage:—

Rs. 10,000 Madras, Jan. 1st, 1925.

Six months after date pay to me, or my order the sum of Rupees Ten Thousand, value received.

(Signed) DEVADATTA.

To

CARELESS, Esq.,
London.

The above Bill of Exchange or demand for Rs. 10,000 is sent to the London customer by the Madras manufacturer when he sends him the consignment of goods for which the payment is demanded. The merchant Careless accepts the liability stated on the bill by writing across it "accepted" with his signature and date. He also notes down the Bank of payment. The accepted bill is then sent back to Devadatta, who may now proceed in either of two ways.

(1) He may retain the bill until it *matures*, *i.e.*, until the date when payment is due from Careless; or (2) if Devadatta should require money at an ealier date, he may get the bill discounted by his Banker or Bill-broker, 'the Imperial Bank.' In this case, the bill is sold to the Imperial Bank which gives him in cash its present value calculated at the current rate of discount for the period from the date of discounting to the date on which the sum falls due.

There arises, now the question as to how much the banker or broker who 'discounts' the bill pays over to Devadatta. If the banker pays him the true present value of the Rs. 10,000 due after the unexpired period of the bill at the current rate of discount, he makes no profit for himself, beyond that of securing an investment for his money. It is a part of the regular business of a banker to discount bills. It is the custom, therefore, in the business world, for the banker to pay over in cash, the face-value of the bill, less the simple interest on it for the number of days the bill has yet 'to run,' *i.e.*, the number of days from the date on which the bill is discounted up to the first of July when the bill becomes due. This kind of reduction which the banker makes is called the practical, or

commercial or banker's discount. Banker's discount is therefore the same as the simple interest on the face-value of the bill for the unexpired time.

As we have seen, the true discount on the bill is the simple interest on the present value of the bill, and as the true present value is always less than the face-value, the Banker's discount is always greater than the True discount. Hence the sum paid to the holder of the bill by a banker after discounting can never give the face-value when put out to interest at that rate for the unexpired time. The difference between the Banker's and True discounts is what the banker gains for obliging the customer with ready cash and this difference is called the banker's gain.

An expression for the banker's gain can be deduced as follows:—

Banker's discount = int. on the face-value of the bill
= int. on (the true present value + true discount)
= int. on the true present value + int. on the true discount.

∴ Banker's discount—true discount = banker's gain = interest on the true discount for the unexpired time of the bill.

In practice true discount is very rarely allowed. Hence in what follows, unless otherwise stated, Banker's discount must be allowed.

Days of Grace. After a bill is nominally due, it is customary to allow three extra days called days of grace before the bill is said to be legally due. Therefore in working examples, the three days of grace should be added on when the form of the question so admits

it, *i.e.*, when the actual dates are given; but otherwise they are supposed to be already included. Months always mean 'Calendar months.'

The detailed working can be understood from the following examples:—

(1) Find to the nearest anna, the discount on a bill for Rs. 3,650 drawn on April 15th, at 4 months, and discounted on May 18th at 6 %.

The bill is nominally due on Aug. 15th and legally (allowing for three days of grace) on Aug. 18th. The question therefore comes to this:—

What discount should be deducted from a debt of Rs. 3,650 due on Aug. 18th, if payment is made on May 18th?

The bill has yet to run from May 18th to Aug. 18th, *i.e.*, May 12 + June 30 + July 31 + Aug. 18 [Aug. 18th included] (only one of the two extreme dates is included) = 91 days.

∴ The banker's discount = the simple interest on Rs. 3,650 for 91 days at 6 % = Rs. 54-9-7.

∴ The banker discounts or buys the bill for Rs. 3,650—Rs. 54-9-7 or Rs. 3,595-6-5. He keeps the bill, and on 18th Aug. presents it at the Bank named in the bill and receives Rs. 3,650-0-0.

The true present value of Rs. 3,650 for 91 days at 6 % can be shown to be Rs. 3,601-9-7 and true discount to be Rs. 48-6-5.

Therefore if the banker pays for the bill Rs. 3,595-6-5 instead of Rs. 3,601-9-7, he makes a profit of Rs. 6-3-2.

This fact can also be expressed in the form that the banker's discount is greater than the true discount by Rs. 6-3-2.

(2) A holds a bill of B's for £1000 due 3 months hence and B holds a bill of A's for £1050 due 8 months hence. They settle matters by exchange of bills and the payment of a sum of money. If the rate of discount is 5% per annum, determine what this sum should be and which man should pay it.

Here we have first to find the discounted value of the two bills. Since the dates are not specified, we have to understand that the 3 days of grace are already included in the time of 3 and 8 months.

The discount on £1000 for 3 months at 5 % = $1000 \times \frac{3}{12} \times \frac{5}{100}$ = £12-10-0. ∴ The discounted value of the bill held by A = £987-10-0.

The discounted value of the bill held by B is similarly seen to be £1015-0-0.

∴ In closing the accounts, A should pay B a sum of £27-10-0.

(3) A banker discounts a 3 months' bill of Rs. 900 at 6 % discount. What rate % per annum is he charging?

The banker's discount on the bill = $900 \times \frac{6}{100} \times \frac{3}{12}$ or Rs. 13-8-0.

∴ The discounted value of the bill = Rs. 886-8-0.

Therefore the banker advances Rs. 886-8-0 in payment of the bill and in 3 months will receive Rs. 900.

∴ Rs. 13-8-0 is the interest on Rs. 886-8-0 for 3 months.

∴ Rate % required = $\frac{100 \times 2}{1773} \times \frac{12}{3} \times \frac{27}{2} = 6\frac{18}{197}$ % per annum or 6·09 %. *Ans.*

(4) A tradesman marks his goods with two prices one for ready money and the other for six months' credit allowing 10 °/₀ discount. If the cash price of an article be Rs. 28-8-0, determine what its credit price ought to be.

From the problem, it is easy to see that the cash price = 95 °/₀ of the credit price. Hence the credit price corresponding to a cash price of Rs. 28-8-0 is equal to $\frac{\cancel{3}}{\cancel{5}} \times \frac{\cancel{5}\cancel{\phi}10}{\cancel{9}\cancel{8}_{\cancel{8}}}$ or Rs. 30.

(5) In how many months' time must a debt of Rs. 960 be due if it may be cancelled by an immediate payment of Rs. 928, the current rate of discount being 8 °/₀ per annum?

We have to determine in what time, the sum of Rs. 960 could give an interest of Rs. 32 at 8 °/₀ per annum.

The answer is easily deduced to be

$$\frac{\cancel{\phi}\cancel{\phi}}{\cancel{9}\cancel{\phi}\cancel{\phi}} \times \frac{\cancel{3}\cancel{5}}{\cancel{8}_{\cancel{\phi}}} \times \cancel{5}\cancel{1} = 5 \text{ months.}$$

∴ The bill must mature in a time of 5 months.

NOTE :—A *promissory note* differs from a Bill of Exchange only in the fact that it is drawn up by the debtor instead of by the creditor. Thus in the case illustrated above 'Careless' might send a Promissory Note to Devadatta in which he makes a formal promise to pay Rs. 10,000 at the date named in the note. Thus the promissory note is in fact, a bill in which the drawer and the acceptor are identical. These notes also may be discounted in the same manner as the bills of exchange.

A bank note is a promissory note issued by a banker payable at sight or on demand and a bank cheque is a bill of exchange drawn on a bank.

EXERCISES.

1. A holds a bill from B for Rs. 10523, payable 9½ months hence, and B holds a bill from A for Rs. 15,964 payable 7 months hence; they agree to settle accounts by A's giving B a new bill payable 3 months hence. Determine the amount for which this bill should be drawn, the rate of interest being 4°/₀ per annum.

2. If a sum of money be 19 times the true discount on it for a given time at a given rate, and if the interest on the same sum for the same time at the same rate be Rs. 32-8-6, find the sum. What other information is necessary to determine the time or the rate?

3. Find the discount on a bill of Rs. 1,850-10 as. due 18 months hence at 6½°/₀.

4. The interest on a certain sum at 5°/₀ per annum, for a certain time is Rs. 58-5-4, and the true discount at the same rate for the same time is Rs 46-10-8. Find the sum and time.

5. A bill drawn on 27th of Jan. '24 for Rs. 1,580 payable 7½ months thence was discounted by a banker on March 20th. What would the banker have paid for it? How much would be gained in this transaction? (Assume the rate of discount to be 6°/₀).

6. Find the practical discount in the following cases;—

(*a*) On Rs. 767-8-0 nominally due on April 10th, 1924, but discounted on February 15th of the same year, the rate of discount being 9°/₀.

(*b*) On £5,250-13-4 due 145 days hence at 6°/₀.

(*c*) On Rs. 7,525-8-0, due 292 days hence at the rate of 4½ pies per Rupee.

7. In discounting a bill for Rs. 5550, a banker pays down Rs. 5383-8 as. If the rate of discount be 9°/₀, determine how many months the bill has yet to run.

8. The banker's discount on £ 450 at 4°/₀ is the same as the discount on £ 459 for the same time and at the same rate. Find the time.

93. EXCHANGE.

Growth of intercourse and trade which now prevail among all the progressive countries of the world, has made it necessary for us to be able to express the

equivalent of a sum of money according to the monetary systems of the different countries.

If a German coming over to India, brings with him some of the gold coins of his own country, and starts a banking account in Madras by paying these coins, over the counter; what ought the Madras banker to allow him for them? This depends wholly on the amount of pure gold they contain and the calculation of what they are worth regarded merely as precious metal, is to be made.

Example:—80 lbs. troy of British standard gold 22 carats fine, (pure gold is said to be 24 carats fine. Therefore one carat $= \frac{1}{24}$th part) are coined into 3738 sovereigns. 1,000 grammes of French standard gold, $\frac{9}{10}$ fine, is coined into 155 napoleons. If 1 napoleon = 20 francs and 1 oz. troy = 31·103 grammes, find in francs and centimes the intrinsic value of £1, and in shillings and pence the value of 1 napoleon.

£3738 contain 960 oz. troy standard gold $= \frac{22}{24} \times$ 960 oz. troy fine gold $= 880 \times 31{\cdot}103$ g. of fine gold;

$\therefore$ 1 sovereign contains $\frac{880}{3738} \times 31{\cdot}103$ gr. of fine or pure gold ... ... (1)

155 napoleons contain 1000 g. of French standard gold $= \frac{9}{10} \times 1000$ gr. fine gold.

$\therefore$ 1 napoleon contains $\frac{900}{155}$ or $\frac{180}{31}$ g. fine gold... (2)

From (1) and (2), £1 $= \frac{880}{3738} \times 31{\cdot}103 \times \frac{31}{180}$ napoleons $= 25{\cdot}221$ francs.

Similarly 1 napoleon $= \frac{180}{31} \times \frac{3738}{880} \times \frac{1}{31{\cdot}103}$ £ or 15*s.* 10$\frac{1}{4}$*d.* (nearly).

94. PAR OF EXCHANGE.

If we work similar problems with the coins of the various countries of the world, we get what is called the *Mint Par of Exchange*, *i.e.*, the sum which would be given for the gold coin of one country at the Mint of another country. The following table gives the result for the most important commercial countries :—

Country.	Coin.	Approx. value in Eng. money.	Approximate value of 1 £ sterling.
Great Britain and Colonies.	1 Sovereign=20 shillings	240*d.*	£1.
India ...	1 Rupee = 16 annas.	16*d.*	Rs. 15.
France, Belgium, Switzerland	1 Franc = 100 centimes.	9½*d.*	25·22 francs.
Italy ...	1 Lira = 100 centimes.	9½*d.*	,,
Greece ...	1 Drachma = 100 lepta.	9½*d.*	,,
Denmark, Norway and Sweden	1 Krone = 100 ore.	13¼*d.*	18·2 krones.
United States ...	1 Dollar = 100 centimes	49¼*d.*	4·87 dollars.
Germany ...	1 Mark = 100 pfennig.	11⅞*d.*	20·43 marks.
Austria Hungary.	1 Krone = 100 heller.	10*d.*	24·0 Krones.
Holland ...	1 Florin = 100 cents.	20*d.*	12·0 florins.
Russia ...	1 Rouble = 100 kopecks	38*d.*	6·30 roubles.
Portugal ...	1 Milreis = 1000 reis.	4*s.* 5¼*d*	4·5 milreis.
Japan ...	1 Yen = 100 sen.	24*d.*	10·0 yens.
China ...	1 Dollar (Hong-Kong) = 100 cents	24*d.*	10·0 dollars.
	1 Tael (Shanghai) = $\frac{1000}{717}$ dollars.	33½*d.*	7·16 taels.

N.B The Mint Par of Exchange is liable to variation, since the values of the different precious metals are not always in the same ratio.

EXERCISES.

A. Using the table of the Mint Par of Exchange, find in Indian money the value of:—(1) 1746 francs 25 centimes. (2) 1500

roubles. (3) 975 dollars, (4) 1073 marks 25 pfennig and (5) 375 yen and 40 sen.

B. From the same table work out the value in foreign money of :—(1) Rs. 75-12-0 in francs and centimes ; (2) £575-13-4 in marks, etc. (3) 1050 in roubles, and (4) £17-6-8 in krones and ore.

95. BILLS OF EXCHANGE.

Suppose, now, that an American wine merchant has ordered in a stock of Bordeaux wines and Rhenish wines from a French wine dealer, and that the French dealer's charge is 62500 francs, how is the English merchant to pay him? He could of course calculate the quantity of American gold required to make up this total in French gold. He could then pack up this amount of gold and send it to France. The merchant would have to incur the cost of carriage and insurance of this large quantity of gold. The French merchant would have to wait for the coining of this gold into his coin by the French Mint. This tedious and inconvenient way is avoided by having recourse to *Bills of Exchange*. The American merchant will go to one of the foreign bill-brokers in New York and purchase a 'bill on Paris' for the requisite amount. This bill will be simply an order from the bill-broker to his correspondent in Paris to pay to the French merchant who will present it, the stated sum in French money either '*at sight*,' *i.e.*, on demand, or at the end of some stated time. The first kind of Bills is called 'the Short Exchange' while other kinds are known as 'the Long Exchange.' The American merchant will have to pay the bill-broker a little for his trouble in undertaking to

transmit the money, and thus, 1 dollar in the United States will not purchase a bill for 5 francs 18 centimes but rather less, say about 5 francs 15 centimes.

But we may ask whether the bill-broker has not to send the gold to France after all. Have we simply transferred the responsibility to the shoulders of the bill-broker? Not at all. Just as the American merchant is getting wine from France, the French merchants are ordering motor-cars from America. Payment is made by them through bills on New York.

Thus the merchants can balance these accounts and bills against both Paris and New York. If both the countries purchased from each other goods of exactly equal value, then no gold or bullion of any kind need be sent from one country to the other. But this exact balance of trade never occurs in practice. When the balance of trade *is 'against France' and 'in favour of America,' i.e.*, when France has purchased more from America than America from France and therefore when France has to send some gold to America, only the balance need be sent. Thus the difficulty of sending gold is minimised if not thoroughly obviated.

96. SPECIE POINTS.

What charge has to be made by the broker on the American merchant? To answer this question, we have to remember that the Bill of Exchange was introduced to lessen the cost of sending gold from one country to another and hence the money which the merchant would be prepared to pay the bill-broker

and his fee for his help can never exceed the charge of sending bullion abroad. For, if this limit is exceeded, the merchant would prefer the alternative method of sending gold abroad as it is more profitable to him. Hence we see that there is a limit to the charge that can be demanded by the broker. Thus, if d be the cost of sending 1 dollar to France, and the bill-broker would give credit to $(5{\cdot}18-d)$ francs only for every dollar received by him. This limit is therefore known as a *Specie point*. There is another limit to this. For if the bill-broker offers more than $(5{\cdot}18+d)$ francs for every dollar to be paid in France, the French merchants would begin to send gold to America. Between these two values, therefore the Exchange of Paris may vary from day to day or even hour to hour according to circumstances.

97. DIRECT AND INDIRECT EXCHANGE.

The fluctuations in the exchange may make it advantageous to pay his French merchant through a German agent. For he may buy bills on Berlin and direct his German agent to remit the money to Paris by buying in Berlin bills on Paris. When a new country intervenes, as Germany in the above case, the exchange is said to be *indirect*. The rate of exchange between New York and Paris *via* Berlin or other cities, is called the *arbitrated rate of exchange*.

Examples.

(1) A person in Paris owes another in Vienna 3375 Austrian florins. Instead of paying direct, he pays through a broker in London and thereby gains,

in English money at current rates, £ 0-12-9 $\frac{14}{25}d$. The exchange between London and Paris being 25 francs for 1£ and between London and Vienna 11·60 florins for 1£, find the rate of exchange between Paris and Vienna.

3375 Austrian florins $= \frac{3375}{11{\cdot}60}$ £ or £ 290-18-11·6.

Since there is a gain of £ 0 12-9$\frac{14}{25}$ by adopting this indirect exchange, the person in Paris would have had to send £290-18-11·6 + £ 0-12-9$\frac{14}{25}$ or £291-11-9·2*d*. if he had remitted direct to Vienna.

∴ 3375 Aus. florins are equivalent to £291-11-9·2*d*. or 7289$\frac{17}{24}$ francs.

∴ 1 florin is equal to $\frac{7289\frac{17}{24}}{3375}$ or 2·16 francs.

∴ The exchange between Paris and Vienna is 1 florin = 2·16 francs.

(2) Find the cost of settling a debt of £750 by buying from a bill-broker 4 months' bills on London at 1*s*. 4*d*. per rupee, discount 5%, brokerage and stamp duty 2 per mille or thousand.

The value of the debt when payment is actually effected is £750 + interest on this £750 for 4 months at 5% $= £\,750 + \frac{\cancel{5}}{\cancel{100}\ \cancel{20}} \times \frac{1}{\cancel{3}} \times \frac{\cancel{75}}{25}$ or £762$\frac{1}{2}$.

(The bills mature only at the end of 4 months.) In addition to this there is the *brokerage* (*i.e.*, the sum charged by the broker for his help) and stamp-duty equal to $\frac{1525}{\cancel{1000}} \times \cancel{1}$ or 1·525 or £764·025.

∴ The equivalent of this debt in Rupees = Rs. 764·025 × 15 or Rs. 11460·375 = Rs. 11460-6-0.

(3) On Aug. 16, 1900, the rate in London on Berlin (sight) was 20·495; on Paris was 25·175; in Berlin for bills on Paris 81·35. A man has to pay 7500 francs in Paris. Ought he to buy bills on Berlin or Paris direct? What difference in English money, is there between the two courses? What is the arbitrated rate between London and Paris?

Direct. £1 = 25·175 francs: The equivalent of 7500 francs = $\frac{7500 \times 1}{25{\cdot}175}$ £ or £297-18-3·5.

Via Berlin. £1 = 20·495 marks and 81·35 marks = 100 francs.

∴ £1 = $\frac{20{\cdot}495 \times 100}{81{\cdot}35}$ francs. ∴ The equivalent of 7500 francs = $\frac{7500 \times 81{\cdot}35}{2049{\cdot}5}$ £ = £297-13-10·2.

Hence we find that it is cheaper to send the money *via* Berlin. The difference is £ 0-4-5·3.

The arbitrated rate of exchange is the value of £1 sent *via* Berlin and that is $\frac{2049{\cdot}5}{81{\cdot}35}$ francs = 25·19 francs.

EXERCISES.

1. A merchant in New York owes 2500 dollars to a merchant in London. He draws on Berlin at 1 dollar for 4 marks 27 pfennig, then on Paris at 4 marks for 5 francs, and then on London at 9·75 *d.* per franc. What does the English merchant get?

2. How many Rupees must I have in order that, if I take them to France and there change them into francs at an exchange of 25·45 francs for 15 Rs, I can spend as many of them as are equal to Rs. 240 and by changing the rest into Rupees on leaving at an exchange of 25·25 francs for 15 Rs, have as many rupees as I started with ?

3. If Rs. 15 = 1£ = 25·22 francs, change 3895 napoleons (each equal to 30 francs) into rupees.

4. One dollar weighs 412·5 grs. and is 21·6 carats fine, 1 lb. Troy of British standard silver 22·2 carats fine is coined into 66 shillings. Find the number of pence in 1 dollar when British standard silver has a marketable value of 5 *s.* 1*d.* per oz.

5. If the cheque of exchange in Berlin on Paris is 100 francs = 66·7 marks and the rate of discount for 6 months' bills in Berlin is at 6% per annum, what debt in Paris can be discharged by a person in Berlin who holds a six months' bill on Berlin for 5250 marks ?

6. If a merchant has to remit upon Paris at 25·63 and to draw upon Lisbon at 56, at what rate he may draw upon Lisbon if the Paris rate falls to 25·25 ?

7. What is the arbitrated rate of exchange between London and Vienna through Lisbon when £ 1 = 20·43 marks and 100 marks = 22·27 milreis. Answer, correct to two decimal places.

8. If napoleons be bought at 16*s.* 7*d.* each in London and sold for 5 thalers 18 groschen each in Berlin where one pound equals 6 thalers 22 groschen, what sum will be gained or lost upon 725 napoleons by this transaction ? Assume 1 thaler to be equal to 29·75 groschen.

9. The par of Exchange between England and India is 1*s.* 4*d.* per rupee. A money-lender gives Rs. 14½ for every sovereign and 1*s.* 2¾*d.* for 1 rupee. A traveller changes Rs. 5750 into pounds and on his return changes the balance of £ 12-17-6 left with him into rupees. Find in rupees the profit made by the money-lender on both the transactions ?

98. CHAIN RULE.

In this connection, there is a particular application of the Unitary Method called the Chain Rule. Its use sometimes saves a little time in calculations.

Ex. If 47 lbs. of coffee are worth as much as 11 lbs. of tea, 20 lbs. of tea are worth as much as 940 lbs. of sugar, a cask of sugar costs 2 guineas and 1 cwt. of coffee costs 8 guineas, what is the weight of a cask of sugar?

We generally arrange our data in such cases thus:—

1 cask of sugar = 2 guineas
8 guineas = 1 cwt. of coffee
47 lbs. of coffee = 11 lbs. of tea
20 lbs. of tea = 940 lbs. of sugar.

Then we deduce at once that 1 cask of sugar

$$= \frac{1 \times 1 \times 11 \times 940}{1 \times \frac{8}{4} \times 47 \times 20}$$

$$= 2\tfrac{3}{4} \text{ cwts.}$$

N. B.—It should be noted here that in the arrangement the left-hand side of one equality refers to the same kind as the right-hand side of the just previous one.

This is really an abbreviated form of 1 cask of sugar is worth $\frac{2}{8}$ guineas but 1 guinea is the cost of $\frac{1}{8}$ cwt. of coffee. ∴ 1 cask of sugar = $\frac{2}{8} \times \frac{1}{8}$ cwt. of coffee. etc., etc. But the notation '1 cask of sugar = 2 guineas' for 'is worth 2 guineas' is misleading and cannot be strongly recommended.

Examples.

(1) When silk is sold at 17·38 francs the metre, find the price per yard in shillings and pence. Assume £ 1 = 25·22 francs, 1 metre = 39·371 inches.

(2) If 1 £ is equivalent to 25 francs, 36 ducats to 390 francs, and 20 ducats to 41 roubles, determine the number of roubles equivalent to £ 75-15-0.

(3) A plot of land is valued at £ 2,250 an acre Find its value in Rupees per cent. (100 cents = 1 acre and 1*s.* $3\frac{1}{2}d.$ = Re. 1.)

99. THE EQUATION OF PAYMENTS.

Suppose several sums of money are due to a person at different times and we are asked to determine the exact time when all these sums may be paid together without loss to either party. The way we adopt to determine this time of payment is called *the equation of payments*. In working out these questions we assume that the sum of the interests of the several debts for their respective times = the interest of the total debt due for the equated time.

Example.

A merchant has purchased goods, the times at the end of which the values are due are as follows :—

Value of goods Rs. 725 credit 3 months.
" " " 460 " $4\frac{1}{2}$ "
" " " 1200 " 5 "
" " " 175 " 20 days.

Find the equation of payments by which a single payment of Rs. 2560 will discharge the debt.

Let the required time be x months credit.

Then the int. on Rs. 2560 (725 + 460 + 175 + 1200) for x months = int. on Rs. 725 for 3 months + int. on Rs. 460 for $4\frac{1}{2}$ months + int. on Rs. 1,200 for 5 months + int. on Rs. 175 for $\frac{2}{3}$ months

= interest on Rs. $(725 \times 3 + 460 \times \frac{9}{2} + 1200 \times 5 + 175 \times \frac{2}{3})$ 1 month.

$\therefore 2560x = 10361\frac{2}{3}$.

$\therefore x = \frac{31085}{3} \times \frac{1}{2560} = 4{\cdot}05$ months nearly.

EXERCISES.

1. Rama owed Krishna Rs. 3250 to be paid at the end of 7 months; he paid Rs. 1500 in 3 months: when is the remainder due?

2. A merchant has purchased goods the due dates of which are as follow:—

March 15th, £ 220, due April 18th.
April 21st, £ 125, due May 24th.
April 27th, £ 200, due June 30th.
May 15th, £ 350, due July 18th.

He wishes to give a bill for the total amount due, the bill to be drawn payable on the average due date. Ascertain this date.

(Assume that this date is x days after the 18th of April the earliest of the due dates and then proceed to calculate the equated time of the several payments).

CHAPTER XXIII.

STOCKS AND SHARES.

100. SHARES.

When a man engages in business on his own account he invests a certain sum called *capital*, with which he trades and any balance that may remain over his expenditure is called his profit. If a number of partners conjointly subscribe the capital of a business, the excess of receipts over expenses is divided among them in the ratio of their capitals. In these days of big enterprises, it very often happens that a business or an undertaking is on too large a scale for one or a few men to subscribe the required capital. In such cases, a *company* is formed by the coming together of a number of persons who, among themselves provide or subscribe the sum of money required.

The capital is then divided into a number of equal parts called *Shares* and the different members of the company are called the *Shareholders*. The shareholders select some of their number to manage or direct the affairs of the company. They are called the *Directors* and they carry on the business. The profits of the business at stated periods (generally half-yearly) are divided among the shareholders and each person's share of the profits is known as his *Dividend*.

Each shareholder is generally required to take up an integral number of shares of the company. It

also happens that the shareholders are not immediately asked to pay up the full value of their shares. A portion of the share money is paid on application, a second portion on *allotment* and the business goes on on the understanding that the shareholders should *pay up* the balance when a demand is made for the same. Before the shares become *fully paid up*, each subscriber is given a *scrip* or a provisional certificate of subscription to the share capital containing the number of shares, and the different instalments made. When the shares become fully paid up, these scrips are exchanged for share certificates or bonds.

101. DEBENTURES.

Debentures form a special kind of shares where a man gets the right to a *fixed* rate of interest according to agreement whatever may be the prospects of the business.

Preference shareholders also have the right to a *fixed* rate of interest before the profits of the business are divided among the holders of *ordinary* shares. But a Preference shareholder has no right against the property of the company as in the case of Debenture-holders.

If the business of the company thrives very well, it will happen that the ordinary shareholders may get a greater rate of interest than either the Debenture-holders or Preference shareholders. It is also generally true, the more satisfactory the security is, the lower will be the rate of interest allowed.

102. SALE AND PURCHASE OF SHARES.

Let us now suppose that a company is formed with a capital of Rs. 400,000 divided into 4,000 shares of Rs. 100 each and that the profits allow of a dividend at 9% per annum. These shares are purchased by the public, in varying numbers, who seek to *invest* their money in the business. The purchaser gets the right to a dividend of Rs. 9 per share of Rs. 100. A holder of 25 shares would get at the end of each year Rs. 225 ($25 \times 9 = 225$). The subscriber could not however get back from the company the Rs. 2,500 invested by him. But he might find a purchaser for his shares to whom on the purchase the right to the dividend passes. Thus the sale and purchase of the shares of the company is purely a private transaction and does not in any way affect the original capital of the company.

If the business of a company is very prosperous, a purchaser may like to pay for a 100 Rs. share more than Rs. 100, the *face-value* of the share, to secure what he considers to be a good investment. While if the company is not working satisfactorily, the price of the shares may go down below the face-value of Rs. 100. Thus owing to circumstances the *market-value* of the shares may be *more than*, *equal to* or *less than* the nominal value. In the first case, the share is said to be *at a premium*, in the second at *par* and in the third at a *discount*. If a Rs. 100 share fetches Rs. 110, the share is at a premium of 10, if only Rs. 100, it is at par and if it fetches only Rs. 75 it is at a discount of 25. The

excess over or the deficit of money from the subscribed sum merely represents the personal gain or the loss of the seller of the shares. The purchaser will be credited in the accounts of the company merely with the nominal value of the shares he is buying. Since the dividend is distributed according to the number of shares held, the rate of dividend declared by the company is very often different from the rate which a purchaser gets for his actual investment.

When the shares of a company are not fully paid up, the purchaser of such shares must contribute the remaining sum on demand. The dividend will then be declared only on that portion of the share which has been 'paid up' and not on the full Rs. 100. Other things being equal, the market-rate of interest also affects the price of shares. For if only a lower rate of interest can be had in the outside market, purchasers will be tempted to pay more than the nominal value of a share for it as they find it a better investment. Thus a purchaser of a Rs. 100 share in a company declaring 9% dividend while ordinarily he is able to get only 8% outside would pay up to 9/8 of Rs. 100 or Rs. 112-8-0 for one share in the company. Similarly if the current market-rate is 10%, the price of the share goes down and the purchaser will offer only 9/10 of Rs. 100 or Rs. 90. The shares will thus be at a discount.

Care should be taken to bear in mind the vital distinction between *shares* and *cash*. Thus a purchaser who buys 20 shares of Rs. 100 each at a premium of 20, invests Rs. 2,400 but the shares

held by him have only a nominal value of Rs. 2,000. Also if by chance, the shares can be sold only at a discount of 25, he can by the sale of his shares, realise only a cash of Rs. 1,500. Hence we see the cash invested, the nominal value of the shares, and the sum of money realisable from sale, are all different.

Examples.

1. If fifty £ 10 shares in a company paying a dividend of 8% are sold for £17-10 each and the proceeds invested in £ 5 shares in another company paying a dividend of 3½% at £ 3-10 each, find what the difference in income would be. What rate of interest does he get for his investment in the 2nd case?

The money realised by the fifty £ 10 shares = £50 $\times \frac{35}{2}$ = £ 875.

The nominal value of the fifty shares = £ 500 and as the income depends only on the nominal value, the income in the first case = £ $\frac{500 \times 8}{100}$ or £ 40.

The nominal value of the shares purchased in the second company is £ $\frac{\overset{125}{\cancel{875}} \times 5 \times 2}{\underset{1}{\cancel{7}}}$ = £ 1250. (He will get 250 shares of £ 5 each).

∴ The income in the 2nd case (note again, this dividend depends only on the face-value of the stock)

= £ $\frac{\overset{25}{\cancel{1250}} \times 7}{\underset{4}{\cancel{100}}}$ or £ $\frac{175}{4}$ or £ 43-15-0.

∴ The income in the 2nd case is greater by £ 3-15-0.

To find the actual rate obtained by the man, we have to take into consideration, the actual sum invested by him and this is here £ 1250. ∴ On an outlay of £ 1250 an income of £ 43-15-0 is made.

∴ The rate is $\frac{\cancel{175}}{\cancel{2}} \times \frac{\cancel{100}}{\cancel{1250}}$ or $3\frac{1}{2}\%$.

2. One company pays $5\frac{1}{2}\%$ on shares of Rs. 100 each; another pays at the rate of $3\frac{1}{2}\%$ on shares of Rs. 10 each; if the price of the former be Rs. 115-8-0 and of the latter Rs. 7-12-0, compare the rates of interest which the shares give a purchaser.

An investment of Rs. 115-8-0 brings an income of Rs. $5\frac{1}{2}$ in the first case.

Hence this works up to a rate of $\frac{11}{\cancel{2}} \times \frac{100}{\cancel{200}\ \cancel{21}}$

or $4\ \frac{16}{21}\%$.

In the 2nd case, an investment of Rs. $\frac{31}{4}$ fetches an income of ·35 Re.

Hence this works up to the rate of $\frac{35}{\cancel{100}} \times \frac{4}{31} \times \cancel{100}$

or $\frac{140}{31}\%$.

∴ Rate in the first case : Rate in the 2nd case ::

$$\frac{100}{21} : \frac{140}{31} \text{ or } 155 : 147.$$

3. A man lays out Rs. 4500 in the purchase of Rs. 60 shares and, after they have risen to Rs. 90, sells out. What is his profit?

The number of shares held is $\frac{4500}{60}$ or 75 and on each share there is a profit of Rs. 30 (90—60) made.
∴ The required total profit is Rs. 30×75 or Rs. 2250.

4. A is the original holder of 150 Rs. 75 shares (Rs. 60 only being paid up) in a company paying 10% per annum.

B purchases these from A at Rs. 85 for each share. What did these shares cost A and B respectively and what % does B get on his outlay.

A pays Rs. 60 for each share held by him. (Note that he is liable for the remaining Rs. 15 only so long as he is a shareholder).

∴ A must have invested Rs. 150×60 or Rs. 9,000.

B pays Rs. 85 for every share of A's. ∴ B must have paid Rs. 150×85 or Rs. 12,750.

But B will be credited in the accounts of the company only with Rs. 9,000 the sum to the credit of A. Hence B gets from the company Rs. $\frac{9{,}000 \times 10}{100}$ or Rs. 900.

To secure this income of Rs. 900, B has invested Rs. 12,750.

∴ B gets for his money $\frac{180}{\cancel{900}} \times \frac{2}{\cancel{100}/\cancel{1275}\cancel{0}}$ or $\frac{360}{51}$ or $7\frac{3}{51}$ % or 7·06 %.

EXERCISES.

1. The Rs. 75 shares of a spinning company are quoted at Rs. 60 each. What is the nominal value, and what is the market value of 37 such shares?

2. I lay out Rs. 8,750 in buying Rs. 150 shares of a company when they are at a premium of 25. How many are bought and what is their nominal value?

3. The holder of 200 Rs. 100 shares sells them when they have gone down to Rs. 80, and invests the proceeds in buying at par Rs. 50 shares in a company paying 3 °/₀ per annum. How many shares does he get and what is his annual income from them?

4. By purchasing Rs 20 water shares for Rs. 24 each, a man gets 4 °/₀ on his outlay. What rate °/₀ is the company paying? What is his dividend if he buys 45 shares?

5 The Rs. 150 shares of a company, of which Rs. 90 only are paid up, are selling for Rs. 175. If the company pays 8 °/₀ what income will a man get who invests Rs. 3,500?

6. I invest Rs. 8,000 in a company paying 9 °/₀ when their 150 Rs. shares can be bought for Rs. 200. My friend invests a certain sum in a company paying a dividend of 6 °/₀ per annum when their 100 Rs. shares are available at a discount of 25. If we derive the same annual income, determine the sum invested by my friend.

7. The holder of 95 £10 shares in a weaving company paying a dividend of 9 °/₀. sells out when they have risen to £20. He invests the proceeds in the purchase of Bank shares, quoted at £47-10-0, when the Bank is paying a dividend of 6 °/₀. If his income is increased by £4-15-0 by the transaction, what is the nominal value of a Bank share?

8. How much must be invested in a company paying 5½ °/₀ whose £40 shares (£ 30 only being paid up) are quoted at £55 so as to obtain a dividend of £82-10-0? What must be paid by the owner of the shares if the remaining £ 10 per share be called up?

9. A gentleman holds 80 shares in a gas company and he sells them at £7-10 shillings each, thereby making a profit of £64

What price did he himself pay for each share? If the company was paying a dividend of 6 °/₀, how much was he receiving?

10. A man buys 750 Rs. 15 shares in a mining company paying 8 °/₀ per annum. They are quoted at 10 °/₀ above par. What is his annual income, and what °/₀ does he obtain on his investment?

103. STOCKS.

Money borrowed by Governments, Town Corporations, Railways and some other large undertakings is known as *Stock*. The public who lend the money are known as *Stock-holders* and receive interest periodically (generally half-yearly or quarterly) as a percentage on the amount of stock held. This interest is sometimes at a fixed percentage and this fixed rate gives the name to the stock. Thus 5 °/₀ stock means that interest at the rate of 5 °/₀ per annum on the amount of stock held is paid. It also happens that the periodical interest is made after a division of profits and in this case it is called a *dividend*.

Distinction between Stocks and Shares:—Only an integral number of shares in a company can be bought or sold. The term stock is given to the capital of a company, any part of which can be bought or sold. Stock capital is not divided into a number of equal portions as in the case of shares. The *market value* of the stock, *i. e.*, its selling and purchasing price fluctuates as in the case of every other commodity. These fluctuations are determined among other things, the prosperity of business, the available rate of interest in the outside market, political and other reasons. Hence the stock as in the case of shares is sometimes at a premium, sometimes at

par, and sometimes below par or at a discount. As in the case of every other commodity, a unit is required in terms of which stock can be measured. This is generally taken to be £ 100 or Rs. 100 stock and this unit is sometimes called the *cent.* of stock. Care should be taken to distinguish between Rs. 100 stock and Rs. 100 cash. The stock-holder can either buy or sell only the right to get the dividends or interests at the stated periods. Any loss or gain that may result in such transaction is purely of a private nature and does not affect the stock capital of the business. The purchaser will be credited only with the nominal value of the stock bought by him and the interest or the dividend has reference only to this nominal value. Hence the rate of interest allowed on the stock capital is not necessarily equal to the rate of interest which the stock-holder gets for his investment.

104. GOVERNMENT LOAN AND CONSOLS.

When the Government of India finds it necessary to raise money, it issues a *Government Loan* or *Government Paper* as it is called, bearing generally $3\frac{1}{2}\%$ but in special cases different rates. When a person buys this Government paper, he is said to invest his money in the Government of India securities.

'Consols' is an abbreviation for consolidated annuities. This term is applied to the Government stocks forming the public debt or national debt.

N. B.—The price of a stock rises sometime preceding the time of the payment of interests or dividends and falls immediately after this payment.

Examples.

A man invests Rs. 2,300 in buying $5\frac{1}{2}$% stock when it is at a discount of 20. Find his annual income. What rate is he getting for his money? Will his income be increased or decreased if he invests his money in buying 8% stock at a premium of 15?

The annual income depends upon the nominal value of the stock held. Hence we have first to determine the stock held in the first case. The stock is at a discount of 20, *i.e.*, a sum of Rs. 80 will enable him to buy a Rs. 100 stock. Hence the nominal value of the stock purchased in $5\frac{1}{2}$% $= \text{Rs. } \frac{575}{2300} \times \frac{5}{80} = \text{Rs. } 2,875$.

Every Rs. 100 stock fetches in an income of Rs. $5\frac{1}{2}$ a year. ∴ his annual income from the $5\frac{1}{2}$ per cents.

$= \text{Rs. } \frac{115}{2875} \times \frac{11}{200}$ or Rs. $\frac{1265}{8}$ or Rs. **158-2-0**.

To calculate the actual rate he is getting on his investment, we must consider the actual sum invested. On Rs. 2,300, he makes an income Rs. 158-2-0. ∴ the required rate on his investment $= \frac{1265}{8} \times \frac{100}{2300}$

$= 6{\cdot}875\%$.

In the second case, an investment of Rs. 115 gives a stock of Rs. 100.

∴ the nominal value of the 8% stock purchased for Rs. 2,300 $= \text{Rs. } \frac{2300 \times 100}{115}$ or Rs. 2,000 stock.

$\therefore$ annual income in this case = Rs. 2,000 $\times \frac{8}{100}$ or Rs. 160-0-0.

$\therefore$ The annual income in the second case is greater than that in the former case by Re. 1-14-0.

(2) Which is the better investment 4% stock at 110 or 6% stock at 150?

The nature of the investment depends on the actual rate obtained on it.

In the first case, an investment of Rs. 110 gives an income of Rs. 4.

$\therefore$ rate % $= 4 \times \frac{100}{110}$ or $3\frac{7}{11}$.

In the 2nd case an income of Rs. 6 is obtained on Rs. 150. $\therefore$ the required rate % $= \frac{2}{\cancel{6}} \times \frac{\overset{2}{\cancel{150}}}{\underset{\cancel{6}}{\cancel{100}}}$ or 4.

$\therefore$ the latter investment works out a higher return and hence is the better investment.

(3) The difference between the incomes derived from investing a certain sum in 5 % at 130, and in $4\frac{1}{2}$ % stock at 110 is found to be Rs. 24-8-0. Determine the sum invested.

Let Rs. x be the sum invested. Then the nominal value of the 5% stock = Rs. $x \times \frac{100}{130}$ or Rs. $\frac{10}{13}x$. The income derived in this case would be Rs. $\frac{10}{13}x \times \frac{5}{100}$ or Rs. $\frac{1}{26}x$. In the 2nd case the nominal value of the $4\frac{1}{2}$%

stock = Rs. $x \times \frac{100}{110}$ or, Rs. $\frac{10}{11}x$. The income comes to Rs. $\frac{10}{11}x \times \frac{9}{200}$ or Rs. $\frac{9x}{220}$.

∴ the difference between the two incomes = Rs. $\left(\frac{9}{220}x - \frac{1}{26}x\right)$ = Rs. $\frac{14x}{26 \times 220}$.

By the question, we have $\frac{14x}{26 \times 220} = \frac{49}{2}$.

∴ $x = \frac{49}{2} \times \frac{26 \times 220}{14}$ or Rs. 10010.

Exercises.

What is the cash value of —

1. £ 4,250 stock at a premium of 15.
2. Rs. 3,750 stock at a discount of 20.
3. £ 6,825 stock at 92½.
4. Rs. 1,255 stock at par.

Find the amount of stock which can be purchased for —

5. £ 2,527, price 95.
6. Rs. 9,213, price at a premium of 11.
7. Rs. 3900, price 97½.
8. I buy Rs. 570 stock for a sum of Rs. 456. Determine the price of the stock.
9. Find to the nearest pie how much was lost by investing Rs. 3,284 in 6 % Railway stock at 135 and selling at 117½.
10. What income will be derived from Rs. 483-12-0 laid out in buying 6 % stock at 107½?
11. I invest £ 10,000 in 6 % stock at 125; find my half-yearly income after an income-tax of 9*d*. in the £, has been deducted.

12. If the current price of a 5 °/₀ stock is 105½, find (1) how much stock, a man must hold to derive from it an income of Rs. 420, and (2) how much cash he must invest to secure an income of Rs. 420.

13. Which yields a better income to an investor : 4 °/₀ stock at 87, or 6 °/₀ at 125 ?

14. What rate do I get for my money invested in Railway 4½ °/₀ stock at a premium of 22½ ? What total income do I derive from my investment of Rs. 2,450 in it ?

15. How much must be invested in 5½ °/₀ stock at 141¼, so that after paying income-tax of 1*s.* 2*d.* in the £, the net half-yearly dividend may be £ 70-8-0 ?

105. BROKERAGE.

A *Broker* is an agent through whom the purchase and sale of stock is conducted in practical life. He charges for his services a commission known as *Brokerage* on every £ 100 stock bought or sold. The brokerage is usually ⅛ °/₀ in England and ¼ °/₀ in India. Thus in investing money the *buyer* pays ⅛ °/₀ more than the quoted price of the stock, and the seller receives ⅛ °/₀ less than the quoted price.

It should be remembered that the brokerage is calculated as a percentage on the nominal par value of the stock.

A stock-jobber makes it his business to buy stock at a low rate and sell it again at a higher rate. The *stock-broker* is a kind of middleman between the stock-jobber and the private investor.

Examples.

(1) How much stock at 97⅞ is sold to get Rs. 4789-12-0 (brokerage ⅛ °/₀) ?

By selling Rs. 100 stock, Rs. $(97\frac{7}{8} - \frac{1}{8})$ cash only is realised.

∴ To get Rs. $97\frac{3}{4}$ cash, Rs. 100 must be sold.

∴ Rs. 4,789-12-0 cash is got by the sale of

$\frac{49}{4} \times \frac{100}{391}$ or Rs. 4,900 stock.

(2) Find the yearly income derived from an investment of £ 12,133-6-8 in $4\frac{1}{2}$ per cents at $149\frac{7}{8}$, brokerage being $\frac{1}{8}$.

To secure an income of $£4\frac{1}{2}$, a man must hold £100 stock.

To buy £100 stock, a man must invest $£149\frac{7}{8} + \frac{1}{8}$ or £150 cash.

∴ An investment of £150 yields an income of $£4\frac{1}{2}$.

∴ An investment of £12,133-6-8 gives an annual income of $£\frac{36400}{3} \times \frac{9}{2 \times 150}$ or £364-0-0.

(3) Rs. 1,100 of $2\frac{3}{4}$ consols are sold out at $108\frac{5}{8}$ and the proceeds are invested in the 4 per cents at $96\frac{3}{4}$. Find (1) how much of the second stock can be bought; and (2) the resulting change in the income. (Brokerage being $\frac{1}{8}$ in each case.)

The income in the first case is Rs. $11 \times 2\frac{3}{4}$ or Rs. 30-4-0.

The sale of Rs. 100 stock gives a net cash of Rs. $(108\frac{5}{8} - \frac{1}{8})$ or Rs. $108\frac{1}{2}$.

∴ The proceeds of the sale of the $2\frac{3}{4}$ per cents = Rs. $11 \times 108\frac{1}{2}$ or Rs. 1193-8-0.

Rs. 100 stock of the second kind can be bought for Rs. $(96\frac{3}{4} + \frac{1}{8})$ or Rs. $96\frac{7}{8}$.

∴ The quantity of stock bought in the 2nd case

$$= \frac{2387}{2} \times \frac{800}{775} = \text{Rs. } 1{,}232.$$

∴ Income from this investment = Rs. $1{,}232 \times \frac{4}{100}$ or Rs. 49-4-6 (correct to the nearest pie).

∴ The result of the transaction is an increase of the annual income by Rs. 19-0-6 (correct to the nearest pie).

(4) A man invests a sum of money, sufficient to pay for the stock and brokerage in the $4\frac{1}{2}\%$ at 110, and receiving the dividend for half a year, sells out at $107\frac{1}{8}$, thus losing £700 on the whole transaction. Determine the sum invested. (Brokerage $\frac{1}{8}$ in each case.)

Let £x be the required sum invested. Then the nominal value of the stock in the first case is

$$£x \times \frac{800}{881} \text{ or } £\frac{800x}{881}.$$

∴ The half-yearly income from this investment

$$= £\frac{1}{2} \times \frac{800}{881} \times \frac{9x}{200} \text{ or } £\frac{18x}{881}.$$

The sale-proceeds would be $£\frac{800x}{881} \times \frac{107}{100}$

or $£\frac{856x}{881}$.

∴ The total sum received by the man is

$$£\frac{18x}{881} + \frac{856x}{881} \text{ or } \frac{874x}{881}.$$

∴ The net loss is $\frac{7x}{881}$. This is by the question £700.

∴ We have $\frac{7x}{881} = 700$ or $x =$ £88,100.

(5) A man invests £461-3-0 in the 7 % at $91\frac{7}{8}$ and £553-17-$7\frac{1}{2}$ in the 5 % at $84\frac{7}{8}$. Show that he will receive the same income which he would have received had he invested the whole sum in the 6 % at $89\frac{7}{8}$. (Brokerage being $\frac{1}{8}$ in each case.)

The nominal value of the stock in the 7 % $= £\frac{9,223}{20} \times \frac{100}{92}$ or £501-5-0. ∴ Income from this investment $= £\frac{2005}{4} \times \frac{7}{100}$ or £35-1-9 a year.

The nominal value of the 5 % stock purchased $= £\frac{88621}{160} \times \frac{100}{85}$ or £651-12-6.

∴ The annual income from this investment $= £\frac{5213}{8} \times \frac{5}{100}$ or £32-11-$7\frac{1}{2}$.

∴ The joint income from the two investments = £35-1-9 + £32-11-$7\frac{1}{2}$ or £67-13-$4\frac{1}{2}$.

∴ The nominal value of the 6 % stock purchased $= £\frac{32497}{32} \times \frac{100}{90}$ or $£\frac{18045}{16}$ or £1127-16-3.

∴ The income from this 6 % stock $= £\frac{18045}{16} \times \frac{6}{100}$ or £ 67-13-$4\frac{1}{2}$.

Hence the question.

(6) Which is more profitable, to buy an estate at 24 years' purchase or to invest in the 6 % at 120?

The statement that an estate is bought at 24 years' purchase means the purchase money is 24 times the annual income from the estate. Hence for every Rs. 100 of the money paid for the estate, the estate gives an annual income of Rs. $\frac{100}{24}$ or Rs. $4\frac{1}{6}$.

∴ This kind of investment yields a rate of $4\frac{1}{6}$ % on the purchase money.

In the second case, an investment of Rs. 120 fetches an annual income of 6 %. Therefore the rate derived from this kind of investment is

$$\not{6} \times \frac{\overset{5}{\not{100}}}{\underset{\not{2}}{\not{120}}} \text{ or } 5\ \%.$$

∴ Clearly the second investment is the better of the two.

EXERCISES.

(Unless otherwise stated, brokerage is supposed to be included in the prices of stocks).

1. A person, having bought a certain amount of 3 % stock at 90, afterwards sold it, and, with the proceeds bought 5% stock He obtained Rs. 1,200 less stock than before, but his income increased by Rs. 60 per year. Determine what sum he must have originally invested and the price at which the second kind must have been purchased.

2. A man invests £832-10-0 in the 3 per cents at $89\frac{7}{8}$. His first year's income he invests in the same stock but at a different price. His next year's income is £28-10-0. What was the price of the stock when he made the second investment? (Assume brokerage of $\frac{1}{8}$ in each case.)

3. A person, who derives an annual income of Rs. 160 from the 5 per cents, sells his share at 10 premium and invests the sale-proceeds in the 7 per cents. He thus increases his annual income by Rs. 96-10-8. Determine the price of the 7 °/₀ stock at the time of his investment.

4. A person invests Rs 4,620 in the $5\frac{1}{2}$ per cents at $115\frac{1}{2}$. Soon after his investment, the price of the stock falls down to $92\frac{2}{3}$. How much more stock could he have got, if he had waited till this time? What change would this have made in his annual income?

5. With 3 °/₀ stock standing at 90, I sold out and bought railway stock at 180 which paid 10 °/₀, and my income was thus increased by £ 10; how much stock did I sell?

6. A man invests Rs. 3,600 in 3 °/₀ stock at 90. He sells out at 80 and lends $\frac{5}{8}$ of his money at 6 °/₀ and the remainder at 7°/₀. How long must the loan last, so that when he re-invests his money in 3 per cents at 90 his gain on interest may exactly equal his loss on principal?

7. Determine the change in income resulting from transferring Rs. 9,000 of stock from the $4\frac{1}{2}$ per cents at 87 to $6\frac{3}{4}$ per cents at 116.

8. A invests Rs. 3,750 in the $2\frac{3}{4}$ per cents, when they are at $62\frac{1}{2}$. What is his income? To what °/₀ on his investment does this correspond?

9. I invest equal sums in a 5 °/₀ stock, and in a $3\frac{3}{4}$ °/₀ stock and get $4\frac{7}{12}$ °/₀ for my money on the whole. The $3\frac{3}{4}$ per cents are at 75. Determine the price of the 5 per cents.

10. What sum must be invested in the 4 °/₀ stock at 85, to bring in a net income of Rs. 2,645 per year, when the income-tax is 8 pies per rupee. (Assume brokerage to be $\frac{1}{8}$.)

11. A man invests £ 1,760 in $4\frac{1}{2}$ °/₀ at 99. How much must he invest in the $5\frac{1}{2}$ °/₀ stock at 75 in order that he may get 6 °/₀ on his investment on an average?

12. A person derives an income of £ 830 derived from capital invested in 4 °/₀ stock. He sells out this stock at 119 and the proceeds he invests in a 5 °/₀ stock. What must have been the price of this stock if his present income comes to £850?

13. If money invested in the 4 °/₀ consols gives exactly 4 °/₀ after paying an income-tax of 6*d.* in the £, what is the price of the consols ?

14. Which is the better form of investment, the 6 per cents. at 135 or the $3\frac{3}{4}$ °/₀ at $86\frac{1}{4}$? What sum of money must a man have with him so that by investing $\frac{2}{5}$ in the former and $\frac{3}{5}$ in the latter his annual income may come to £975 ? What difference in income would have resulted if he had chosen the more profitable of the investments alone ?

15. If a spinning company conducts business out of money raised by a 5 °/₀ loan at 87 and realise $7\frac{1}{2}$ °/₀ on actual outlay, find whether the company has gained or lost and at what rate ?

16. I have with me a sum of Rs. 27,391. How must I divide it so that by investing one part in $3\frac{1}{2}$ °/₀ at 90 and the rest in the $7\frac{7}{8}$ per cents. at 120, I may derive the same income from both.

17. A person desirous of purchasing enough 5 °/₀ stock to yield an annual income of £ 2,500, when the stock is 90, delays till the price falls down to 85. How much has he saved ?

18. I invest a certain sum, $\frac{2}{3}$ in 4 °/₀ stock at par and the rest in a 6 °/₀ stock at 120. If I had invested them in the two stocks in the ratio 3 : 7, I would have obtained Rs. 64 more than in the former case. What sum had I to invest ?

19. A man invests £ 5,830 in a certain stock at $99\frac{1}{4}$ and an equal sum when the price has risen to 103. He subsequently sells out the whole of this stock, and obtains only £ 4 more than what he originally invested. Find the price at which he sold out reckoning $\frac{1}{8}$ as brokerage.

20. I buy Rs. 4,125 stock and after paying income-tax at 6 pies in the rupee have an income of Rs. 232-8-0. What rate °/₀ was the stock paying ?

21. I sell out $3\frac{1}{2}$ °/₀ consols at 94 and by investing the proceeds in shares which pay an annual dividend of Rs. 5 per share, raise my income by 5°/₀. Determine the price of each share to the nearest pie.

22. A man has Rs. 11,770 cash in the $4\frac{3}{4}$ per cents. at a premium of 10. If the income-tax is at the rate of 5 pies in the rupee, find his net income.

23. I sell a part of my stock at a discount of $3\frac{1}{2}$ and realise Rs. 19,300 cash which I forthwith invest in the $4\frac{1}{2}$ per cents. at a premium of 1. I thus increase my income by Rs. 131-4-0. What rate of interest was I receiving on the former stock?

24. A person invests Rs. 5,200 in $6\frac{3}{4}$ % stock at 125 and Rs. 8,750 in the $7\frac{1}{2}$ per cents. at 130. Which is the better investment? Find the average rate of interest on the whole investment.

25. The income of a Railway Company would allow a uniform dividend of 10 % if all the shares were ordinary shares. But some of the shares are preference shares entitled to a fixed rate of interest of $7\frac{1}{2}$ %. and the remaining Rs. 2,50,000 consists of ordinary shares getting 12 %. Determine the whole stock of the company.

26. A manufacturing company was started with its capital subscribed in the following way. *viz.*, 2,500 Preference shares of Rs. 100 each, carrying fixed interest at $7\frac{1}{2}$% and 5,200 ordinary shares of Rs. 75 each. In a year the gross profits of the business amounted to $3\frac{1}{2}$ lakhs of rupees. From this sum, the working expenses and other contingencies have to be met to the extent of Rs. 75,000. Determine the rate of dividend that can be declared to an ordinary share-holder.

27. The following is an extract from the prospectus of a ginning company :—

Each share will cost Rs. 100 and the amount shall be subscribed as follows :—

Payable on application	...	15 %.
On allotment ...	...	20 %.
1st July 1922 ...	...	20 %.
1st October 1922 ...	...	25 %.
1st Dec. 1922. ...	...	20 %.

At the end of the year 1922, 1,500 shares were subscribed for according to these conditions. At the end of 1923 as a result of one year's business, the company declared a dividend of 5 %. A man buys shares which gave him Rs 500. Determine his several instalments.

28. I propose to settle on my son an annuity of Rs. 250 by investing in the $3\frac{1}{2}$ °/o consols when they are at $92\frac{3}{8}$. What is the sum I have to invest, if brokerage comes to $\frac{1}{8}$?

29. I bought 30 shares of a certain company for Rs. 1,500 each, the paid-up capital for each share being Rs. 1 200. During the year I got a dividend of $6\frac{3}{4}$ °/o on the paid-up capital and a bonus of Rs. 400 on all my shares. Determine the rate of interest I get on my investment.

30. I buy 5°/o stock at 90. I receive one half-year's dividend and afterwards sell my stock at 96. I then find that I have gained Rs. 1,210 on the whole. What sum did I originally invest?

31. By buying 6 °/o stock at a certain price, I find I make $6\frac{1}{2}$ °/o on my money. I then find on paying an income-tax of 6 pies in the rupee I have a net income of Rs. 3,131 left. Find the amount of stock purchased by me and the price at which the purchase was made.

32. I have a sum of money to invest in 4 °/o Government paper. I have to pay an income-tax of 5 pies in the rupee. But my ideal is to get clear $4\frac{1}{2}$ °/o on my money. What must be the price of the Government paper if this is to be possible?

33. The capital of a company consists of 6,000 shares of Rs.50 each and there are debentures to the amount of Rs. 80,000. The profits of a certain year are Rs. 57,268-0-0. The following dues are to be met from the gross profits:—(1) An income-tax of 1 anna in the rupee. (2) The transference to next year's account of a sum not less than Rs 3,000. (3) Interest of 6 °/o (free of income-tax) is to be paid on the debentures, and (4) the remaining sum together with Rs. 2,225-4-6 carried forward from the last year's account is to be distributed as dividend on the ordinary shares so that the dividend on each share is an exact number of pence. Determine the largest dividend that can be paid on each share under these conditions. What is the exact sum that is carried forward to the next year's account?

34. On a certain day, $2\frac{3}{4}$ °/o consols were at $97\frac{1}{8}$ and $4\frac{1}{2}$ °/o debentures at 105. A man who had a certain sum to invest, calculated that, if he invested it in one of these stocks, his

income would be greater by Rs. 78 in the one case than in the other. What sum had he to invest?

35. I have on hand Rs. 17,500. I have two courses of investment open to me either in the $4\frac{1}{2}$ °/₀ stock at 105 or in a mining company paying a dividend of 2 °/₀ per annum of which the Rs. 100 shares are quoted at 55. Determine which investment will be more advantageous and by how much.

36. I am a holder of 5 °/₀ stock which yields me an income of £ 500 a year. I sell half of it at 92, and invest the proceeds in railway debentures at 46. What dividend per cent. per annum ought the latter stock to pay so that the transaction may increase my income by £ 66-13-4?

37. A speculator buys Rs. 8,000 stock at 65 and his Bank lends him money to the extent of 40 °/₀ of the actual value, charging 5 °/₀ on the loan. He holds the stock for a year, draws out the dividend of 3 °/₀, and sells out at 70. Has he gained or lost in the transaction? How much?

38. A firm of barristers instructed their brokers to purchase sufficient India 4 °/₀ stock to enable them to pay a sovereign per week from the dividends to an old servant. India stock was then at $97\frac{7}{8}$, brekerage $\frac{1}{8}$, and income-tax 1 *s.* in the pound. What was the sum put into the hands of the brokers?

39. How must a sum of Rs. 5,750 be divided into two parts so that by investing one part in the 8 °/₀ stock at 95, and the other part in the 5 °/₀ stock at 90, an average of 7 °/₀ on the investment may be made?

40. I instruct my broker to buy the following stocks at the respective rates and with the given brokerage. Calculate what sum is to be placed in the hands of my broker to enable him to do so. Also determine what exactly the broker gets for his services and what annual income I derive from all these investments.

(1) £ 1 750 India 4 °/₀ stock at 93 [Brokerage $\frac{1}{8}$].

(2) £ 2,500 Canada 5 °/₀ stock at $95\frac{1}{2}$ [Brokerage, $\frac{1}{4}$].

(3) £ 3,225 Mexican 6 °/₀ Railway stock at 95 [Brokerage $\frac{1}{2}$].

(4) £ 250 Debentures ($3\frac{1}{2}$ °/₀) at par (no Brokerage).

(5) £ 1,000 Government 4 % consols at a premium of 20 (no Brokerage).

106. GRAPHS APPLIED TO STOCKS.

The fluctuations in the market-value of a stock can be represented graphically. The graphs in these cases serve the purpose of visualisation in making a forcible appeal to our eyes and bringing home to our minds the changes in the prices. The question arises whether the plotted points are to be joined by broken straight lines or by a smooth curve or whether the plotted points are to be left isolate. The answer to the question is suggested when we analyse the value of the fluctuations themselves. If the changes in the market-value given for plotting are chosen at such intervals of time as make it possible for intermediate values of the kind plotted to exist between these consecutive intervals of time, then we can be justified in passing a free curve through the plotted points. If on the other hand, say, we plot the changes in the price per hour and if the price does not change between the end of one hour and that of the next, we have to leave the plotted points isolate as no intermediate variation can be thought of. Sometimes in the cases of second kind of problems, recourse is had to broken straight lines, and this, though mathematically inaccurate, helps us to see at a glance the nature of the fluctuations.

Example.

The following table gives the price of the 5 % stock during the year 1920 :—

Months.	Jan.	Feb.	Mar	Apl.	May	June	July	Aug	Sep.	Oct.	Nov	Dec.
Price.	97½	95	93	91	89	92	94	96	98	99	100	105

Exhibit the relation graphically. Assuming that during the respective months, intermediate variations are possible, draw a smooth curve through the plotted points.

The graph to be drawn is to connect the time of the year with the corresponding prices of the stock. Hence plot along the horizontal axis months taking $\frac{1}{2}''$ to represent 1 month. Along the vertical axis show the prices. In choosing a scale for the prices, we note that the price ranges from 89 to 105. Hence the origin can be conveniently made to represent 89. Then this arrangement helps us to draw the figure on a large scale of $1'' = 4$. Plot the corresponding pairs of values and join them by a smooth curve as shown in the fig 196. H. S. $1'' = 1$ month. V. S. $1'' = 4$ units.

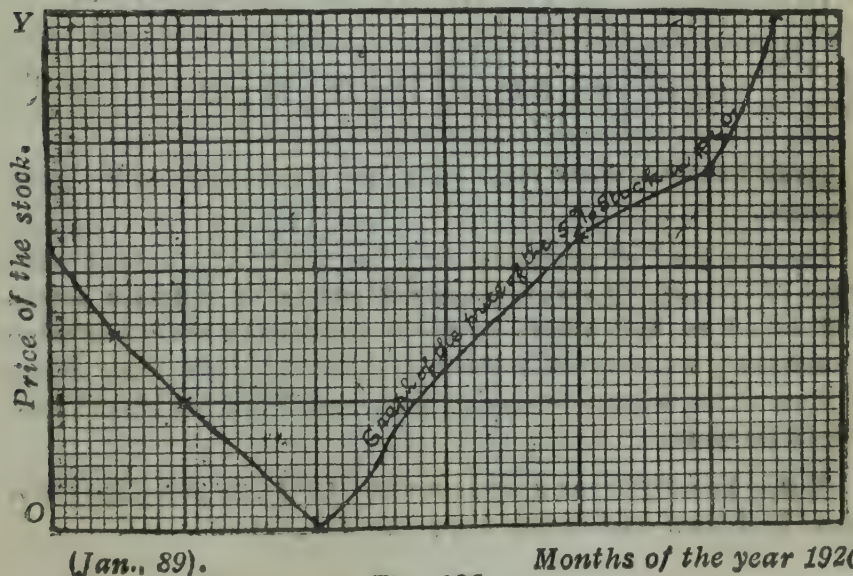

Fig. 196.

EXERCISES.

1. Represent the following fluctuations graphically and assuming that intermediate variations are admissible, draw a smooth curve:—

	Year.	Average Price.	Year.	Average Price.
	1910	91	1918	97
	1911	93	1919	101
	1912	94	1920	103
6 °/₀ Debentures.	1913	89	1921	100
	1914	92	1922	98
	1915	98	1923	95
	1916	100	1924	92
	1917	95		

2. The highest and lowest prices of the consols for the years 1895 to 1904 were as shown in the following table :—

Year	1895	'96	'97	'98	'99	1900	'01	'02	'03	'04
Highest	108	114	114	113	112	103	98	98	94	91
Lo west	104	105	111	107	98	97	91	93	87	85

Make a chart to show these variations graphically on the same diagram.

Another kind of problems may be graphically solved with great advantage. If we are given the prices of different stocks and asked to find which of them will be the most profitable investment, we can easily draw graphs and from their slopes at mere sight, we can determine the comparative benefits of such advertisements.

N.B.—The different graphs for the purposes of comparison must be drawn on one and the same diagram.

Examples.

(1) Determine graphically which of the following is the best form of investment :—

4 % Stock at 80.
7 % Debentures at 150.
3½ % Consols at par
and
6 % Government Paper at 105.

Here the graphs are to connect the prices and the rates. The rate is in each case directly proportional to the sums invested and hence the graphs are straight lines and since the zero sum in each case corresponds to zero interest, the graphs are all straight lines passing through the origin. Let us now represent prices horizontally on the scale 1″ = 30 and interest vertically on the scale 1″ = 2%. [Fig. 197.]

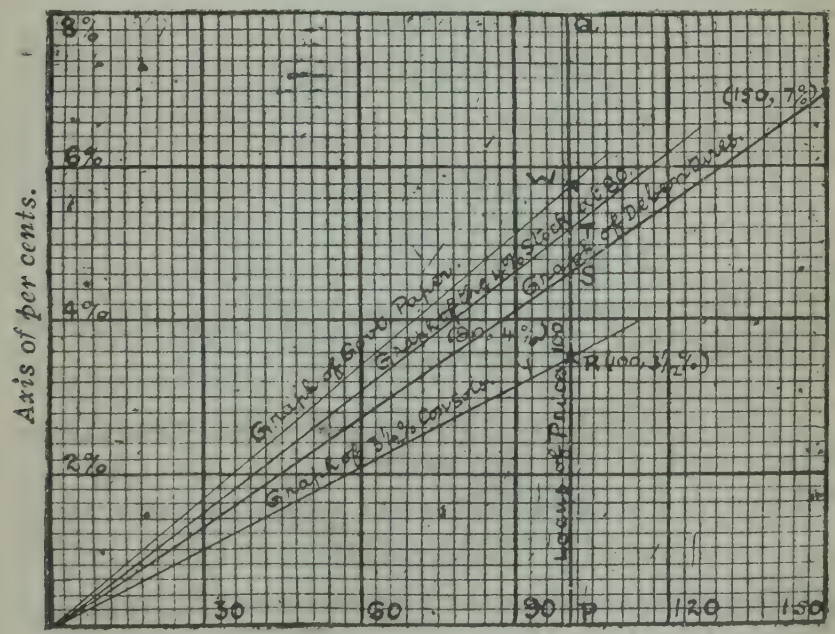

Fig. 197.

We readily see from the graph from the inclinations of the different straight lines, that the 6 % Government paper is the most profitable investment. If we require the actual rates arising from each of these investments we make the graphs cut PQ the locus of all points showing prices of 100. The points of intersection are R, S, T and W in the figure. The point R tells us that the rate of the investment in the Consols is $3\frac{1}{2}$ %; S tells us the rate from the investment in the Debentures is 4·7 % nearly; T shows that the similar rate from the 4 % Stock is 5 % and from W we see that the Government paper gives a rate of 5·7 % nearly. Hence from the point of view of the actual rate an investor can get from these, they can be arranged in the following descending order :—

6 % Government Paper,
4 % Stock,
7 % Debentures
and
$3\frac{1}{2}$ % Consols.

(2) Find graphically what stock at 120 and 80 will fetch the same rate of interest as the 3 % stock at 90

Plot the straight line through the origin representing the graph of the 3 % stock at 90. To find the resulting rate of interest, let the graph cut the locus of prices 100 at M. Produce OM to meet the locus of prices at W. Then the ordinate of W gives the nature of the stock at 120 giving the same rate as the 3 % stock at 90.

From the fig. the ordinate of W is 4 %.

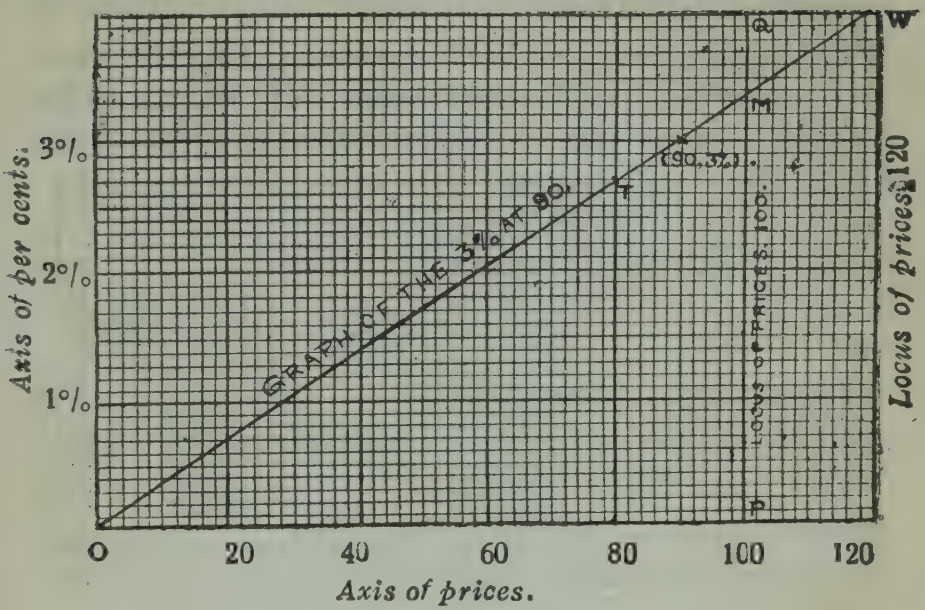

Fig. 198.

OM similarly cuts the locus of prices at 80 at T and the ordinate of T, *viz.* $2\frac{2}{3}$ per cents. (from the fig.) gives the nature of that stock which at 80 yields the same rate of interest as the 3 % Stock at 90.

(3) Find graphically what sum I must invest in the 5 % at 95 to get an annual income of Rs. 28.

Here the things to be connected are the sum invested along one axis and the corresponding income along the other axis. Since the income earned is directly proportional to the sum invested, the graphical representation of the relation is a st. line Since zero investment means zero income, this st. line passes through the origin. Hence to draw the graph, join the origin to the point representing Rs. 95 and

5 (for, according to the condition of the question, an investment of Rs. 95 corresponds to an income of Rs. 5.) To read the required answer, find the reading corresponding to an income of Rs. 28. The required sum is from the figure found to be Rs. 562.

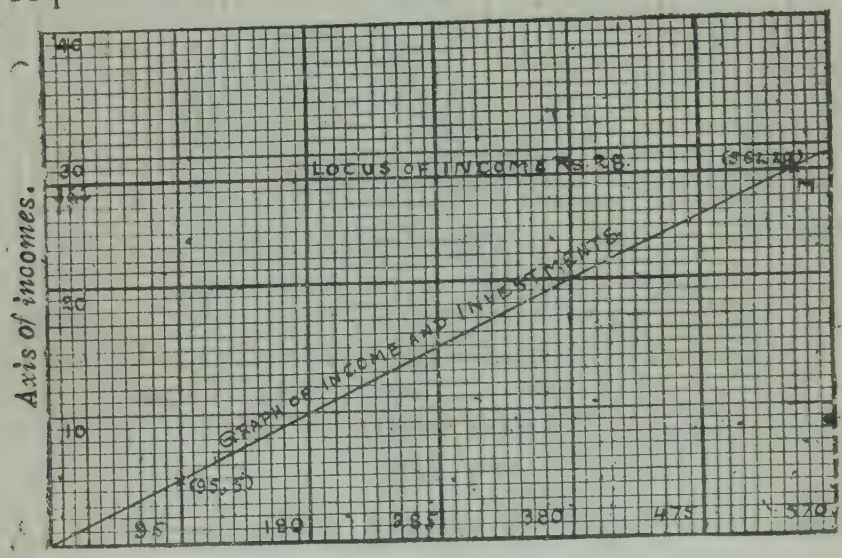

Fig. 199.

(4) Find graphically how a sum of Rs. 100 is to be divided between the 4 per cents. at 90 and the 6 % at 110 so that the average rate may be 5 %.

Draw thro' the origin as in Example (1) the graphs of the 4 % at 90 and the 6 % Stock at 110. (See the fig.) Then the two graphs are to be combined so as to give 5° /₀ on the sum of 100 to be invested. Take a point P representing the given conditions, *viz.*, 100 and 5 %. Thro' P draw a ‖ to one of the graphs to meet the other in Q. Then the point Q gives the required answer. From the fig. we see that Rs. 45 must

be invested in the 4 °/₀ and the remaining Rs. 55 in the 6 %.

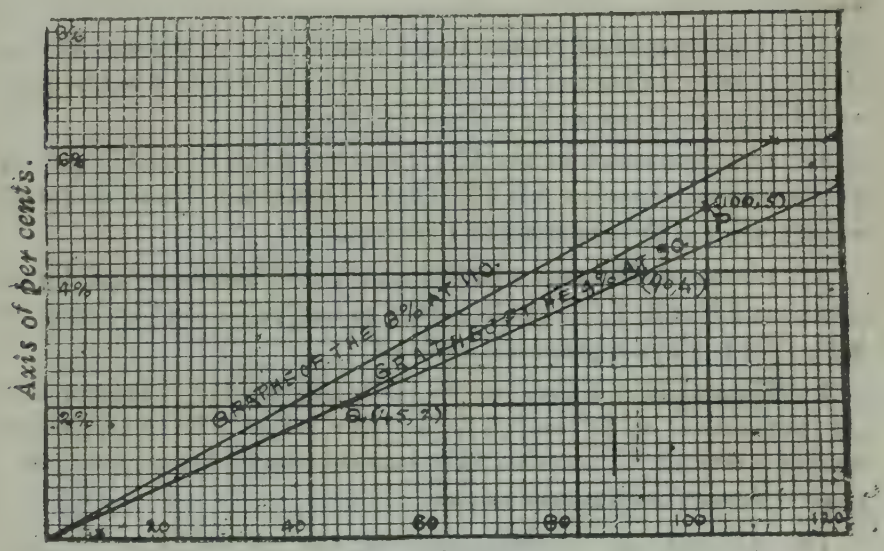

Fig. 200.

EXERCISES.

Determine graphically which of the following is the best form of investment :—

1. 6 °/₀ stock at 120
   9 °/₀ stock at 150
   and
   3½ °/₀ stock at 80.

2. 5 °/₀ stock at a discount of 10
   8 °/₀ consols at a premium of 20
   and
   6 °/₀ debentures at par.

3. Rs. 5 dividend on shares of Rs. 40 each
   Rs. 7-8-0 dividend on shares of 60 each
   and
   7 °/₀ stock at 70.

4. Determine graphically what stocks at 90 and 115 will fetch the same rate of interest as the 4½ per cents. at 105.

5. Find by plotting what stocks at 75 and 130 will fetch the same rate of interest as a dividend Rs. 3-12-0 on shares of Rs. 60 each.

6. Find graphically what sum I invest in 6 °/₀ debentures at 110 to derive an annual income of (1) Rs. 29 and (2) what income will be derived from an investment of Rs. 275.

7. I hold as much of the $5\frac{3}{4}$ °/₀ stock at 85 as to enable me to get an annual income of £25-17-6. Determine from a graph what I can get by disposing of the stock.

8. How should a sum of £500 be divided between the 5 per cents. at 75 and 7 °/₀ stock at 120 to get an annual income of £ 31-13-4? Give a graphical solution.

9. How should a sum of Rs. 100 be divided between the $4\frac{1}{2}$ per cents. at 80 and 6 °/₀ debentures at 120 to derive an average rate of $5\frac{1}{4}$ on the two investments? Determine your answer from your squared paper.

10. I invest my capital partly in the $6\frac{1}{2}$ consols at 95 and partly in the 5 per cents. at 85. On the whole I find that I get on an average $6\frac{1}{2}$ °/₀ for my money. Find from a graph the ratio of the sums invested in the two cases.

CHAPTER XXIV.

TIME AND DISTANCE.

107. SPEED OR VELOCITY.

The speed of a moving particle is measured, when it moves uniformly, by the distance traversed in a unit of time. When we say that a variable speed *at any moment* is s miles per hour, we mean that, if the body continued at that rate throughout one hour, it would describe a distance of s miles Practically no journey is done at a uniform speed. Taking the familiar case of a train, its rate is small when it leaves a station, some time later it reaches its high speed and then reduces again its speed at the approach of the next station. A man similarly walks during the first part of a journey at a fast rate, then his rate slows down as a function of exhaustion and again reaches its normal value after sufficient rest. But in all elementary numerical work, calculation is made on the assumption of uniformity of motion.

108. RELATION BETWEEN TIME AND DISTANCE.

Time varies directly with distance, *i.e.*, the longer the distance to travel, the longer the time for a given rate. Similarly at a given rate, greater the time of motion, the more the distance travelled will be. Hence this sort of relationship between the time and the distance travelled, supposing the rate of motion to be uniform and constant, has for its graphical representation a straight line.

109. RELATION BETWEEN RATE, TIME AND DISTANCE.

Now supposing the distance travelled to be fixed, if the rate is decreased, time is correspondingly increased. Hence for a given distance rate varies inversely with time and similarly time varies inversely with rate.

110. RELATIVE VELOCITY.

The rate of approach or separation of two moving bodies is called the relative velocity of either with respect to the other, and must be carefully distinguished from the *actual velocity* either of them has. *This relative velocity is the sum of the two rates when the two bodies are moving in opposite directions and is the difference of the two rates when the two bodies are in the same direction.* The word *overtake* is used to denote two moving bodies coming together when moving in the same direction and the word *meet* is used in a similar sense when the motion is in opposite directions.

111. MOTION IN A CIRCLE.

This kind of motion differs from motion in a straight line in the persons coming periodically to the starting point at the end of every circuit. Here also motion may be in the same direction or in opposite directions. When two persons move round a circle starting from the same point on the circle and in the same direction, the faster gains over the slower and they will be together again at the starting point when he gains an exact number of rounds over the other. When this time is to be the least, *i.e.*, when we are asked

to find when they will *next be together at the starting point*, we have to determine when the faster *gains only one round* over the slower. When under similar circumstances, motion is in opposite directions, when they meet, they together traverse the length of the whole circuit.

Note.—When a speed in feet per second is to be converted into miles per hour, make use of the relation that a speed of 44 ft. per second is 30 miles per hour.

Examples.

(1) A boat goes up-stream at 3 miles an hour and down-stream at 4 miles an hour. I travelled in the boat from P to Q up-stream, halted at Q for 4 hours and then returned to P. I took 18 hours in all. Find the distance between P and Q.

The total time spent in travelling is 14 hours. The time spent in travelling up-stream is $\frac{4}{7} \times 14$ or 8 hrs. and the remaining time of 6 hrs. is spent in the downward journey. ∴ The distance between P and Q is 24 miles (8 × 3).

(2) A train starts from A to B, 72 miles distant, its proper speed is 20 miles an hour, but after going 24 miles, it meets with an accident which delays it for 10 min. and with its speed reduced it reaches B 58 min. behind time. Determine the reduced speed of the train.

The normal time over the journey of 48 miles of the latter part is 2 hrs. 24 min. But the extra time taken for travelling alone due to the reduced speed is 58 — 10) or 48 min. ∴ The time taken for the latter

part of the journey due to the reduced speed is 3 hrs. 12 min. Hence the reduced speed is $\frac{3}{\cancel{48}} \times \frac{5}{\cancel{16}}$ or 15 miles per hour.

(3) A straight railway runs across an open plain. A man on foot finds from his map that the nearest station is 7 miles from the nearest point on the railway. To reach the station he will have to walk 3 miles farther than to reach the railway at the nearest point. How far is he from the station?

Let P be the position of the man, Q the nearest

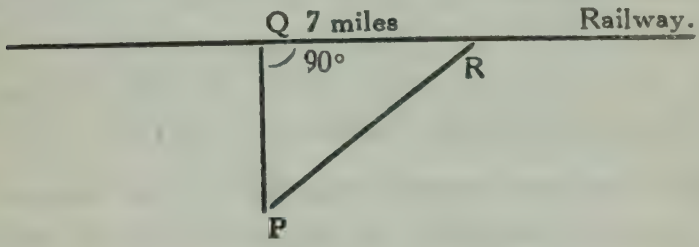

Fig. 201.

point on the railway and R the nearest station on the railway. Then by the question, QR = 7 miles. Let PQ be x miles. Then $(x + 3)$ is the distance he has to walk to reach the nearest station on the railway. Then from the rt.-$\angle$ d. $\triangle$ PQR we have $(x + 3)^2 = x^2 + 49$.

$\therefore\ x^2 + 6x + 9 = x^2 + 49.\quad \therefore\ x = \frac{40}{6}$ or $6\frac{2}{3}$ miles.

Hence PR the distance required is $\sqrt{(\frac{20}{3})^2 + 49}$ or $\sqrt{\frac{400}{9} + 49}$ or $9\frac{2}{3}$ miles.

(4) Two friends walking at 4 and 5 miles an hour respectively start at 6 a. m. from two towns 45 miles

apart. At what time will they meet and how far will each of them have travelled?

The rate of approach is (4 + 5) or 9 miles an hour. They will meet at a time when they have together travelled a distance of 45 miles. Hence the time required is $\frac{45}{9}$ or 5 hrs. after starting. ∴ The time of meeting is 11 a. m. They would then have travelled 20 and 25 miles respectively.

(5) If I walk from my home to my office at the rate of 3 miles an hour, I am 20 min. late. If I run at 5 miles per hour, I arrive 15 min. before time. Find the distance between my house and office.

The rate in the second case is 5/3 of the rate in the first case. Hence the time in the second case must be 3/5 of the first time, *i.e.* 2/5 of the first time less. This is according to the question 35 min. (the difference of time in the two arrivals.) Hence the first time is $\frac{35 \times 5}{2}$ min. Since the rate for that time is 3 miles per hour, the distance required is $\frac{35 \times \cancel{3}}{2} \times \frac{\cancel{3}}{\cancel{3}\cancel{4}}$ or $4\frac{3}{8}$ miles.

(6) Three cyclists *A*, *B* and *C*, ride round a circular track at 8, 9 and 10 miles per hour respectively. They start together from the same point, but *C* rides in an opposite direction to *A* and *B* and meets *A* one minute after passing *B*. What is the length of the track?

Let us suppose that *C* meets *B* in x min. and consequently *A* in $(x+1)$ min. The sum of the distances

travelled by C and B in x min. must be the length of a circuit. Similarly C and A together in $(x + 1)$ min. must have travelled once round the track. Hence equating these two values, we have $1672\,x = 1584\,(x + 1)$.

Whence $x = 18$ $\therefore$ the length of the track $=$ 18×1672 ft. or 6×1672 yds. or 5 miles 1232 yds.

EXERCISES.

1. Rama walks p miles in q hours. Krishna walks 2 miles per hour less than Rama. How far does Krishna walk in $(q + 5)$ hours?

2. If I go to a place M from a place N via a third place R, I have to travel 520 miles in all. R is 4 times as far from N as it is from M. What is the distance from N to R as the crow flies?

3. I drive towards a village which I was told was 56 miles away, at 8 miles an hour with the intention of reaching it in 7 hours. After going a miles, I learn that the journey is 12 miles more than I was originally informed. Give a formula for the amount by which I must increase my speed so as to arrive at my destination as I had originally intended. Evaluate your result when $a = 22$.

4. Two towns are connected by two railways, one route being 10 miles longer than the other. A train on the first railway averaging 40 miles an hour takes 10 min. longer to the journey than a train on the second railway averaging 30 miles an hour. What are the lengths of the two routes?

5. The journey between 2 towns by one route consists of 215 miles by rail followed by 126 miles by sea; by another route it consists of 387 miles by rail followed by 36 miles by sea. If the time occupied on the journey by the second route is $\frac{4}{5}$ of the time occupied by the first route, find the average speed by rail, assuming it to be the same by each route and 25 miles an hour faster than the average speed by sea.

6. On my home, I go by train to a junction, and either walk 3 miles home, or wait for 15 min. travel 10 min. by train and then

walk a mile. I get home at the same time by the two methods. At what speed do I walk?

7. A train after travelling 1 hr. is detained 45 min after which it proceeds at 3/4 of its former rate and arrives 24 min. late. If the detention had taken place 5 miles farther on, the train would have arrived 3 min. sooner than it did. Find the original rate of the train and the distance travelled.

8. A cyclist travelling with uniform velocity along a straight level road on a foggy day, when objects were just visible a furlong off sees a mile stone in front of him for ½ a min. Afterwards, he overtook a carriage, the back of which he saw for a min. and a half. With what velocity (supposed uniform) was the carriage going?

9. A vessel was ordered to steam from P to Q 120 miles away in 6 hours. A break-down in her engines occurred on the way, this delayed her 45 min. and then she was able to proceed at only 3/4 of her former rate. If she reached Q 1½ hours late, where did the breakdown occur?

10. Two trains going in opposite directions meet at 1 p. m. when each is punctual. One day when the first train is 15 min. late and the second 5 min. late they meet at 9 min. past one. If the speed of the first train were 40 miles an hour, determine the speed of the second. Assume that the trains are travelling at their normal speeds.

11. Two boys Parthasarathy and Srinivasan start to run from school to a shop 650 yds. away, and back. Srinivasan returning meets Parthasarathy 50 yds. from the shop and gets back to school two minutes before him, What was the time taken by Parthasarathy?

12. I wish to catch a train 10 miles away and have one hour to do it. I can cycle at 15 miles an hour but at that speed my cycle is likely to break down, in which case, I will have to walk with the machine at 3 miles an hour. What is the least distance I must cycle before I can feel that I will not miss the train?

13. A balloon ascends from a point P on the seashore and is carried by the breeze until it stops perfectly still, vertically above a lighthouse L, when the balloon is found to be exactly 2·5 miles

above the sea-level. The flash of a gun fired at P is noticed in the balloon and the report is heard exactly 21 seconds later. If the speed of sound be taken as 1056 ft. per sec., determine the distance between P and L measured on the ground. Assume that light travels with infinite velocity.

14. A traveller walks a certain distance. If he had gone $2\frac{1}{2}$ miles an hour faster, he would have walked the distance in $\frac{8}{9}$ of the time; if he had gone $1\frac{1}{2}$ miles an hour slower, he would have been 2 hrs. longer on the road. Find the distance travelled.

15. Two men A and B can walk at 4 and 5 miles per hour and cycle 10 and 12 miles an hour respectively. Having only one cycle between them, they have to travel 75 miles, starting and arriving simultaneously. They agree that A shall ride first, then dismount, leave the cycle and finish the journey on foot, when B reaches the cycle he will rest 2 hours and then complete the journey on the machine, Find the time taken on the journey.

16. A train runs from P to Q a distance of 160 miles. There are 6 intermediate stations, at each of which the train stops for $7\frac{1}{2}$ min. The time allowed for the journey is the time that it would have taken if the train ran through at 30 miles an hour without stoppages. The train leaves the last station 10 min. late. If this station is 32 miles from Q at what rate must the train run this distance if it were to arrive at Q in time ?

17. In going the shortest way from A to B, a man had to go back one mile to pick up something he had dropped, and took $3\frac{1}{2}$ hours over the walk. He went back by a route which was $\frac{1}{2}$ a mile longer and took 3 hours over the return walk. Find the rate of walking and the shortest distance from A to B.

18. The diameter of each front wheel of a carriage is x ft., while that of each back wheel is larger by 1 ft. 3″. How far does the carriage advance for each revolution of the front wheels? The carriage on a journey passes through three places A, B and C. Between A and B the front wheels make 600 complete revolutions, and between A and C the back wheels make 600 complete revolutions. What is the distance between B and C?

19. A sailing boat starting from a certain fixed position sailed a certain distance North, then half as far again due South. After

going about and sailing North again for 10 miles, she reached 12 miles South of her original starting point. How far did she sail North in the first case?

20. Two motorists driving uniformly but at different rates start at 12 noon, one from to A to B and back and the other from B to A and back They meet at 4 p. m. for the second time at a point 20 miles from A. If B is 60 miles from A, find when and where they meet for the first time.

21. Two towns P and Q are connected by three different lines of railway of the same length. Three trains leaving P at 12, 12-10, and 12-20 respectively and travelling uniformly are observed to pass a signal-box at Q exactly abreast of each other at 1 o'clock. If at 12-30 the sum of the distances travelled by the three trains be 42 miles, calculate the distance between P and the signal box at Q.

22. Two men run in opposite directions round a circular track at an equal and uniform speed and complete the circle each in 2 hours. 15 min. After the meeting, one man finds that his average speed from that meeting to the next meeting is $\frac{2}{3}$ of what it was before. When and where will they next meet?

23. The railway between two towns A and B is 64 miles long. The 7-30 a.m express from A arrives at 10-24 a m.; and the 6-45 a.m. passenger from B arrives at A at 11-45 a m. Assuming that the rate of each train is uniform, determine when and where they meet.

24. Three places, Egmore, Pallavaram and Chingleput are passed by a train at 8—30, 9—6 and 9—45 respectively. Another train passes Egmore and Pallavaram at 7—40 and 8—50 respectively. If both the trains travel uniformly find when the second train reaches Chingleput. If the rate of the second train can be assumed to be 12 miles an hour. what is the rate of the first?

25. A B is a straight railway and C a place 7 miles from the nearest point B of the railway. Suppose goods to be sent from A to a siding s by rail and thence by road to C, s being x miles from B. The cost by rail is 8 as. per mile and by road Re. 1-0-0 per mile. The distance AB is 16 miles. Find an expression for the total cost of carriage. Give a graphical representation of this

relation. Determine from your graph where the sidings must be located in order to keep the cost as low as possible.

26. A and B start together to go to the top of the mountain at Thirupathi. A could reach the summit 30 min. before B. but missing his way goes a mile and back needlessly, during which he walks at twice his usual pace. He does the rest of the walk at his ordinary rate and reaches the summit only 5 min before B. C. starting 24 min. after A and B and walking at 2 miles per hour, reaches the summit 19 min. after B. Determine A's and B's rates of walking and the distance from the foot to the summit of the mountain.

27. Two man walking from P to Q and Q to P respectively and starting at the same time meet in 9 hours at a point $35\frac{1}{3}$ miles from P, and are 20 miles apart in 5 hours. Determine the distance P Q and the rates of the two men.

28. I set out from Triplicane to walk to Saidapet at 4 miles an hour. Half-an-hour later, my nephew set out from Saidapet to meet me at 3 miles an hour. When and where will we meet if the distance between Triplicane and Saidapet be 7 miles ?

29. On a certain journey, I walked at 4 miles an hour, until the number of miles left to be walked was 5 less than the number of hours I had been walking. Then I walked the rest of the way at 3 miles an hour. If the whole journey occupied 12 hours 36 min. find the length of the journey.

30. A can round a circular path 4 times while B can run round another circular path 6 times. Their speeds are as 5 : 3. Compare the lengths of the courses. If A by increasing his speed runs round his path 7 times while B with his unaltered speed runs round his 7 times, determine by how much °/₀ of the original speed, A has now increased his speed.

31. Two travellers start towards each other from 2 places, one walking 2 miles an hour faster than the other. They meet 3 miles from the middle of the distance and find that the time of the faster over the whole distance would be $4\frac{1}{2}$ hours. Find the distance.

32. At 8 p. m. the Ceylon Boat-mail leaves Egmore for Chingleput and travels at the rate of 27 miles per hour. At

8-30 p.m. a passenger train left Chingleput for Egmore at 21 miles per hour. If they meet exactly midway between Egmore and Chingleput, determine the distance between Egmore and Chingleput.

33. A, B and C walk at the rates of 4, 5, 6 miles an hour respectively and they begin to walk from the same place along the same road at 3, 4, and 5 p. m. respectively. When B overtakes A, A sends him back with a message to C. Determine the time when C gets the message.

34. Though I ride from P to Q and back again at uniform speed, I seem to do the journey from P to Q at 8 miles an hour and the return journey at 6 miles an hour, because the clock at Q is 20 min. slower than the clock at P. Determine my actual rate and the length of my journey.

35. A person walking along a road in a fog meets one waggon and overtakes another which is travelling at the same rate as the former. He observes that between his first seeing and passing the waggons, he walks 15 yds. and 45 yds. respectively. Determine how far he can see in the fog, and compare his rate of walking with the rate of each waggon.

36. Four people wish to be conveyed as quickly as possible to a place 5 miles distant. They can walk 4 miles an hour, and have a motor car that can go 18 miles an hour but holds only two besides the driver. Two set off on foot. The car takes the other two, a certain distance, sets them down to continue on foot, and then returns to bring the other two. At what point must the car set down those it took first in order that the four may arrive at the same instant? Find also the time taken to do the journey.

37. Three motor cycles A, B and C leave Madras for Chingleput. B starts 30 min. later than A and travels 3 miles an hour faster. C starts 45 min. later than A and travels 5 miles an hour faster. If they all reach Chingleput at the same time, find their respective speeds as well as the distance between Chingleput and Madras along the road they travelled.

38. A passenger steamer left Colombo for Bombay and maintained a uniform speed for 36 hrs when her engines broke down,

causing a delay of 4 hours After temporary repairs, she proceeded on her journey but at $3\frac{1}{2}$ miles per hour less than before, and hence arrived at Bombay 25 hrs. late. If she had travelled the whole distance at the reduced rate, without any accident, she would have arrived 3 hours later than she did. Find the distance between Colombo and Bombay.

39. I leave home at half-past two and walk to Town which I reach when the clock on the Ripon Buildings indicates a quarter past three. I stay 25 min. in Town and return home by a route $\frac{1}{4}$ as long again as the route by which I went to Town, at a rate twice as fast as I walked and reach home at 4 o'clock. How much was the Ripon Buildings clock wrong?

40. A marksman firing at a target 800 yds. away hears the bullet strike the target 7·2 seconds after he fires. A by-stander 600 yds. from the target and 500 yds. from the marksman hears the bullet strike the target 5·1 seconds after hearing the report. Calculate the velocity of sound. (Assume x ft. per second to be the velocity of firing and y ft. per sec. the velocity of sound and proceed.)

41. A marksman firing at a target 500 yds. away hears the bullet strike the target 5·25 seconds after he fires. A by-stander equidistant from the target and the marksman hears the bullet strike the target 3·75 seconds after he hears the report. Calculate the velocity of sound. Show that if the by-stander moves so as always to be equidistant from the target and the marksman, the interval of time between his hearing the report and the target struck will be constant whatever may be his distance from the target,

112. PROBLEMS ON UP AND DOWN STREAMS.

Here we have to take into account the rate of the stream. When we *go up or work against the stream*, the current is to our disadvantage and hence hinders us and retards our motion. Thus the net rate up stream is the difference between the rate in still water and the rate of the stream. It is therefore clear that

only when the rate of the man in still water is greater than the rate of the stream itself, he can ever think of going up, otherwise the current gets the upper hand and always puts him back. Similarly when we go down stream we are carried by the current, the current here is to our advantage and our motion is accelerated. Hence the *net resulting rate* down-stream is equal to the sum of the rate in still water and the rate of the current. Thus it is clear that different times will be taken to go up a certain reach in a river and come down the same distance. In this respect therefore motion in rivers and streams where we have a flowing current differs from motion in still water

EXERCISES.

1. A boat goes up-stream three miles an hour and down stream at 4 miles an hour. I travelled in the boat from P to Q up stream, halted at Q for 4 hours and then returned in the boat to P. I took 18 hours in all. Find the distance between P and Q.

Sol. The total time taken for travelling alone is (18—4) or 14 hours. Of this the time for the upward journey : time for the down journey :: 4 : 3. (inversely as the rates.) Hence the time for the upward journey is $\frac{4}{7} \times 14$ or 8 hours. Hence the distance between P and Q is 8×3 or 24 miles.

2. An aeroplane has to bomb fort 220 miles due north. The wind is due south. The speed of the aeroplane is 75 miles an hour in still air. The aeroplane returns in 6 hours. Determine the velocity of the wind.

Let x miles per hour be the velocity of the wind. Then the net rate when going north is $(75-x)$ miles per hour and when going south the net rate comes to $(75+x)$ miles per hour. Hence by the Question we have, $\frac{220}{75-x} + \frac{220}{75+x} = 6$.

3. A man rows against a stream flowing at 4 miles an hour to a certain point and then turns back stopping $1\frac{3}{4}$ miles short of the place whence he originally started. If the whole time occupied in rowing be 4 hrs. 15 min and if his uniform speed in still water be 7 miles an hour, determine how far upstream he went.

4. A man rows to a place $16\frac{1}{2}$ miles distant down a river and back in 6 hours 40 min. If the rate of the river had been twice as great, he would have taken 6 hrs, $52\frac{1}{2}$ min. Find the rates of the river and the man.

5. A man rowing against a stream, meets a log of wood, which is being carried down by the current. He continues rowing in the same direction for a quarter of an hour longer and then turns and rows down the stream overtaking the log 1·5 miles lower down than the point where he first met it. Find the rate of the stream. What peculiarity do you note about the rate of the man?

6. A steamer goes p miles up stream in q hours, and then c miles down in d hours. The stream flows e miles an hour and the boat could go 't' miles an hour in still water. Express p and c in terms of q, d, e, and t. if $p = 28$, $q = 4$, $c = 28$, $d = 1\frac{1}{2}$, what are e and t?

7. A man rows down stream a certain distance and returns. His rate in still water is r miles an hour, Compare in a ratio form the time he takes to what he would have taken if there had been no current. Is your answer dependant on the distance travelled? Give a reason.

8. On a stream, P lies between and equidistant from Q and R. A man can swim from P to Q and back in 3 hrs. 20 min and from Q to R in 5 hrs. 45 min. How long would he take to swim from R to Q?

113. PROBLEMS ON RACES.

Two or more competitors are said to start level when they start together. When one is given some advance over the others either in time or in distance he is said

to have a start. When all the competitors reach the goal or destination at the same instant, then the race is said to be a dead-heat. If one competitor reaches the goal before the others, he is the winner in the race and he is said to beat his rivals by a certain distance or by a certain time, the distance being the balance of the distance which the other competitors have yet before them to complete, and the time being the extra time required by them severally to complete the racing distance.

Examples.

(1) A, B and C run a race which A wins, beating B by 10 yards and C by 21 yards. Afterwards B and C run over the same course and B beats C by 12 yds. What is the length of the course?

Sol:—Let the length of the course be x yds. Then A runs x yds., B, $(x-10)$ yds. and C, $(x-24)$ yds. in the same time. Hence $\frac{\text{B's rate}}{\text{C's rate}} = \frac{x-10}{x-21}$. Hence while B completes his x yds., C would have run only $\frac{(x-21)x}{(x-10)}$ yds., which is by the question $(x-12)$ yds., for he is beaten by 12 yds. Hence we have the equation $\frac{x(x-21)}{(x-10)} = (x-12)$ whence determine x.

(2) Govind can give Gopal 15 yds. start in a hundred yards' race. The sum of the times taken by both of them for a quarter mile is $5\frac{1}{2}$ min. Determine Govind's time over the race.

Sol.—Govind runs 100 yds. while Gopal runs only 85 yds. in the same time. Hence Govind's time : Gopal's time for the same distance : : 85 : 100 or 17 : 20.

Since their joint time for a quarter mile is $6\frac{1}{2}$ min., Govind's time for the quarter mile is $\frac{17}{37} \times \frac{11}{2}$ min. Hence his time over the race is what he would have taken at this rate to run 100 yds. This is $\frac{\overset{5}{\cancel{100}}}{\underset{4}{\cancel{440}}} \times \frac{17}{37} \times \frac{\cancel{11}}{\cancel{2}}$ or $\frac{85}{148}$ min., or $\frac{85}{\underset{37}{\cancel{148}}} \times \frac{15}{\cancel{60}}$ or 34·5 seconds only.

EXERCISES.

1. A and B ride a race of 32 miles on cycles. The driving-wheel of A's machine makes 4200 revolutions per hour, and has a circumference of 158 inches; that of B makes 4,325 revolutions per hour and hes a circumference of 150 inches; which will win and by how much?

2. A hare sees a hound 220 yds away from her, and runs off in the opposite direction at a speed of 15 miles an hour; 36 seconds later, the hound perceives her and gives chase at a speed of 20 miles an hour. How soon will he overtake the hare and at what distance from the spot whence the hare took flight?

3. Two boats start to row a race at 4 o'clock. The race is over at 10 min. past 4, the losing boat being 60 yds. behind at the end. At 6 min. past 4, this boat was 575 yds. from the winning post. Determine the racing distance and the speed of each boat in miles per hour.

4. A can just give B a start of 40 yards and C a start of 54 yards in a walking contest of half a mile. How much can B give C in a quarter mile contest?

5. A and B run a race; B has 60 yds. start, but in the time A runs 25 yds., B runs 20 yards, determine the length of the course if A comes 5 yards ahead of B.

6. In a cycle race Rama goes a ft. per second and Krishna goes $\frac{3a}{7}$ miles per hour. After 5 min. Rama is 110 yards ahead of Krishna. What is the value of a?

7. Rama and Govind start to run a race to a certain post and back again. Rama returning meets Govind at 100 yds. from the post, and arrives at the starting point 2 min. before him. If he had immediately run again to meet Govind, he would have met him when Govind had still to run $\frac{1}{8}$ of the distance between the post and the starting place. Find the length of the race and the time occupied in it.

8. A and B start to run a race. When A has run p ft., B is q yds. behind, B then increases his pace by 5 : 3. The race lasts 8 min. longer and results in a dead heat. Find the distance and the rates of running.

9. In a walking race, A gives B a start of $\frac{1}{8}$ a mile. After A had walked for 45 min., he overtakes B but 20 min. later he meets with an accident which detains him 20 min. On resuming the race, A walks for 1 hour 15 min. and again overtakes B one mile from the winning post. Find the length of the course.

10. I take 9 steps while my friend takes 11, but 5 of my steps are equal in length to 7 of my friend's. Who is the quicker walker and who will win in a race of 100 yds. and by how much?

11. Rama can walk 5 miles in $1\frac{1}{2}$ hours, and Krishna 6 miles in 1 hr. 50 min. They start to walk a match over a distance of 20 miles. Who will arrive first and by how many min. will he win?

12. Two persons undertake to run a mile race. One of them runs uniformly throughout at 11 yds. in $2\frac{1}{4}$ seconds while the other runs partly at 6 yards in $1\frac{1}{4}$ seconds. The latter finds that he is 60 yds. behind his rival and so with an increased speed wins by 10 seconds. Determine his speed during the latter part of the race.

13. Govind and Krishna run a race. Govind runs uniformly at 5 yds. per second and Krishna at 6 yds. over the first half and at only 4 yds. over the other half. Determine which would win and by how much. If the distance by which the race is won is 10 yards, determine the length of the course.

14. I cycle a mile in $7\frac{1}{2}$ min. and my friend cycles at $7\frac{1}{2}$ miles an hour. The faster wins the race by 6 min. Find by how many yards he will win.

15. In a race, B has a start over A, and C an equal start over B. If A passes B in 10 min. and 7 min. afterwards passes C, in what time from the start will B pass C?

16. A can give B 17 yds. and can give C 35 yds. start in a quarter mile race, and B can give C 5 seconds start in the same distance. How long does each take to run the quarter of a mile?

17. P and Q run a race which P wins by 'a' min. Determine in feet the distance run, if P's rate is 'b' miles an hour and Q's rate is 'c' miles an hour.

114. PASSING OF TRAINS.

The length of a train means the distance between the two extremes of the trains, *i.e.*, generally the distance between the front part of the engine and the rear of the guard-van. Sometimes problems arise in which the moving trains cannot be treated as particles but due attention is to be paid to the dimensions of the moving body. Such problems occur when one train passes a man or telegraph post or an extended body at rest as a platform or an extended body in motion as in the case of a second train in motion. Passing of a man or a telegraph post is said to commence when the engine is opposite to the man or the post, and it is said to be complete when the guard-van is opposite to the same. To do the same, the engine, or

for our purposes, the train has to gain a distance equal to its own length. This is done at the rate of the train itself when the man is at rest or at the difference of the rates of the two bodies when the motion is in the same direction or at the sum of the rates when the motion is in opposite directions. When the second body is an extended body but stationary as in the case of a platform being passed by a train, the train has to travel a distance equal to its own length and that of the platform and this is done at its rate. But when one train passes another in motion, the passing train has to gain over the other a distance equal to the sum of the lengths of the two trains and this is done either at the sum or the difference of the rates of the two trains as the case may be.

Illustrative Examples.

(1) A train whose length is 165 ft. is moving at the rate of 30 miles an hour, and overtakes a second train 143 ft. long which is moving on a parallel line at 20 miles an hour. Find how long the first train will take (1) to pass a mile-stone, (2) to pass a passenger in the second train and (3) to pass the second train.

Sol. :—In such problems the first step is to express the rates in feet per second. The rate of the first train is 44 ft. per second and that of the second is $\frac{2}{3}$ of 44 or $29\frac{1}{3}$ ft. per second. The first train will pass a mile-stone when it travels a distance of 165 ft. (its own length) with its actual velocity. The time

required for this $\frac{165}{44}$ seconds or $3\frac{3}{4}$ seconds.

(2) The passenger in the 2nd train partakes of the motion of the second train and hence the rate of the passenger is $29\frac{1}{3}$ ft. per second.

The train has to travel *a distance equal to its own length* to pass the passenger in the 2nd train and this is done at the difference of the two rates as the two trains are moving in the same direction (as indicated by the word overtakes).

∴ The time required for this is $\frac{165}{(44-29\frac{1}{3})}$ seconds

or $\frac{165 \times 3}{44}$ or $11\frac{1}{4}$ seconds.

(3) The time taken by the first train just to clear the second train is the time that it takes to travel *a distance equal to the sum of the lengths of the two trains* at the difference of the two rates. The required time is therefore $\frac{165 + 143}{(44 - 29\frac{1}{3})}$ seconds

or $\frac{308 \times 3}{44}$ or 21 seconds.

(2) A passenger train, moving at the rate of 45 miles an hour, overtakes a mineral train, $1\frac{1}{2}$ times as long and moving on a parallel line at the rate of 27 miles an hour, and passes it completely in 25 seconds. How long would the passenger train take to pass completely through a station 165 yds. in length?

Sol.:—The net rate of gaining is (45 — 27) or 18 miles per hour or $\overset{22}{\cancel{44}} \times \frac{\overset{6}{\cancel{18}}}{\underset{5}{\cancel{30}}}$ or $\frac{132}{5}$ ft. per second.

The passenger train has to travel a distance equal to the sum of its length and the length of the mineral train or $2\frac{1}{2}$ times its own length, as the mineral train is $1\frac{1}{2}$ times as long as the passenger train.

$\therefore$ $2\frac{1}{2}$ times the length of the passenger is $25 \times \frac{132}{5}$ or 660 ft. $\therefore$ The length of the passenger is $\frac{2}{\cancel{5}} \times \overset{132}{\cancel{660}}$ or 264 ft.

In the second case the passenger train has to travel (264 + 495) or 759 ft. to pass the station. This the train does with its actual velocity. $\therefore$ The time required for this $\frac{\overset{23}{\cancel{759}} \times 1}{\underset{2}{\cancel{66}}}$ or $11\frac{1}{2}$ seconds.

(3) A train was noticed to pass a station 440 ft. long in $6\frac{2}{3}$ seconds, and another station 220 yds. long in $9\frac{1}{3}$ seconds. How long was the train, and how fast was it moving?

Sol.:—In the second case the extra time taken is $2\frac{2}{3}$ seconds. This is because the train has to travel an additional distance of (220 × 3 — 440) or 220 ft. Hence in $2\frac{2}{3}$ seconds the train travels 220 ft. Hence the rate of the train is $\frac{3}{8} \times 220$ or $82\frac{1}{2}$ ft. per sec

or $\frac{\overset{15}{\cancel{30}} \times \cancel{180} \overset{15}{\cancel{9}}}{\cancel{7} \times \cancel{44}}$ or $56\frac{1}{4}$ miles per hour.
$\quad 4$

Again if the train travels $\left(\frac{165}{2}\right)$ ft. per second, then in $6\frac{2}{3}$ seconds it must have travelled $\frac{\overset{55}{\cancel{660}}}{\cancel{7}} \times \frac{\overset{10}{\cancel{70}}}{\cancel{6}}$ or 550 ft.

∴ The length of the train is (550 — 440) ft. or 110 ft.

(4) A man standing on a platform notices that a train going in one direction takes 3 seconds to pass him, while a train of the same length in the other direction 4 seconds. How long will they take to pass one another?

Sol.:—The first train travels its own length in 3 seconds. ∴ In 1 second it travels $\frac{1}{3}$ of its length. Similarly the second train travels in 1 second, $\frac{1}{4}$ of its own length. ∴ In going in opposite directions the two clear per second $(\frac{1}{3} + \frac{1}{4})$ or $\frac{7}{12}$ of the length of either, since the trains are of equal length. But to clear twice the length of either (*i.e.*, the sum of the lengths of both the trains) the time taken will be $\frac{2 \times 12}{7}$ or $3\frac{5}{7}$ seconds or 3·4 seconds nearly.

EXERCISES.

1. Two trains are travelling in opposite directions at uniform speeds. They pass each a telegraph post in the same time and they pass each other in 4 seconds. If the lengths of the trains

added together be 220 yds. and the faster train runs 5/4 times as fast as the slower, determine the length and speed of each train.

2. A train going at the rate of 20 miles per hour passes a man walking in the same direction at the rate of 4 miles per hour in 12 seconds. Determine the length of the train.

3. Two trains start simultaneously from Madras and Dhanush kodi respectively and travel towards each other at 30 and 20 miles per hour respectively. When they meet it is found that the Madras train has travelled 90 miles more than the other. Find the distance from Madras to Dhanushkodi.

4. A train, while travelling at the rate of 30 miles per hour, whilst inside a tunnel meets another train of $\frac{3}{4}$ its length, travelling at 45 miles an hour, and passes it completely in $4\frac{1}{2}$ seconds. Find the length of the tunnel, if the first train passes completely through it in 15 seconds.

5. A person on the platform of a station which is 132 yds. long, notices that a train, which takes 3 seconds to pass him occupies $7\frac{1}{2}$ seconds in passing completely through the station. What is the speed of the train ?

6. A train whistles both on entering and on leaving a tunnel 165 ft. long and the engine-driver on another train, which is approaching the tunnel from the opposite side at 45 miles an hour, hears the whistles at an interval of 3 seconds. Determine the rate of the first train in miles per hour assuming that sound travels at 1056 ft. per second.

7. Two trains are travelling in opposite directions at uniform speeds. They each pass a man at rest in the same time, and they pass each other in 10 seconds. The sum of the lengths of the two trains comes to 330 yds. and their speeds are as 5 : 6. Determine separately the length of each train and its speed in miles per hour.

8. A train, going at 36 miles an hour, overtakes another train 288 ft. long going in the same direction on a parallel line at 27 miles an hour and completely passes it in $22\frac{1}{2}$ seconds. Find the time in which the trains would have completely passed each other, if they had been travelling in opposite directions. What is the length of the first train ?

9. A train 66 yds. long overtook a person walking along the line at the rate of 3 miles an hour and passed him completely in 8 seconds. It afterwards met a cyclist and passed him in 3 seconds. At what rate was the cyclist riding?

10. A train 88 yds. long overtakes and passes A who is walking in the same direction at the rate of 5 miles an hour in 5 seconds; it passes B in 3 seconds. In what direction and at what rate is B travelling?

11. Trams start every quarter of an hour from a certain place and run at a uniform valocity of 7 miles an hour without stopping; a man walks along-side the tram-line at the rate of 3 miles an hour. Find the interval that elapses between the passing of consecutive trams if he walks (1) in the same direction as, and (2) in the opposite direction to that in which the trams are going.

12. One day I walked to a tram teminus to catch a car within sight and when I was at a distance of 30 yds. from the terminus, the car started and I began to walk faster, for I knew, that it should halt for a couple of minutes at a place 110 yds distant from the terminus. I caught the car at the halting station just, starting, in three min. from the time I began to walk faster. Find the rate of the car, and the rate at which I ran, supposing them to be uniform.

13. A train starting for a regular journey of p miles, maintains a uniform speed greater by s miles than its speed on previous journeys and thereby saves n hours on the whole. Form an equation from which its speed r miles on former journeys can be found. Evaluate your answers when $p = 252$, $s = 2$, and $n = 3$.

14. A boy found he had a hours leisure before him. He rode out on his cycle at b miles an hour but when he was about to return, his cycle punctured and he had to walk back to school at $\frac{1}{2}\,b$ miles an hour and thus he found he was 20 minutes late to school. Determine in terms of a and b how far he must have travelled out. If the break-down had not occurred would he have been in time if a were $1\frac{1}{2}$ and $b = 6$?

15. Rama and Gopal travel along the same road in the same direction. Rama at p miles and Gopal at q miles an hour. At a certain instant of time, Gopal is r miles ahead? Gopal is overtaken h hrs. later at a place 'n' miles off. Express 'r' and 'n' in terms of p, q and h. Evaluate your answers when $p = 9$, $q = 4{\cdot}5$ and $r = 18$.

CHAPTER XXV.

TIME AND DISTANCE—GRAPHS.

In all time and distance problems, whenever motion is uniform, graphs are straight lines. If motion is continuous, the graph also is a continuous straight line. Halts or breaks in motion are represented by breaks in the graphs, the discontinuity being represented by breaks parallel to the time-line of such lengths as to represent the duration of the stoppage. When the time and distance values at the origin represent the time of starting and the place of starting, the graphs in such cases alone will pass through the origin. In other cases the graphs should not be wrongly taken through the origin. The usual precautions of drawing the figure on as large a scale as possible on appropriate scales specified clearly should be attended to here also. The points to be joined to get a st. line graph should be as far apart as possible to get the graph accurately.

EXERCISES.

1. Two boys start from Triplicane to the Madras X'ian College at an interval of 48 min. and reach it at the same time walking at $1\frac{3}{4}$ miles and $2\frac{5}{8}$ miles an hour respectively. Draw a graph of their motions and thence deduce how far the college is from Triplicane along the road they travelled. (The graph of the slower boy who must be assumed to start earlier would pass through the origin if the origin is taken to represent Triplicane and his time of starting. The graph of the second boy cannot pass through the origin for even though the origin represents Triplicane, his place of starting,

the time at the origin is not the time of starting of the second boy. The point where the two graphs cross fixes the Xian College in the diagram.)

2. A and B set out from X to Y walking at 9 and 7 miles an hour. If A starts when B is 8 miles from X, when will he overtake B and where? Give a graphical solution.

3. A and B start from P to Q at an interval of 2 hours 15 min. B cycles at 8 miles an hour and A walks at 4 miles an hour. If they both reach Q at the same time, find graphically the distance PQ.

4. A beats B by 10 yds. and C by 25 yds. in a hundred yards race. Determine graphically how many yards' start could B give C in a hundred yards' race.

(Here no time is mentioned in the question. In such cases, assume any convenient length to represent A's time over the race. Then draw the graph of A which is a straight line through the origin. The graphs of B and C are also straight lines through the origin, since they start from the same place and at the same time as A. When A is at the goal, B is 10 yds. behind and C 25 yds. behind. Thus the positions of B and C at this time are fixed. These positions joined to the origin give the graphs of B and C. The graph of B is now produced to cut the locus of 100 yds. and from the graph of C his position is determined at the time given by the point of intersection of B's graph and the locus of 100 yds. This distance by which C lags behind B is the greatest start which B can afford to give him without himself losing the race.)

5. One man walks up a street 600 yds. long at the rate of 200 yds. in 3 min. but stops for 3 min. in a soda-shop half-way up the street. Another man, starting from the same end as the first, but 7 min. later walks after him at 100 yards per min. Draw a graph of their motions (1" = 4 min. and 1 in. = 100 yds.) and thence find when and where the second man will overtake the first.

6. At noon, a boy begins to walk along a road at 4 miles an hour, and at 2 p. m., a cyclist rides after him at 10 miles an hour.

Draw a graph of their motions and find when the cyclist overtakes the boy.

7. A train has a journey of 125 miles to perform in 4 hrs. 10 min. If the train is delayed 40 min. in starting, find graphically how many miles an hour faster must it travel so as to arrive at the proper time.

8. A burglar having broken into a house, is interrupted and makes off at 4 a. m. towards a town 15 miles away alternately running for 15 min. and walking for 15 min. at the rates of 10 miles and 4 miles an hour respectively. The owner of the house, having ascertained the direction in which the thief had gone, pursues on his cycle at 15 miles an hour starting at 4-50 a.m. Was the burglar caught before reaching the town? If so, at what time and how far from the house?

9. A hare starts to run at 12 miles per hour when a dog is 44 yards off. After $\frac{1}{2}$ a min., the dog sees it and pursues at 16 miles per hour. How soon will the dog catch the hare? Give a gtaphical solution. (Express the rates in yds. per min., and then proceed).

10. A hare and a tortoise agree to run a 5 mile race. The tortoise runs at a uniform rate of 3 miles an hour and the hare runs at 8 miles an hour for 5 min., and then sleeps for 15 min., then runs for another 5 min, and so on alternately running and sleeping. Represent their motions graphically and thence deduce who wins the race.

11. A and B ride along the same road starting together at 8 and 10 miles an hour respectively. A quarter of an hour later, C pursues them and catches B 26 min., after catching A. Determine from a graph C's rate

12. A man leaves a house and walks along a road at 5 miles per hour. 10 min. later the owner of the house starts in pursuit on a cycle at the rate of 10 miles per hour. but he has to dismount after the first 5 min. and wastes 3 min. blowing up his tyre and then continues the chase. How long does the chase last?

13. A walks a distance of 24 miles at the rate of 4 miles an hour and B starting an hour later does the distance in 3 hours less. Draw a graph of their motions and thence determines

(1) when and where B overtakes A, (2) their distance apart after B has been walking $2\frac{1}{2}$ hours (3) the times when they are 2 miles apart, and (4) when they are not more than 2 miles apart.

[Sol. :—The graphs of A and B are drawn as usual, and the meeting point of these two graphs gives the place and time of their meeting. The second answer is deduced by drawing a line parallel to the distance line at $3\frac{1}{2}$ hrs. from the origin and noting the intercept on this line by the 2 graphs. To deduce answer (3). proceed as follows. Take any point P on any one of the two graphs,

V. S. $1'' = 10$ miles. H. S. $1'' = 1$ hour.

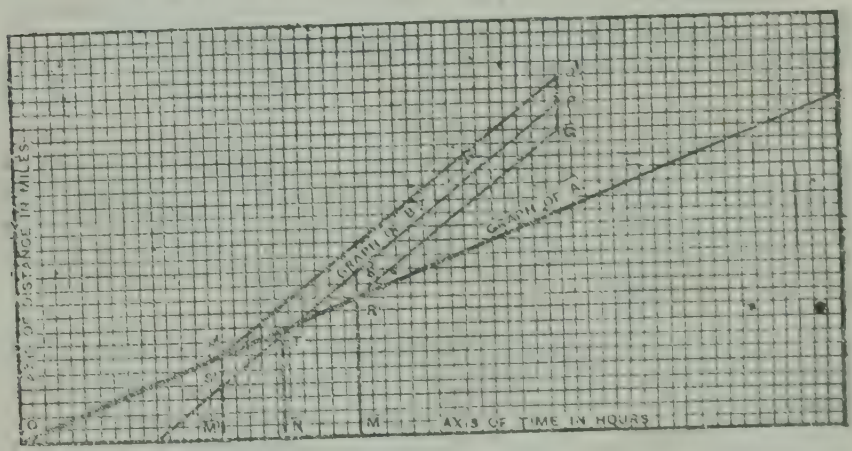

Fig. 202.

Draw PQ parallel to the distance line to represent the required distance apart. Draw QR parallel to the graph on which P lies. Draw SRM parallel to PQ to complete the parallelogram PQRS. Then at time OM, A is at R and B at S. The distance between them is SR= PQ = 2 miles. By drawing PQ' vertically upwards and completing the construction as above we get another time OM' when the distance between A and B is R' S' or 2 miles. Hence there are two distinct times when the distance between A and B has a given value. Closer observation shows that these two positions lie on opposite sides of the meeting point T of the two graphs. From OM' to ON the distance between them goes on diminishing till it is zero at N. Then the distance between them goes on increasing or the two men are getting away farther and farther apart. At time OM the distance is equal to 2 miles. ∴

From time OM'toOM the distance between them is either 2 miles or less than 2 miles and hence during this interval of time, they are not more than 2 miles apart. Thus we deduce answer (4).]

14. P motors at 16 miles an hour, starting at noon, and stopping for ½ hr. at the end of each hr. Q starting at 2-30 p.m. motors without stoppages, at 40 miles an hour. Draw a graph of their motions and thence find when and where he passes P.

15. A starts walking at 4 miles an hour, and 15 minutes later B starts at the rate of 8 miles an hour. Find graphically when and where B overtakes A.

16. A walks at 4 miles an hour, but takes a rest of half-an-hour at the end of every 4 miles. B starting at the same time, and walking at a uniform rate, without any rest catches A up just as he is starting after his third rest. Find graphically B's rate of walking.

17. A travels at 5 miles an hour, but takes a rest of half-an-hour at the end of each hour B starting 2 hours after A and travelling uniformly without resting overtakes A 17½ miles from home. Find graphically B's rate of walking.

18. A motorist starts to do a journey of 8 miles in ½ an hour, but after travelling for 22½ min. finds himself behind time. He quickens his pace to 24 miles an hour, and just completes his journey in time. Find his initial rate of travelling.

19. A motorist does a journey of 80 miles in 6 hours. During the first part of the journey, he travels at 10 miles an hour and during the latter part of the journey, he travels at 18 miles an hour. Find graphically how far he travels at each rate. (Adopt the construction suggested in the Chapter on the Graphical Representation of Stocks and Shares.)

20. A train leaves A at 4-15 p m. and arrives at B at 6-6 p.m. Another train leaves B at 4-45 p m. and arrives at A at 6-50 p m. Assuming the trains to travel uniformly, find graphically at what time they will pass each other.

21. A train starts from Manchester for Liverpool (30 miles distant) at 8-30 a.m. and travels at 40 miles an hour. Another train starts from Liverpool for Manchester at 8-45 a.m. and travels

at 48 miles an hour. When and where do they pass each other? (2) When is one train just 5 miles away from the other? (3) For how long a time, are the trains not more than 5 miles apart.

22. The table below shows the distance in miles from London of certain stations and the times of two trains, one up and one down. Suppose each run to be made at constant speed and show by a graph the distance of each train from London at any time between those given. ($1'' = 20$ miles and 3 inches to represent 1 hour.)

0 London.		4-30 p. m.		7-0 p.m.
$5\frac{1}{2}$ Willesden	Arr.	4-38 p. m.		
	Dep.	4-42 p. m.		No intermediate stop.
66 Northampton	Arr.	5-50 p. m.		
	Dep.	5-54 p. m.		
113 Birmingham		7-0 p. m.		5-0 p m.

23. A, B, C and D are four stations on a railway. The distance AB is 10 miles, BC = 10, and CD = 8. The following is an extract from a time-table. Plot in one diagram, graphs to show the positions of both trains at any time between these given, assuming that they run uniformly between the stations. When and where do they pass each other?

	Up.		Down.	
A	Dep.	7-5 a. m.	D	Dep. 8-29 a. m.
B	Arr.	8-15 a. m.		
	Dep.	8-18 a. m.	C	———
C	Arr.	8-37 a. m.	B	———
	Dep.	8-40 a. m.		
D	Arr.	8-55 a. m.	A	Arr. 9-10 a. m.

24. I live a mile from my railway station and take 16 min. to walk the distance. I am accustomed to meet at a certain point some children going in the opposite direction to school. The children take 25 min. to the mile. One day when I am at my usual time, I met them 200 yds. further on. Find graphically how much later are the children than on ordinary days.

25. Two men start at noon to walk; the one from A to B, the other from B to A. If A and B are 20 miles apart and the men walk at 3 miles and 2 miles an hour respectively, construct a graph which will enable you to determine when and where they meet. Read off from the graph their distance apart at 1-30 p.m. and also find at what time they are first at a distance of 6 miles from one another.

26. A starting at noon from a town P and walking 4 miles an hour, reaches a town Q at 6 p. m. B starts from Q at 1 p. m. and meets A at 3-15 p. m. At what time will B reach P?

27 A travels from P to Q a distance of 60 miles and back again at the rate of 9 miles an hour. On his way back, he meets B who travels at the rate of 6 miles an hour and who started at the same time from P. Find graphically the distance of their meeting point from P.

28. A road runs from P to S passing through Q and R. PQ is $10\frac{1}{2}$ miles and QR = 7 miles. A starts from P at 6 a.m. and walks to S at a certain rate halting at Q for $\frac{1}{2}$ hr. and at R for 1 hr. B starts from P at 7 a.m. on his cycle at 7 miles an hour. He returns to Q where business detains him for $1\frac{1}{2}$ hours. He then goes straight to S which he reaches at 4 p.m. Find the distance from P to S, A's rate of walking and the time when A reaches S. Represent the above graphically and find from the graph when and where A and B will be together.

29. A man did a journey of 11 miles in the following manner. He walked part of the way, then cycled at the rate of 11 miles an hour and finally completed the journey by walking for 15 min. If his walking was at the rate of 3 miles an hour and he took 3 hours over the journey, find how far he cycled.

30. A man walked a certain distance at the rate of 4 miles an hour, and then ran part of the way back at the rate of 6 miles an

hour, walking the remaining distance home in 15 min. The whole journey took him 1 hr. 35 min. How far did he run, and what distance? If a second man does the reverse double journey uniformly starting at the same time and occupying the same time, find graphically when and where they will meet.

31. A man walked from A to B at the rate of 4 miles an hour, without delay ran back at the rate of 6 miles an hour for $\frac{1}{2}$ an hour then waited for 39 min; and then completed the journey home by walking the rest of the way in an hour. How far is it from A to B if the whole journey occupied $3\frac{1}{2}$ hours, and what distance did he walk in the last part of the journey?

32. The following is a programme of a pilgrim party 1975:—

(1) Start at 4 a. m. from Madras by aeroplane, straight to Dwaraka (900 miles). Stay 2 hours.

(2) Visit to Bombay (950 miles from Madras) by motor and back to D. Stay at Bombay 1 hour.

(3) Return to Poona (800 miles from Madras) by aeroplane. Stay 1 hr.

(4) Return to Madras in aeroplane. Represent these journeys graphically and find when the party will reach Madras, supposing that the aeroplane travels at 180 miles an hour and the motor at 40 miles an hour.

33. A passenger train starts from Egmore at 19 o'clock and runs at the rate of 20 miles an hour, stopping for 5 min. at the end of every half hour. Three and half-hours later on, an express train leaves Egmore and runs at the rate of 30 miles an hour stopping for 10 min. at the end of every 25 miles. With the help of a graph, find approximately when and where the express overtakes the passenger train. To avoid a collision, where will you detain the passenger train if a loop-line is provided at the end of every 5 miles from Egmore? (Note that time is measured as 24 hours in Railways, from midnight to midnight.)

34. A man in a launch at a point L is 30 miles from the nearest point A of a straight coast-line and wants to reach a point B on the coast 60 miles from A. He can either go by water the whole way or make for any point on the coast between A and B

where a motor-car is running and watching to pick up and finish the journey by land. Fill up a table like that below and draw a graph to show how his total time from L to B varies according to the point of the coast at which he lands. And from your graph find what point he ought to strike in order to execute the whole journey in the least possible time. Assume the speed of the launch to be 10 miles an hour and that of the motor 20 miles an hour. Draw a diagram to measure distances instead of having recourse to calculation.

Distance from A of landing point in miles.	0	10	20	30	40	50	60
Total time in hours.	—	—	—	—	—	—	—

35. A man rows against a stream, flowing at $2\frac{1}{2}$ miles an hour to a certain point and then turns back stopping 3 miles short of the place whence he originally started. If the whole time occupied in rowing be 5 hrs. 15 min. and his uniform speed in still water is $4\frac{1}{2}$ miles an hour, find graphically how far upstream he went.

H. S. 1" = 2 miles. V. S. 1" = 2 hours.

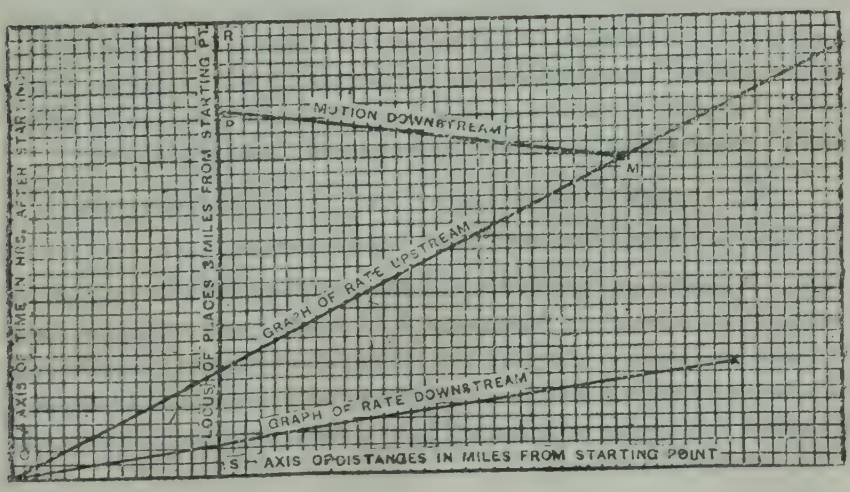
R
MOTION DOWNSTREAM
M
GRAPH OF RATE UPSTREAM
GRAPH OF RATE DOWNSTREAM
LOCUS OF PLACES 3 MILES FROM STARTING PT.
O
AXIS OF TIME IN HRS. AFTER STARTING
S AXIS OF DISTANCES IN MILES FROM STARTING POINT

Fig. 203.

Note:—Draw thro' the origin the two st. lines representing rates of $(4\frac{1}{2}, + 2\frac{1}{2})$ and $(4\frac{1}{2}, -2\frac{1}{2})$ miles an hour respectively. Take P in the fig. to represent the point where he stops finally. This point is 3 miles away from the starting point and hence lies on RS the locus of all such points. The man arrives there $5\frac{1}{4}$ hours after starting. This additional consideration helps us to fix P. Through P draw PM || to the graph of the rate down-stream. Then M the point of intersection of this line with the graph of rate up-stream determines the point where the man must have ceased to row upward and commenced to row down-stream. The distance co-ordinate of M gives the required answer. It is found to be approximately 8·8 miles).

36. A and B are two towns 90 miles apart. A goods train leaves A at 7-45 a. m. and travels uniformly without any stoppages, at 15 miles an hour. An express leaves A at 8-45 a.m. and reaches B at 11-45 a.m. Draw a graph of their motion and thence determine—

(1) When and where one passes the other,

(2) When one train is just 8 miles from the other, and

(3) The interval of time during which the trains are not more than 8 miles apart.

37. A motor car travelling at the rate of 40 miles an hour starts from a place P to travel on a road running beside a railway to a place Q, a distance of 150 miles. Fifteen minutes after the car has started, a train travelling at the rate of 60 miles an hour leaves P for Q. If the train stops for 10 min. at a station 50 miles from P, draw a graph of their motions and thence deduce—

(1) at what distance from P the two are together, and

(2) at what time they are 10 miles apart.

38. A motor car on its way to Madras overtakes a cyclist at 8 a.m. the car reaches Madras at 9-30 a.m., and after waiting for one hour returns meeting the cyclist at at 11 a.m. Supposing the speeds of the car and the cyclist to be uniform, find graphically when the cyclist will reach Madras, also compare the speeds of the car and the cyclst.

39. I have 40 min. before me to do a journey of 6 miles. I get into a tram going at 12 miles an hour, but after going some

time, the current fails. Therefore I get down and walking at my usual rate, I reach the destination just in time. If my usual rate of walking is 4 miles an hour, find from a graph, how far I walked.

≅40. A body is subject to either of two forces, one giving an acceleration of 20 ft. per second and the other a retardation of 7 ft. per second. Only one of the two forces can act at a time and if under the action of these two forces the body travels 119 ft. at the end of 7 seconds, find from a graph, for what length of time the force of retardation must have acted on the body.

41. A cyclist had to ride 56 miles. He rides for a time at 8 miles an hour and then alters his speed to 10 miles an hour and takes in all 6 hours, 12 min. to cover the distance. Find graphically when and where he changed his speed.

NOTE. Draw through the origin lines to represent 8 and 10 miles an hour respectively. Then take a point to represent that a journey of 56 miles is completed in 6 hrs. 12 min. A st. line is then drawn parallel to the 10 mile rate through this point to cut the graph of 8 miles rate. The point of intersection gives the point where the speed was changed. If it does not matter whether the man first rode at 8 miles an hour and then changed to 10 miles or *vice versa*, through the point chosen as above, draw a parallel to any one of these rates to meet the other. But when the rate at which the man is approaching the destination is material as in this question, we should draw a parallel to the approaching rate to meet the other rate.

42. A man starts at 7-50 a.m. to catch a 10-10 a.m. express at a station 25 miles away. He walks at the rate of $3\frac{3}{4}$ miles an hour until overtaken by a bus travelling at the rate of 15 miles an hour. If he manages to arrive at the station just in time, find graphically when and where he got into the bus.

43. I start at 9 a. m. to catch the noon express at a station 30 miles away I cycle at 12 miles an hour; but on the way the cycle breaks down and then I get into a jut going at 6 miles an hour. If I am just in time to catch the train, determine from a graph, when and where the break-down must have happened.

44. The cost of coal at a certain place A is 15*s*. a ton. The cost of carriage along a railway through A is 3*d*. per ton per mile.

Draw on squared paper a diagram from which may be found the cost of coal delivered from A at any place on the railway less than 80 miles from A. Eighty miles from A, on the railway there is a place B at which the cost of coal is $18\frac{1}{2}$ *s.* a ton. Find from the graph, the position of a town on the line AB at which the cost of the coal is the same whether it is brought from A or from B.

[Here the two things to be connected are distance from A (taken as the origin) and the prices of coal at various distances from A. Since the cost of carriage is proportional to this distance, the graph is a st. line. The origin is here chosen to represent station A and the price of 15*s*. MN contains all points at distances of 80 miles from A and hence MN is the locus of station B. The Y-axis is for a similar reason the locus of station A. Point P is taken on MN to represent 35*s*., the price of a ton of coal carried there from station A. Then OP represents the graph of coal carried from station A. Similarly Q (80 miles, $18\frac{1}{2}$*s*.) joined to R (0 miles from A, $38\frac{1}{2}$*s*.) gives the locus of coal carried from B to A. The two cross at S. At S therefore the cost of the coal is the same whether the coal is carried from A or from B. This point S is found from the graph to be at a distance of 47 miles from A and the price of the coal there is found to be $26\frac{3}{4}$*s*.]

H. S. $1'' = 15$ miles and V. S. $1'' = 80d$.

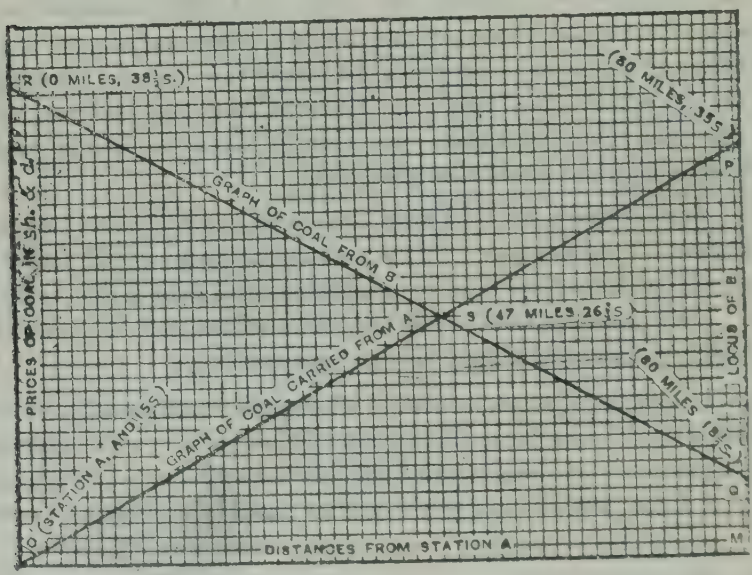
R (0 MILES, 38½S.)
(80 MILES, 35S.)
GRAPH OF COAL FROM B
S (47 MILES 26¾S.)
(80 MILES, 18½S.)
GRAPH OF COAL CARRIED FROM A
O (STATION A AND 15S.)
DISTANCES FROM STATION A
LOCUS OF B

Fig 204.

CHAPTER XXVI.

PARTNERSHIP.

In these days of big enterprises and commercial development, it is not possible for a single person to subscribe the capital necessary for a business. Hence a number of partners join together, each subscribing a portion of the required capital. At the end of the year the resulting profits or losses, as the case may be, must be divided,in fairness, proportional to the capitals invested by each partner. Sometimes it so happens that the capital instead of remaining fixed during the year under consideration undergoes a change by either additional investments or withdrawals. In such problems, the element of time also is to be considered. Here the distribution of profits must be proportional to the product of each partner's capital multiplied by the corresponding time during which the said capital is used. The unit of time in these cases can be taken to be a month or a year as may be convenient. The products obtained as above, to which the profits are distributed proportionately, are known as *equivalent capitals*.

More complicated problems arise where only one of the partners acts as a manager or an active or working partner and the other-partners are merely sleeping or passive partners. The latter set of partners have merely contributed a share of the capital. In such cases, a fixed allowance generally a percent

age of the annual profits is set apart initially before the allotment of dividends. It is only the balance that is available for proportionate distribution. This proportionate distribution is implied when the profits are directed to be treated as interest on the capital.

N.B.—The above form of procedure is general and hence ought to be adopted unless other ways of distribution of the profits are specially agreed upon by the parties concerned; in which case, the profits are to be apportioned according to those special conditions.

Illustrative Examples.

(1) Two partners A and B provide Rs. 7500 and Rs. 2500 respectively of the capital of a business. Of the profits, B first receives 20% as manager, interest is then paid on the capital at 4% per annum, the remaining profits being equally divided. In a year in which A receives Rs. 562, what does B receive?

∴ Let x Rs. be the total profits. Then A's share is Rs. 300 (as interest) + $\frac{1}{2}$ ($\frac{4}{5}x$ — 400) Rs. which is by the question Rs. 562. Hence we have $\frac{2}{5}x$ + 100 = 562. ∴ $x = \frac{462 \times 5}{2}$ or 1155. ∴ B's share of the profits (including his allowance as manager) comes to Rs. 1155—562 or Rs. 593.

(2) Two partners started in business, A contributing Rs. 12,000 and B, Rs. 18,000. B was to have 15% of the profits for his share as manager. At the end of 7 months, A withdrew $\frac{1}{3}$ of his capital, and 2 months later B withdrew one-half of his. The profits of the year amounted to Rs. 3130. What sum of money ought each to receive?

[Here the capital changes during the year. Hence the equivalent capitals ought to be first determined. The convenient unit of time is a month. A's equivalent capital is Rs. (7 × 12000 + 5 × 8000) or Rs. 1,24,000 and B's comes to Rs. (9×1,80,000 + 3× 9000) or Rs. 1,89,000. The profits available for distribution comes to Rs. $\frac{\cancel{34}^{17}}{\cancel{100}_{2}} \times 313$ or Rs. $\frac{313 \times 17}{2}$.

∴ A's share is Rs. $\frac{\cancel{313} \times 17}{\cancel{2}} \times \frac{\cancel{124}^{62}}{\cancel{313}}$ or Rs. 1054.

∴ B's share ought to be Rs. (3130—1054) or Rs. 2076.]

EXERCISES.

1. Three partners contribute sums of Rs. 250, Rs. 500, and Rs 750 respectively towards a venture on the understanding that the profits shall be divided in such a way that the *rate* of interest which each receives shall be in proportion to the amount of his contribution. If the profits for a year amount to Rs 490, how much will each of them receive?

2. A tradesman began business on 1st Feb. 1924, with a capital of Rs. 10000. On 21st August of the same year he was joined by a partner who brought Rs. 7500 to the business. At the end of December, the profits were found to be Rs. 5325. Find to the nearest pie, the share of each.

3. A and B enter into business together, A providing $\frac{3}{5}$ of the capital; at the end of the first year, they have made a profit of 7% on the capital, at the end of the 2nd year a profit of 10% on that year's capital and at the end of the third year, they are bankrupts and can pay only Re. 0-12-0 per rupee. If they are now worth Rs. 33,327-4-0, how much did each contribute? (Assume that the profits of each year are also used in the next year's business).

4. A and B are partners in a business. A's capital being Rs. 6750 and B's Rs. 11250. A is to have 10% of the profits for managing the business and the rest of the profits is to be divided in proportion to their capitals. If B's share of the profits is Rs. 2367-8-0, what does A receive ?

5. A and B enter into partnership on the following conditions *viz.* (1) A is to subscribe a capital of Rs. 7500 and B Rs. 6000. (2) A who is the working partner is to be paid Rs, 75 a month for managing the business and (3) the remaining profits, if any, should be treated as interest on the capital. At the end of 4 months, A withdraws Rs. 3000 of his capital, and this is supplied by B. At the end of the year, A got Rs. 400 more than B. Determine each man's share of the profits.

6. A, B and C start a business contributing to the capital in the ratio of 6, 9 and 5 respectively. At the end of the first year, A withdraws Rs. 1900 of the capital which is replaced by B. At the end of the second year B's share of the profits for the 2 years is Rs. 1450 and C's Rs. 700. Assuming that the same profit was made in each year, find the sum originally invested by each.

7. A is a working and B, a passive partner in a business. Their capital amounts to Rs 9600 of which Rs. 3600 belongs to A and the rest to B. The profits are to be divided in the ratio of of their capitals, after allowing A 5% of the profits for managing the business. If A's profits be Rs. 360 less than B's, find each man's share.

8. A and B enter into partnership on the following terms *viz.* (1) Each is to receive 6% per annum on his capital. (2) The remaining profits, if any, were to be divided equally. At the end of a year, A receives Rs. 3250 and B Rs. 2750. If is found that B thus gets Rs. 180 less than what he would have obtained if the whole profits were divided in the ratio of their capital. Determine each man's share of the capital.

9. Three authors propose to publish a book on Algebra : the first spends 5 months at it, the second 6 months, and the third 9 months. They respectively pay Rs. 62-8-0, Rs. 150 and Rs 75 for printing charges. The other expenses come to Rs. 1560 which is to be divided among them, 5200 copies of the book are

printed and 13 copies are counted as 1 dozen and sold at Re. 1-8-0 each copy. If the profits are to be divided proportional to the time and tho money spent by each, find what sum each will receive.

10. A, B and C entered into partnership and provided capitals of Rs 5500, Rs. 6500 and Rs. 8500 respectively. Some months later, Rs. 2500 extra capital being required, it was supplied by B. At the end of the business year, the total profit was Rs. 1047-6-0 and A's share thereof Rs. 259-14-0. When did B supply the extra capital ?

11. The weights of three diamonds are to one another as 3 : 5 : 7 The weight of a fourth diamond is $1\frac{1}{3}$ times the total weight of the three diamonds. Assuming that the values of these diamonds are proportional to the squares of their respective weights and that the heaviest diamond costs Rs. 1500, find the values of the other diamonds.

12. If the diametes of a penny, half-penny, and farthing were in the ratio of $p : q : r$ and their weights in the ratio of $a : b : c$, find the ratio of their thickness. Find it when $p = 6$, $q = 5$, $r = 4$, $a = 10$, $b = 6$ and $c = 3$.

13. Three men A, B and C, have together completed a piece of work for which they receive Rs. 64-14-0. A has worked for 4 days of $10\frac{1}{2}$ hrs., B for 11 days of 7 hours, and C for 6 days of 9 hours each. How much money ought each to receive ?

14. Three men A, B and C, go away together for a holiday and agree to share all general expenses equally. A starts with Rs. 310, and takes charge of B's money Rs. 243-10-8 and C's money Rs 215-12-0 A buys some photographs for himself for Rs. 13-8-0, a new cycle bell for Rs. 2-8-0 and a pair of shoes for C for Rs. 13-8-0 and the remaining outlay being for general expenses. At the end of the trip, A finds there is Rs. 23-6-8 left. How should they settle their accounts ?

15. A sum of s annas is divided among m men and w women, a man's share being to a woman's share in the ratio $p : q$. What does each receive ?

CHAPTER XXVII.

PROBLEMS ON WORK AND TIME.

When we have to combine the work done by several agents, or when we have to compare their work, it is necessary first to ascertain the amount of work each can do *in the same time*, usually one day, one hour, or one minute, according to the condition of the problems. When some agents do negative work we should pay attention to the nature of the work when the joint work of the several agents is considered. The addition therefore in such problems is algebraic addition.

Illustrative Problems.

(1) A boy can do a piece of work in p hours and another boy can do the same piece of work in q hours. If they both work at it, find how soon the work will be finished.

(**Sol** :—The first boy does in 1 hour $\frac{1}{p}$ of the work and the second boy $\frac{1}{q}$ of the work. $\therefore$ their joint work comes to $\left(\frac{1}{p} + \frac{1}{q}\right)$ or $\frac{p+q}{pq}$ per hour. Hence to complete the work, they would together take $\frac{pq}{p+q}$ hrs.)

(2) A cistern can be filled by a supply pipe in k min. and emptied by a waste-pipe in l min. If both pipes are opened when the cistern is empty, how long will it be before the cistern is full? Also if the cistern can hold r gallons, how long would it be before exactly 't' gallons had run in? Evaluate your answers when $k=10$, $l=16$, $r=240$ and $t=72$.

[**Sol.**—Here the work turned out by the waste-pipe is treated as negative work. ∴ the joint work turned out by both the pipes in 1 min. $= \left(\frac{1}{k} - \frac{1}{l}\right)$ or $\frac{(l-k)}{kl}$ of the cistern being filled. ∴ the time for the filling of the whole cistern is $\frac{kl}{(l-k)}$ min. The corresponding numerical answer is $26\frac{2}{3}$ min. In the 2nd case, the part of the cistern to be filled is $\frac{t}{r}$. ∴ the time required for doing this $\frac{klt}{r(l-k)}$. The corresponding numerical answer is $\frac{8\cancel{0}}{\cancel{3}} \times \frac{7\cancel{2}}{\cancel{240}} \overset{8}{\underset{\cancel{3}}{}}$ or 8 min.]

(3) Three persons A, B and C working alone can do a piece of work in 12, 16 and 24 days respectively. B and C work together for 4 days and then A takes C's place. How soon will the work be finished?

[**Sol.**—The part of the work done in the first 4 days by B and C $= 4\left(\frac{1}{16} + \frac{1}{24}\right)$ or $\frac{5}{12}$. The remaining

work that is to be done by A and B is $(1 - \frac{5}{12})$ or $\frac{7}{12}$. But A and B can do in 1 day $(\frac{1}{12} + \frac{1}{16})$ or $\frac{7}{48}$ of the work. ∴ the required time is

$$\frac{7}{12} \times \frac{48}{7} \text{ or } 4 \text{ days from the time A took C's place.}$$

(4) Rama and Krishna can reap a field in 3 days, Rama and Gopal in 4 days and Krishna and Gopal in 6 days. All of them work together and reap the field. They thus jointly earn Rs. 27-0-0. How ought this sum to be divided among them?

[**Sol.** :—Rama + Krishna reap in 1 day $\frac{1}{3}$ of the field.
Rama + Gopal ..... $\frac{1}{4}$ "
Krishna and Gopal ..... $\frac{1}{6}$ "

∴ adding 2 times the work turned out by Rama, Gopal and Krishna together = $(\frac{1}{3} + \frac{1}{4} + \frac{1}{6})$ or $\frac{3}{4}$.

∴ Rama, Krishna and Gopal must have been working together for $\frac{8}{9}$ days in reaping the field. Now the wages earned must be proportional to the work turned out by the respective workmen.

Rama + Gopal + Krishna = $\frac{3}{8}$ in 1 day.
Rama + Gopal alone = $\frac{1}{4}$ "

∴ Krishna alone in 1 day can reap $\frac{1}{8}$ of the field and proceeding similarly we find that Rama can do in 1 day $\frac{5}{24}$ of the work and Gopal $\frac{1}{24}$ of the work.

∴ the total work turned out by Rama $\frac{5}{24} \times \frac{8}{3}$ or $\frac{5}{9}$

and therefore he has earned Rs. 15. Similarly Krishna

has earned Rs. $\frac{1}{8} \times \frac{8}{3} \times 27$ or Rs. 9 and Gopal has

earned Rs. $\frac{8}{3} \times \frac{1}{24} \times 27$ or Rs. 3.]

(5) If 25 men with 10 boys can do in 3 days as much as 21 men and 30 boys can do in $2\frac{1}{2}$ days, compare the rates of working of a man and a boy; also how many boys must help 40 men to do the same piece of work in 2 days?

[**Sol.**:—The work of 25 men and 10 boys in 1 day $= \frac{1}{3}$.

Similarly the work turned out by 21 men and 30 boys per day $= \frac{2}{5}$.

$\therefore$ from the first condition to finish the work in 1 day we must have 75 men + 30 boys and from the second condition to finish the work in 1 day we must have $52\frac{1}{2}$ men + 75 boys.

Since the work done in both the cases is the same we must have 75 men + 30 boys = $52\frac{1}{2}$ men + 75 boys or $22\frac{1}{2}$ men = 45 boys.

$\therefore$ 1 man does twice as much work as a boy. This answer might have been obtained also algebraically thus:—Let us suppose that a man does $\frac{1}{x}$ of the work in 1 day and a boy $\frac{1}{y}$ of the work in the same time.

Then the conditions of the problems reduce to the following equations, *viz.* $\frac{25}{x} + \frac{10}{y} = \frac{1}{3}$. and $\frac{21}{x} + \frac{30}{y} = \frac{2}{5}$.

Solving for $\frac{1}{x}$ and $\frac{1}{y}$ we get $\frac{1}{x} = \frac{1}{2}$. Hence the result.

To get the second answer we proceed as follows:—
The first condition tells us that 60 boys can turn out the work in 3 days.

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$\therefore$ $\frac{180}{2}$ or 90 boys can finish it in 2 days. But there are already employed 40 men or 80 boys. $\therefore$ The required additional number of boys = (90—80) or 10.]

(6) A cistern partly full, into which a steady stream of water is flowing, has a number of equal holes in the bottom, which can be opened or closed at will. If 10 were opened, the cistern would be emptied in 20 min.; if 8 were opened, it would be emptied in 35 min. Twelve are opened, how soon will it be emptied?

[**Sol**:—In this and similar problems we have to take into account a uniform supply, uniform waste and an initial store. Such questions are solved as follows. Let us suppose that the cistern originally contains x min. flow. Then the quantity of water flowing out of the cistern per min. though each waste-pipe is $\frac{x+20}{200}$ min. supply. Similarly the same result from the second condition comes to $\frac{x+35}{280}$. Hence we must have $\frac{x+20}{200} = \frac{x+35}{280}$; whence $x = 17\frac{1}{2}$, *i.e.*, the cistern originally contained $17\frac{1}{2}$ min. flow of water. Substituting this value of x in the expression for a minute's waste through 1 pipe, we have $\frac{37\frac{1}{2}}{200}$ or $\frac{3}{16}$ min. flow. Hence when 12 of these waste-pipes are opened, the quantity of water that gets out of the

cistern per min, is $\left(12 \times \frac{3}{16}\right)$ or $2\frac{1}{4}$ min. flow, *i.e.*, $1\frac{1}{4}$ min. over and above the quantity of water that gets into the cistern per min At this rate it would take $\frac{17\frac{1}{2}}{1\frac{1}{4}}$ or 14 min. for the cistern to be emptied.]

EXERCISES.

1. The materials of an old house were sold in auction for Rs. 2250. The understanding was that they should be removed within 25 days and a penalty of Rs. 10 for everyday exceeding the above 25 days should be paid. The cost of removing comes to Rs. 12-8-0 per day. The materials were finally sold for Rs. 2942-8-0, realising a profit of 10% on the total outlay. How long did it take to remove the materials ?

2. In what time will a tube 4 ft. by $2\frac{1}{2}$ ft. through which water flows full bore at 4 miles an hour fill a tank 440 ft. long, 40 yds. broad and 15 ft. deep ?

3. a men can do a piece of work in c days, but with the assistance of b women can do the same in d days. Find the time in which 1 woman can do the work. Evaluate your answer when $a = 5$, $b = 8$, $c = 10$ and $d = 6$.

4. A certain sum of money would pay A's wages for p days, or B's wages for q days or C's wages for r days. How long will the money last if A, B and C are employed together ?

5. A labourer is hired for p days on condition that he would be given $q$as. for every day he works and fined Re.0-5-4 every day he was idle. If, at the end of the period, he has earned m Rs., find how many days he has been idle. Evaluate your answer when $p = 48$, $q = 12$ and $m = 23$.

6. a men and b women can reap x acres in a week while c men and b women can reap y acres in the same time. How many acres will a man reap in a week ? How many weeks will it take d men to reap z acres ? Evaluate the two expressions when $a = 20$, $x = 8$, $c = 12$, $y = 6$, $d = 30$ and $z = 15$.

7. A and B take a days to do a piece of work. They work together for b days and then call in C, and the three finish the work in p days from that time. Given that C does in m days as much work as A in n days, determine in how many days B can do the work working at it alone. Evaluate your answer when $a = 30$, $b = 10$, $p = 12$, $m = 2$ and $n = 3$.

8. A labourer was engaged for x days on condition that he should get a as. for every day he worked and pay a penalty of b as. for every day he was absent. He worked for d days, and at the end of the contracted period received c as. Express x in terms of a, b c, and d. Give a numerical result when $a = 4$, $b = 2$, $d = 12$, and $c = 8$.

9. Three boys working alone can do a piece of work in 4 hours, a hours and $2a$ hours respectively. If all work simultaneously they can finish the work in 30 min. Determine the value of a.

10. A steam plough ploughs an acre in x min. Express, in hours, the time it will take to plough a field m ft. by n ft.

11. A mine is flooded and water continues to pour in at a uniform rate. If 24 men take to pumps, the mine can be freed of water in 30 min., and if 27 men work at the pumps, the mine becomes dry in 25 min. How many men must work at the pumps if the mine is to become dry in $12\frac{1}{2}$ min.?

12. A contractor has undertaken to put up an embankment 350 yds. long whose vertical cross-section is a trapezium having its top and bottom widths 18 and 27 ft. respectively. The slopes are each inclined at an angle of 45° with the horizontal. If he has undertaken to finish the work under a contract of Rs. 60,000 and if he pays Re 0-15-6 for every 7 cubic ft. of work turned out, find the net gain or loss in this undertaking.

13. At the siege of Port Arthur in the Russo-Japanese War. it was found that a certain length of trench could be dug by the soldiers and coolies in 6 days; but that when half the number of coolies were present, it required 10·5 days to do the same length of trench. Compare the work turned out by the coolies and the soldiers.

14. The wages of a man are 10 as. a day and of a woman 5 as. a day and 3 women can do as much work as 2 men. A piece of work can be done by 40 men in 60 days, find the difference in the cost if 10 men and 30 women are employed to do the work instead of 40 men.

15. A certain number of men can do a piece of work in 10 days. If there were 3 men more, it could be finished in 2 days less. Find the number of men at first.

16. A, B and C can together do a piece of work in 24 days. A does the same work as B in the same time. Had either A or B been absent, then the two others would have accomplished $\frac{7}{9}$ of the work in 28 days. In what time can each do the work separately?

17. A can do a piece of work in 10 days, B in 8 days, and C in 12 days. They all begin together, but only C continues until the work is finished; A leaves after working for $1\frac{1}{2}$ days, and B after working for 2 days. How long will the work be on hand?

18. A field can be reaped by 10 women in 4 days, or by 6 boys, in 10 days, or by 2 men in 12 days. One man, three boys and three women are employed. Find the total expense, if the daliy wages of a man, a woman and a boy are 8 as. 5 as. and 3 as. respectively.

19. Water enters a tank by a tap A which can fill it in x hours and leaves it by two taps B and C which can empty it in y hours and z hours respectively. If the cistern is full, find in what time, it can be emptied.

20. A, B and C can do a piece of work in 60 days; after working for 10 days, A withdraws and B and C continue to work at the same rate for 20 days more, B then withdraws and C completes the work in 96 days working $\frac{1}{3}$ longer each day. Working at his former rate C can do the work in 222 days. In what time, can B alone do it?

21. A man having contracted to construct a railway in one year employs 750 men for $5\frac{1}{2}$ months and then finds he has done $\frac{1}{5}$ of the work. Will he able to keep good his contract? How many more men must be employed for the remaining period in order to complete his contract as agreed?

22. A contractor undertook to carry out a certain work in 100 days, reckoned from the date of the commencent of the work. He employed 240 workmen and after 45 days one-third of the work was done. Owring to unforeseen difficulties work was suspended for 25 days, and it was found that owing to the particular seasonal difficulties 5 men would be required to do as much work in 1 day as was first done by 3 men in 1 day. How many additional men must be employed in order to finish the work within the stipulated time ?

23. If the grass of a field grows uniformly, and 15 cows consurme it in 8 days, while 9 cows eat the grass of the same field in 16 days, how many cows will eat it in 12 days ?

24. If the grass of a meadow grows uniformly, and 42 oxen consume in 30 days the grass of a meadow of 71 acres, and 74 oxen in 20 days the grass in a meadow of 10 acres, how many oxen will in 21 days consume the grass of a meadow of 14 acres ?

25. A contractor receives every week a certain sum, which he uses for paying wages. His capital together with weekly subsidy, would just enable him to pay 20 men for 52 weeks. If he had 40 men at the same wages, his capital together with the weekly subsidy would just suffice for $6\frac{2}{3}$ weeks. How many men can he maintain for 7 weeks on the same principle ? (Answer to the nearest integer.)

26. A house which lets for Rs 750 a year takes 8 months to build by day labour alone. But if a second set of workmen be employed by night, it is found that although night labour costs 40 °/₀ more than day labour, the house is finished in 4 months and a net gain of Rs. 100 results by employing the two sets of men. Determine the cost of labour in each case.

27. If 30 Indian workmen each earning Re. 1-12-0 per day, can do the same piece of work in 27 days that takes 48 English workmen, each earning 3 *s.* a day to complete in 20 days ; taking the value of a pound to be Rs. 10-0-0, determine which class of workmen is more profitable to employ ? If a piece of work done by the Indians costs Rs. 2750, what would be the cost of the same work done by English workmen ?

28. After a certain number of men had been employed on a piece of work for 36 days, and had finished $\frac{1}{3}$ of it, 24 men more were set on, and the rest of the work was completed in 24 days. How many men were employed at first. Determine the cost of finishing the work if a man's daily wages be Re. 1-7-0 ?

29. To do a piece of work, a contractor can employ two classes of workmen, whose wages are in the ratio of 17 to 13. If he employs the higher paid men (who work the faster), he pays Rs. 2247-8-0 in wages, being Rs. 160 less than what the lower paid workmen would have cost him. Compare their rates of work,

30. There is a well containing 3000 c. ft. of water; two pumps raising 25 and 40 c. ft. per min. respectively are employed to empty it, while it is constantly supplied by a spring which can refill it in 45 min. The two pumps work together for 20 min. ; when that of the smaller capacity ceases work for $12\frac{1}{2}$ min. ; the two pumps then work together until the well is empty. How long will each pump have been employed ?

31. I find that I can engage 30 workmen for 11 wks ; or 62 workmen for 5 wks. ; at uniform wages, and in either case pay the wages exactly by means of the interest now accumulated on a certain sum of money and that which will arise during the particular period of contract. How many workmen could I engage on the same principle for 20 wks.?

32 I earn Rs. 15400 a year. I remain at my head-quarters all the year except that I take a holiday at Ooty. Rent and taxes come to Rs. 695-0-0, clothing comes to Rs 350 a year, and travelling to and from Ooty costs Rs. 105. How long is my stay at Ooty if my weekly expenses while at home come to Rs. 125 and while at Ooty Rs to 300 and if I save Rs. 475 at the end of the year?

33. Rama can do a piece of work in 5 days which Krishna can do in $6\frac{2}{3}$ days. If Rama demands Rs. 3-8-0 a day and Krishna Rs. 3-2-0 a day, what would Rama charge for doing a piece of work for which Krishna demands Rs. 125-8-0 ?

34. Two pipes together fill a cistern in $3\frac{1}{3}$ hours. Running singly one takes 4 hours longer than the other to fill it, Determine the time taken by each to fill it.

35. A works $6\frac{1}{2}$ hours a day for 5 days, B $4\frac{1}{2}$ hours a day for 8 days and C 7 hours a day for $3\frac{1}{2}$ days. A, B and C have thus together earned Rs. 27-12-0. Assuming that the wages earned are proportional to the respective times of labour, determine what A, B and C would have earned separately.

36. Rama can build a wall in 45 days and Gopal can build the same wall in 30 days. Rama begins to work at the wall but after a time he is replaced by Gopal and the wall takes in all 25 days. Determine for what time Rama worked at it.

37. I find that 6 men and 8 boys reap 27 acres in 9 days and that 4 boys turn out as much work as 3 men. I engage 12 men and 12 boys to reap my estate of 63 acres. The daily wages of a man are Re. 1-2-0 and of a boy are Re. 0-12-0. Find the cost of reaping the estate. If I engage purely boys to reap the estate in the same time, what would I have had to pay? What would it have cost if men alone were engaged for an equal period?

38. A can do a piece of work in 6 days, B in 10 days and C in 12 days. They all commence at it together, but A and B staying away $1\frac{1}{2}$ days and $2\frac{1}{4}$ days before the work is completed while C continues to the very end. Determine how long the work was on hand.

39. Two walls are built; the first takes half as long again to build as the second and twice as many men are employed in building the first. Their wages are 20% higher and they work 9 hours a day and 6 days a week, while the others work only $7\frac{1}{2}$ hours a day and 5 days a week. The wages in building the first wall come to Rs. 875, what would be the corresponding wages paid to the other set of workmen?

40. There are two equal cisterns P and Q. P is full and Q is empty. The waste pipe can empty the cistern P in 20 min. and the supply pipe can fill the empty cistern in 25 min. The pipes are opened at the same time. When will the water stand at the same level in both, and when will there be $1\frac{1}{2}$ times as much water in P as in Q?

41. A cistern is constantly supplied with water; when it is full, it is found that 24 equal taps opened together will empty it in $5\frac{1}{3}$ min. and that 15 of them take 13 min. How many of these equal taps should be opened to empty it in 33 min.?

43. If 13 locomotive engines each of 290 horse power working 7 days a week for 11 hours a day can convey 7315 tons of goods to a distance of 221 miles in a given period, how many hours' work a day for 6 days a week must be done by 7 locomotives of 319 horse power each in order to convey 4845 tons of similar goods to a distance of 154 miles in twice the time.

SECTION B.

GRAPHS APPLIED TO PROBLEMS ON WORK AND TIME.

Work done is directty proportinal to time. Hence the graphical represention of work and time is a st. line since zero work corresponds to zero time, the graphs pass through the origin. Broadly considering, time and work problems fall into 2 classes *viz.* class (1) where the several agents add to the work turned out and class (2) where some agents do positive work and some other agents do negative work.

Illustrative Examples.

(1) A can do a piece of work in 4 days and B in 5 days. Determine from a graph how long they would together take to do the work.

H.S. 1″=1 day V.S 25″=1 work.

Locus of one full work.

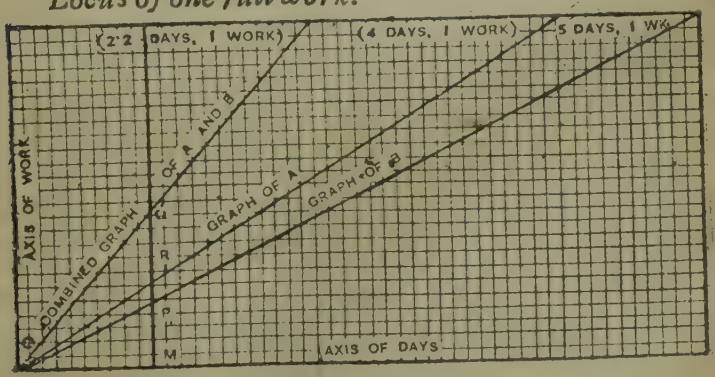


Fig. 205.

Sol.:—Plot days along one axis and work along the other. Days are here plotted along the horizontal axis on a scale 1″ to 1 day, vertically any convenient length can be taken to represent 1 work. Here 2·5″ have been used. Draw the separate graphs of A and B. These are lines through the origin. Plot the point (4 days, 1 work) and join it with the origin. Then this line gives the graph of A. Similarly join the origin (O) with (5, 1). This gives the graph of B. To combine these graphs, consider any time OM. At this time MR represents A's work and MP represents B's work. ∴ If both A and B be employed, they would turn up a work equal to MP + MR. Hence step off RQ = MP. Then MQ represents the joint work turned out by them in time OM. Hence Q is a point on the joint graph of A and B, since the joint graph must be a line passing through the origin the combined graph is obtained by joining Q with O, the origin. Produce this combined graph to cut the locus of one full work. Then the time taken by both A and B when they work together is obtained. From the graph it is seen to be 2·2 days. To find from the same graph when they will complete a part of the work, determine where the joint graph cuts the locus of the required part of the work. Similarly if we have several agents all doing positive work the corresponding bits of work done by each are placed side by side and the point thus obtained when joined to O gives the joint graph.

(2) A bath can be filled by a pipe A in 2 hours and emptied by another pipe B in 5 hours. How long

will it take to fill the bath if both the pipes are opened together?

Horizontal Scale 1″=1 hour. Vertical Scale 2·5″=1 bath.

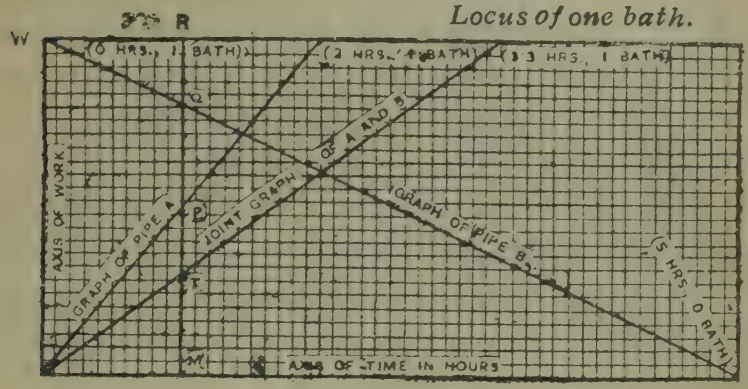


Fig. 206.

Sol.:—Here one pipe does positive work and the other negative work. Represent hours horizontally and take any convenient length vertically (use 2·5″) to represent one full bath. Join the origin O with the point (2 hrs. 1 bath). Then this gives the graph of the filling pipe A. To draw the graph of the emptying pipe B, the origin is the point (0 hrs. 1 bath). Note that for purposes of graph, the emptying pipe is supposed to be opened when the bath is initially full and at the end of 5 hours the bath is emptied. This point (0, 1 bath) when joined to (5 hrs, 0 bath) gives the graph of the emptying pipe. To combine these two graphs, consider any time OM. Then at this time from the corresponding graphs, we see MP is the portion filled in and RQ is the portion emptied. Hence the net work will be (MP—RQ). Hence step off PT equal to RQ. Then MT=MP—RQ. Hence MT is the net work at

time OM when both the pipes work together. Thus T is a point on the joint graph of the two pipes. Thus the joint graph is the line joining the origin O and T. Produce this joint graph to cut the locus of one bath. The point of intersection gives the time taken by both the pipes to fill the bath. This is found from the graph to be 3·3 hrs. The same procedure is adopted when we have a number of filling pipes and a number of waste-pipes. The bits of work of the positive agents are graphically added and from this sum the bits of work of the negative agents are subtracted to get the final point on the joint graph. For all negative agents the origin is W in the figure and for all positive agents, the origin is O.

(3) A man receives 4*s*. for every day he works but is fined 1*s*. 6*d*. for every day he is absent. After 25 days, he receives £2-16 0 as wages. Find graphically for how many days he worked and for how many days he was absent.

Sol.:—The two things to be connected are number of days along one axis and number of shillings along the other. This problem resembles the model (2) supra in having both positive and negative agents of work but differs from it in the following respect While the two pipes, the filling and the waste pipe, can be thought of as working together, in the question before us we cannot say that a man is both present and absent at the same time. Hence one agent here (absence) excludes the other agent, *viz*. work. Plot

days along the horizontal axis and number of shillings along the vertical axis.

Draw separately the graphs of wages and fines. The graph of wages is a straight line passing through the origin (since wages are proportional to number of days of work and since zero days correspond to zero shillings) —To draw this graph, join the origin to (25 days, 100 *s.*) If OK represents the number of days of work then RK represents the number of days of absence. To draw the graph of fines, join (0 days, $37\frac{1}{2}$ *s.*) with (25 days, 0 *s.*) To read the required answer from the graphs we reason as follows. Take any point on one of these graphs. Here the point P on the graph of wages is taken. Draw PM parallel to the axis of shillings to represent the net earnings of 56 shillings. Draw MN parallel to the graph on which P lies to meet the graph of fines. Draw WNT parallel to PM. Then OT gives the required number of days of work and consequently TR represents the number of days of absence. For at time OT, the graph of earnings gives the wages TW. For the corresponding time TR of absence the graph of fines shows the fines to come to TN. Hence the resulting earnings come to WN = PM (for figure PMNW is a parallelogram) = shillings 56 by construction. The answer is found to be 17 days. Similarly for the unaltered rates of fine, wages and contracted period, the same set of graphs can be used to get the number of days of work or absence for any net earnings. The point of intersection of the two graphs of fine and

wages gives the number of days of work and absence corresponding to zero net earnings.

V. S. 1″=40 *s*. H S. 1″=5 days.

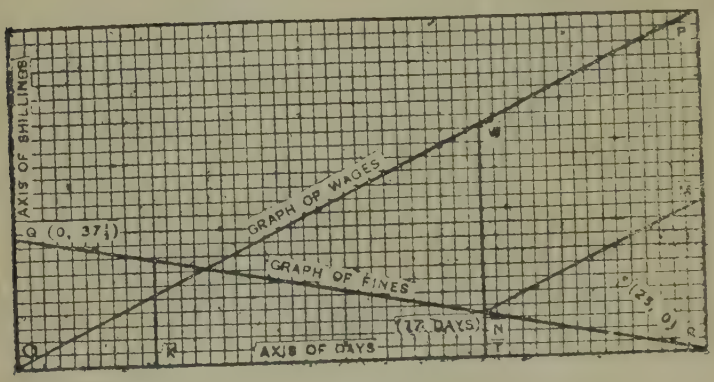


Fig. 207.

EXERCISES.

The models supra and the various principles of graphs explained and illustrated in the earliar chapters will be sufficient to enable the student to work the following questions :—

1. A man starts digging a garden at a rate which would complete the work in 10 days. After working for 4 days, he takes 2 days' rest and after that, working at a faster rate still does the work in the intended 10 days. Find graphically how long he would take to do all the garden working at the faster rate.

2. A labourer is hired for 40 days on condition that he is to be paid Re. 0-12-0 for every day he works, and that he is to pay a fine of Re. 0-4-0, for every day he is idle. If he receives Rs.18-0-0 at the end of the period, find graphically the number of days he worked.

3. A tap would fill a cistern in 10 hrs. When the cistern is half-full, a second tap is turned on and the cistern is full in one hour more. How long would the second tap, running alone take to fill the cistern ? Give a graphical solution.

4. A bath can be filled by a pipe A in 3 hours and emptied by another pipe B in 7 hours. How long will it take to fill the bath if the pipes are opened together ?

5. A alone can do a piece of work in 15 days and B in 12 days. A begins alone and after a certain interval B joins him and the work is finished in $7\frac{1}{2}$ days. Find graphically how long A and B were working together.

6. A certain number of men can do a piece of work in 8 days. If there were 6 men more, it could be finished in 3 days less. Find graphically the number of men at first.

[H. S. $1'' = 2$ days

V. S, $2''$ to represent the first set of men.

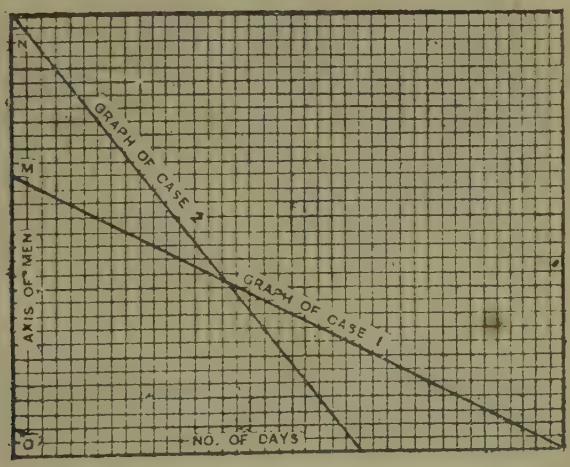


Fig. 208.

Sol.:—Draw the graphs of cases (1 and 2) as shown in the fig. Then MN represents the six additional men on the scale on which OM represents the number of men at first. On this analogy OM represents 10 men. Hence the initial number must have been 10.]

7. A cistern can be filled by a pipe A in 25 min. and emptied by B in 40 min. They are opened simultaneously but A is closed at the end of every 5 min. for 3 min. and B for 2 min. at the end of every 4 min. When will the cistern be full? Illustrate graphically.

8. A can reap a field in 10 hrs., B in 5 hrs., and C in 15 hrs. Find graphically the time taken to complete the work if all of them work at it together.

9. Rama can build a wall in 10 days and Krishna in 8 days. They commence to work at it together. But after some time Krishna stops away. Then Rama continuing to work at it alone finishes the wall in $5\frac{1}{2}$ days from that time. Determine graphically for what time they were working together.

CHAPTER XXVIII.

PROBLEMS ON PROFIT AND LOSS.

When a manufacturer enters upon his business, he hopes to sell his goods at a higher price than their cost of production. He sells *wholesale* or in all large quantities to the retail tradesmen who also hopes to sell them at a higher price than that which he invested in buying them (including, if necessary, the expenses of his business). In both cases they are said to be selling *at a profit.* If as may sometimes happen, the goods are sold at a price less than the cost price, they are said to be sold *at a loss. The actual loss or gain is the difference between the cost price and the selling price.* This difference, for purposes of ready comparison, is very often expressed as a *percentage on the cost price.* When the profit is p %, the C. price is $\frac{100}{(100+p)}$ of S.P. and S.P. is therefore $\frac{(100+p)}{100}$ of the C. price. When there is a loss of l % the C. price is $\frac{100}{(100-l)}$ of the S.P. and the S.P. is $\frac{(100-l)}{100}$ of the C. P.

Illustrative Examples.

(1) A wine merchant mixes together one pipe of wine (126 gallons) at £80, one at £90, and one at £100 and sells one-third of the mixture at 13*s.* 4*d* a

gallon. At what price per gallon, must he sell the remainder so as to gain $12\frac{1}{2}\%$ on his total outlay?

Sol.:—The total outlay is £270. To gain $12\frac{1}{2}\%$, the total S.P. must be $\cancel{270} \times \frac{\cancel{203}}{\cancel{8}} \times \frac{3}{\cancel{2}}$ or $£\frac{609}{2}$ or £304-10-0.

The money realised by selling the first one-third is $\cancel{126} \times \frac{2}{\cancel{3}}$ or £84. ∴ The balance to be recovered from the sale of the remaining 2 pipes or 252 gallons is £220-10-0.

∴ The reqd. S. P. per gallon of the remainder is

$$s\ \frac{\cancel{441}}{\cancel{7}} \times \frac{1}{\cancel{252}} \times \frac{5}{\cancel{10}}$$ or 17s.-6d.

(2) A ton of ice is made at 6 ps. per lb. and sold at 9 pies per lb. A certain quantity is kept for sale at the factory and the remainder is sent to branch shops. If the average loss from melting, etc., be $12\frac{1}{2}\%$ in the former and 25% in the latter, and if the profit on the ton made is Rs. 17-8-0, determine the quantity sold in the branch shops.

Sol.: The outlay is Rs. $\frac{2240 \times 1}{32}$ or Rs. 70. Let us suppose that x lbs. of the ice are sent to the branch shops. Then $(2240-x)$ are sold at the factory. The

money got by selling $\frac{3}{4}x$ at the branch shops (since 25% melts away) is Rs. $\frac{3}{4}x \times 3/64$ or Rs. $\frac{9}{256}x$. The money got by the sale in the factory is Rs. $\frac{7}{8}$ (2240—x) $\times$ 3/64 or Rs. $\frac{21}{512}$ (2240—x). $\therefore$ Total S. P. = $\frac{9}{256}x$ = Rs. $\frac{9}{256}x + \frac{21}{512}$ (2240—x). This must be by the question equal to the C.P. + gain. $\therefore$ We have

$$\frac{9}{256}x + \frac{21}{512}(2240-x) = 87\frac{1}{2} \text{ whence } x = \frac{35 \times 64}{3}.$$

But the answer required is, what was sold in the branch shops which is only $\frac{3}{4}x$. $\therefore$ The required answer is $\frac{3}{4} \times \frac{35 \times 64}{3}$ or Rs. 560.

EXERCISES.

1. A wine merchant, buys in Bordeaux a cask of claret and the freight to Madras is Rs. 15-12-0. It contains 208 litres of wine at 80 centimes the litre, and for the cask itself, he has to pay Rs. 7-8-0. He pays also a duty of Re. 0-15-0 per gallon and bottles the wine at a cost of Re. 1-14-0 a doz. bottles. If he sells it at Rs. 13-8-0 a dozen bottles, determine the percentage of profit made by him on his outlay. (Assume Rs. 15=25 francs=2500 centimes; 6 bottles=1 gallon=4·55 litres.)

2. A merchant buys 8 maunds of coffee seeds at Re. 1-4-0 per viss and 10 maunds at Rs. 1-8-0 per viss and mixes them. He sells the mixture at a uniform price and gains $12\frac{1}{2}$ °/₀ on his outlay. Find the S. P. of each viss of the mixture.

3. A man buys 45 bushels of apples at 2*s*. 6*d*. a bushel. A third of them is spoilt; of the remainder, he sells a third at 2*d*. a pound and the remainder at 4*s*. a bushel. (A bushel of apples weighs 60 lbs.) What °/₀ of profit does he make?

4. A cycle dealer invests Rs. 35,000 in buying 100 ordinary cycles and 50 motor cycles. He sells the ordinary cycles at a profit of 15°/₀ and the motor cycles at a profit of 20 °/₀ and gains

Rs. 6,500 on the whole. Find the price be paid on an average for an ordinary cycle and for a motor cycle.

5. A merchant buys mangoes wholesale at Rs. 15 per thousand. He has to pay Rs. 2-8-0 per thousand for railway and other charges and he finds that 16 °/₀ of the mangoes get damaged and cannot be sold. At what price should be sell the remainder so as to gain 25 °/₀ on his outlay?

6. I bought a horse and a carriage for Rs. 950 : I sold the horse at a gain of $33\frac{1}{3}$ °/₀ and the carriage at a gain of 8 °/₀ and gained 20 °/₀ an the whole. Find (1) the price I paid for the horse and (2) the price for which I sold the carriage.

7. A man borrowed Rs. 8000 at $7\frac{1}{2}$ °/₀ C. I. and forthwith lent the money as followa:—

Rs. 500 at 12 °/₀ S. Interest

Rs. 4000 at 9 °/₀ C. Interest and the rest at $7\frac{1}{2}$ °/₀ S.I. If at the end of 2 years all the transactions were closed, find his gain.

8. A house is offered for sale at an increase of 10 °/₀ in its original price; but not being disposed of, it is sold for Rs. 50 less than what the owner had originally demanded. If he at this price makes a profit of $6\frac{2}{3}$ °/₀, find the cost price.

9. A merchant sells firewood at a gain of 2 as. in the rupee of the cost price, which he shows in his accounts, while the prices marked there are 12 °/₀ more than the actual cost. In weighing, he so arranges 10/11 of a ton is sold as 1 ton. Find his gain °/₀.

10. A farmer bought 448 sheep at Rs. 2-8-0 each and sold $\frac{1}{8}$ of them at a gain of 4 °/₀. He then lost 133 sheep. At what rate °/₀ profit must be sell tbe remainder so as to gain 10 °/₀ of the whole outlay?

11. A tea dealer sold tea at Re. 1-8-0 a lb. and made a profit of 20 °/₀; the cost price was reduced 4 as. a lb. and he then started to sell at Re. 1-4-0. Weat is his new percentage of profit?

12. The purchaser of a farm lets it at a rent which after deduction of 15 °/₀ for repairs and necessary outgoings will return 5 °/₀ on the purchase money. What is the return in a year when the owner allows the tenant 25 °/₀ reduction of rent?

13. A sold a horse to B, gaining $7\frac{1}{2}$ °/₀ on what it cost him. B sold it to C for] £70-19*s*. gaining 10 °/₀ on what it cost him. What did A pay for the horse? Explain year reasoning.

14. A firm has a capital of Rs. 90,000. The gross annual sales are Rs. 37,500. The cost of materials and manufacture is 35 °/₀ of this. Rent, rates and taxes amount to Rs 3870; advertising Rs 9,400; office expenses Rs. 1,360; manager's salary Rs. 2,000 together with one °/₀ commission on the gross sales. Find the rate of interest that can be paid on the capital. If the advertising is doubled, and in consequence annual sales increase 35 °/₀, while the other expenses remain unaltered, what increase will there be in the net annual profit?

15. A grocer mixes two qualities of sugar at 2 as 4 ps. and 3 as. 4 ps. per lb. respectively and gains 20 °/₀ on his outlay by selling the mixture at 3 as. a lb. In what ratio does he mix the two qualities?

Note:—This and similar questions on mixtures of only two ingredients can be easily and orally solved in the following manner. The S. P. 3 as. a lb. gives a profit of 20 °/₀. The cost price of the mixture = 5/6 of Re. 0-3-0 or Re. 0-2-6. Henca if every lb. of the mixture be disposed of at the rate of $2\frac{1}{2}$ as., there will be neither gain nor less.

But the former variety will give a gain of 2 ps per lb. and the latter a loss of 4 ps. per lb. ∴ The mixture must be so constituted as to have the loss in one case compensated by the gain in the other. Hence we must mix 2 lb. of the former variety with every lb. of the latter. Sometimes when the required answer is to be expressed as a ratio, some difficulties are experienced by beginners, such difficulties can be avoided if in writing the ratio, the same order is preserved in one part of the proportion as in the other. Thus, for example, we say either, former variety : latter variety :: 2 : 1 or latter variety : former variety :: 1 : 2].

16. A grocer sells his best quality of tea at 3*s* 6*d*. per lb. and thereby gains a profit of 5 °/₀: he sells his second quality at 2*s*. 6*d*. per lb. gaining a profit of 8 °/₀. In what proportion does he mix the two qualities, if by sellihg the mixture at 2*s*.9*d*. per lb. he gains 10 °/₀?

17. The owners of a weekly paper receive one anna per copy but its cost of production is $1\frac{1}{3}$ anna. 20 °/₀ of the papers printed are not sold and 96 °/₀ of the selling price only is collected. The editor receives Rs. 161-5-4 per week for advertisements and the net gain per week is Rs. 100. How many copies are being sold weekly?

18. A merchant bought 60 measures of castor oil at Re. 1-2-0 per measure and purified it at a cost of 2 as. per measure. During the process 5 °/₀ of the oil was lost and the rest was bottled at a cost of Re. 1-0-0 per dozen bottles. If the marked price of a bottle was 12 as. and a discount of $12\frac{1}{2}$ per cent. was given for wholesale dealers, find the merchant's gain °/₀ if he sold $\frac{1}{4}$ of the bottles in retail and the rest wholesale. (Assume 1 measure = 4 bottles.)

19. A grocer buys 50 lbs. of tea and 30 lbs. of coffee for £8-5-0. By selling the coffee at a loss of 5 °/₀ and the tea at a gain of 10 °/₀, he makes a profit of 10*s.* 6*d.*; what was the cost price of coffee and tea per lb.?

20. A man having bought a lot of goods for £150-0-0, sells $\frac{1}{3}$ at a loss of 4 °/₀. By what °/₀ must he raise that selling price in order that by selling the rest at the increased rate, he may gain 8 °/₀ on the whole transaction?

21. The manufacturer of an article sells to an agent at a profit of 20 °/₀. The agent's wholesale price to a shop-keeper is at a profit of 10 °/₀ and the shop-keeper retails his goods at a profit of $12\frac{1}{2}$ °/₀. Find (1) the cost to the manufacturer of an article bought in the shop for Rs. 212-12-0. (2) The profit °/₀ which the manufacturer would make if he sold direct to the customer at the shop-keeper's price.

22. A person buys some goods in London for £45, and pays £4-5-0 for freight and insurance to Madras. On the arrival of the goods, he pays 7 °/₀ duty on the declared value which was the London price turned into rupees at the current rate of exchange. If he had bought them in Madras, he would have paid 40 °/₀ above the declared value. How much did he gain by buying the goods in London, the rate of exchange being Rs. 14-13-0 a sovereign?

23. Ten people start a chit fund, one of them is the agent privileged to take the first collection without bidding and without any deduction. If the half-yearly subscription be Rs. 100 from each member and the amounts bid for in the successive half-years commencing from the second be Rs. 800, Rs. 820, Rs.820, Rs.840, Rs. 850, Rs. 870, Rs. 920, Rs 970 and Rs. 1000 respectively, the difference between these amounts and the actual collections being distributed among those members who had not bid for the chit till then, find who is the gainer—the first bidder or the member who waits till the last half-year, calculating simple interest at 7·5 °/₀ per annum.

24. A bungalow is offered for sale at an increase of 12 °/₀ on its cost price, but not being disposed of, it is now offered for Rs. 2400 less than before; it being still unsold, this second offer is reduced by $2\frac{1}{2}$ °/₀ of itself and it is then sold. If a profit of 10°/₀ is made by this sale, determine the original cost price of the bungalow.

25. A man purchases two estates, for the larger of which he pays 3 times as much as for the other. He presently sells them and in doing so gains 20 °/₀ on the cost price of the smaller and 25 °/₀ on that of the larger estate. If he received Rs. 75,280 in all, find what he paid for each estate

26. A trademan puts 2 prices on his goods, one for cash and the other for credit. If the credit price of an article be Rs. 53-2-0, credit being calculated for 6 months at $12\frac{1}{2}$ °/₀ per annum, what is the cash price?

27. One man sells his articles so as to gain 10 °/₀ on the cost price, while another sells so that every 10 °/₀ of his selling price is his gain. An article which costs each of them the same sum is marked $2\frac{1}{2}$ as. cheaper by one of them. For what price can it be had in the two shops?

28. The capital of a company is Rs. 25,750; its working expenses are 35 °/₀ of its gross receipts, and 12 °/₀ of its net profits are put to a reserve fund. What must be the yearly gross receipts of the company to pay the shareholders $7\frac{1}{2}$ °/₀?

29. I buy goods and find that the cost of carriage is 5 °/₀ on the cost of goods. I am compelled to sell at a loss of 10 °/₀, on my total outlay. If, however, I had received Rs. 46-8-0 more than I did, I would have gained 2¼ °/₀. What did I pay for the goods?

30. A man buys 20 reams of paper at 1 anna per sheet. He sells at the rate of Rs. 2-4-0 per quire and those he cannot sell he returns getting 7 as. a dozen for them. If he gains Rs. 100 in the whole transaction, determine how much he was able to sell.

31. A bookseller buys books to the nominal value of Rs. 1092, but he gets 9 ps. in the rupee discount. How much does he spend? If in addition to the discount named, the publisher sends him 13 copies at the price charged for 1 dozen, and the book-seller sells the books at a discount of 6 ps. in the rupee, determine his profit in the transaction.

32. A man bought 100 copies of Algebra and 75 copies of Geometry for Rs. 518-12-0. By selling the former at a gain of 25 °/₀ and the latter at a loss of 20 °/₀ he made a profit of Rs. 20. Find the cost of one copy of each kind.

33. At the close of a musical entertainment held under the auspices of a School Literary Society, the Secretaries found that 350 reserved tickets at Re 1-8-0 each, 500 ordinary tickets at Re. 0-12-0 each and 75 ladies' tickets at Re. 0-8-0 each were sold. The expenses they had to incur came to Rs. 450-10-0. Find how much the society profited by this performance.

34. A tradesman bought some sugar at 11 as. a viss and sold it at Rs. 4-0-0 a thulam. He assumed that a viss was $\frac{1}{8}$ of a thulam, whereas in reality a thulam was to a viss as 100 to 20. What profit per cent. did he think he made, and what profit per cent. did he actually make?

35. A man buys 40 articles for Rs. 1750, but 13 of them get soiled. He determines to sell each soiled one at $\frac{4}{5}$ of the price at which he sells a new one. What must this price be in order that he may make a profit of 45 °/₀ on his outlay?

36. The capital of a railway is 4 millions sterling; $\frac{3}{4}$ is borrowed on mortgage at 6°/₀: the remainder forms the subscribed capital.

The working expenses are 35°/₀ of the gross receipts, 20°/₀ of the remainder is put by as the reserve fund; find to the nearest penny what the average monthly income should be to pay the subscribers 7½ °/₀.

37. An armourer undertakes to supply 4000 swords at Rs. 10-12-6 each and he estimates that if 10 °/₀ fail to stand the required tests and are worthless, the profit will be 15 °/₀ on his whole outlay. At the trial 35 °/₀ of the swords prove worthless, how much does the armourer lose by the contract?

38. A manufacturer estimates that he can make a certain patented article at Rs. 170 per hundred. Find (to the nearest pie), the retail price of each article which must be fixed so that, after paying the patentee, a royalty of 10°/₀ on the retail prices, and giving tradesman a discount of 33⅓ °/₀ on the retail price, the manufacturer may make a profit of 25 °/₀ on the cost of making it. If the retial price is fixed to be Rs. 3-0-0, what would be the actual amount of profit made by the manufacturer on each article?

39. In a laboratory there are 96 ordinary gas-burners each burning 3½ cubic ft. of gas per hour for an average of 15 hours per week from July 7th to April 7th. They try a new patent burner advertised to save 33⅓ °/₀. If the price of gas is Rs. 2-8-0 per 1000 cubic ft., how much ought they to save in that time by using the patent?

40. I purchase some garden land, and reservjng 3/7 of it for my own use. sell the rest in such a way that the actual cost to me for my own portion is reduced to ¾. Had I sold it at Rs. 125 more, I would have been a gainer by the whole extent of land which I reserve for myself. Determine how I bought the land and sold it.

41. A wine merchant buys a gallons of wine of one kind at Rs. l per gallon and p gallons of another kind at Rs. s per gallon. He mixes them together and adds some water. If by selling the mixture at Rs. t per gallon, he gains m °/₀: determine how much water has been added. Evaluate your answer when $a=10$, $l=1$, $p=10$, $s=1\frac{1}{2}$, $t=1\frac{3}{8}$ and $m=10$.

42. A grocer sells a certain kind of tea at a pies per lb. To increase his profits, he mixes $(p - 6)$ lbs. of this tea with 6 lbs. which cost him p pies per lb. less and sellst he mixture at the same price of a pies per lb. as before. By how much does he increase his profit on each pound of tea sold? Give a numerical result when $p = 10$, and $a = 192$.

43. The manufacturer's list price of an article exceeds the cost of making it by r °/₀; and discount of s °/₀ is allowed on this price. Determine the rate of profit made by the manufacturer.

44. By selling articles at the rate of k articles for b pies a dealer incurs a loss of l °/₀. What °/₀ would he gain or lose if he sold them at the rate of l articles for k pies?

45. In Georgetown x pencils cost a rupee but in Triplicane the same pencils are sold at Re. $\frac{1}{6}x$ per dozen. By buying 144 pencils in Georgetown and selling them in Triplicane a profit of Rs. 2-0-0 is made. Find the sum invested in the transaction.

46. Rama borrows Rs. P. at 100 k °/₀ per annum simple interest for 2 years and lends it for the same time at 100 l °/₀ per annum compound interest payable at the end of every year, Determine the profit made. For what value of l will this profit be zero?

47. The Gokhale Hall in the Armenian Street, Georgetown, is licensed to accommodate 1000 people. 1000 people were present at a lecture, for which k of them paid a rupee each and the remainder Rs. three each. Find the total sum collected If the k persons had paid Rs. 3-0-0 each, and the rest a rupees each, how much greater would the receipts have been? Evaluate your answers when $k = 650$. Suppose again that the total cost of the lecture was Rs. 1350. How many of the two kinds of tickets would have had to be sold just to cover expenses, the total number sold being 1000?

48. I bought a house and 3 years afterwards sold it for Rs. 4725 less than what it cost me, and thus received 20 °/₀ less than what the purchase money would have amounted to at 6 °/₀ C. I. What sum did I pay for the house?

49. If radium costs Rs. 10,000 per gram and 125 milligrammes of radium are required to illustrate a lecture, the charge for admission to which is Rs. 2-8-0, how many tickets would have to be sold to pay the cost of providing the radium. If 2000 tickets were sold, determine the profit °/₀, if the other expenses of the lecture come to Rs. 500.

50. I can save 20 °/₀ of my income. If my income were raised Rs. 25-0-0 a month and my expenses were raised by 20 °/₀, my yearly savings would go up by Rs. 62-8-0, determine my monthly salary.

51. A barters some sugar with flour which is worth 2*s*. 3*d*. a stone (14 lbs.) but in weighing his sugar uses a false stone weight of 13½ lbs. B, on discovering this, says nothing, but raises the price of his flour. What value should B set on his flour so that the exchange may be fair? If B fixes his price at 3*s*. a stone, what °/₀ of profit does he make?

52. A man has to glaze 72 picture frames, each of which requires a piece of glass 17″ by 9″. He cuts these from sheets of glass, measuring 18″ by 25″. How many such sheets must he use? If he buys these sheets at the rate of Re. 0-2-6 per square foot and sells the glass actually put into the frames at Re. 0-4-0 per square foot, what profit °/₀ does he make?

53. A milk-dealer purchases 2½ measures of pure milk at 8 as. a measure. How much water must he add to it so that he may sell at 1 anna an ollock and obtain a profit of cent. per cent.

54. A milkman buys one measure of milk for 7 as. and adds an equal quantity of water to it. If he sells the diluted milk at 9 pies an ollock, using a false ollock which is only ⅘ of the real ollock, what is his gain °/₀?

Madras Secondary School-Leaving Certificate Public Examination.

1916.

N. B.—Only **nine** *questions are to be attempted.*

1. The following table is a classification of the number of people, in thousands, supported by agriculturists and actual workers in agriculture for the years 1910 and 1911. Find to the nearest integer the decrease or increase per cent. in the figures for 1911, as compared with those for 1910 under each head :—

Description.	Supported by agriculturists.		Actual workers in agriculture.	
	1910	1911	1910	1911
Non-cultivating land-owners	34	30	23	19
Cultivating land-owners	461	512	426	484
Non-cultivating tenants	6	2	4	1
Cultivating tenants	225	167	207	151
Labourers	279	284	340	453

2. On March 1, 1913, a person borrowed from *A* Rs. 1,450 at 8 per cent. simple interest, and lent the sum on the same day to *B* at 8 per cent. compound interest : if both the transactions were closed on March 1, 1915, find his gain.

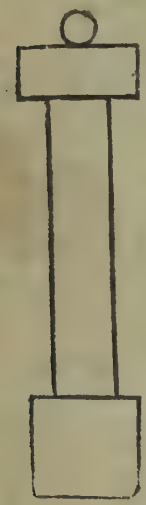
3. Determine by graphs which of the following is the most profitable investment :—

(1) $3\frac{1}{2}$ per cent. Government of India paper at 98 :

(2) 4 per cent. Bank deposit at par :

(3) Rs. 20 Bank shares yielding 10 per cent. dividend at Rs. 48.

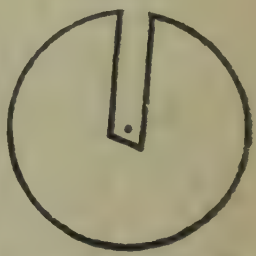
4. A school grants its Games Association Rs. 216 a year, which is distributed among its three branches : Tennis, Football, and Badminton, in the proportion of 14 : 6 : 1. The Tennis branch is 14 strong, and each member pays a subscription of Rs. 2 per half-year. If the branch pays Rs. 6 a month to the pickers and spends Rs. 32 per annum on repairs to the court, find how many balls a month at Rs. 12 per dozen the branch can supply itself with, as not to work at a loss.



5. If a bookseller fixes his prices so as to include a profit of 30 per cent. on the cost price and offers a reduction of 10 per cent. on the published price for cash payments, find his gain per cent. If the cost price of 34 copies of a book be Rs. 50, what is his average gain per copy when all are sold for cash ?

6. Find the volume of a gate pillar consisting of a cylindrical column 6′ high and 1′ in diameter, placed on a cube of 2′ edge and surmounted by a square slab 1′ thick and 2′ side with a sphere of 6″ radius placed on the top of it.

7. A cast-iron cylindrical weight is 16″ in diameter and has a slot 2″ wide and 10″ long as in the figure. Find, correct to 2 places of decimals, its thickness in inches when its weight is 100 lbs. (One cubic inch of cast-iron weighs 0·26 lb.)



8. The following table gives the power (P lb.) to be applied in order to move the weight (W lb.) for a certain machine :—

W	50	100	200	300	400	500
P	106	122	152	184	216	246

Draw a graph showing the relation between P and W, and find to the nearest integer the value of P when $W = 350$, and of W when $P = 158$.

9. If a person buys p dozen mangoes at a as. per dozen, q dozen mangoes at b as. per dozen, and r dozen mangoes at c as. per dozen, and sells $\frac{3}{5}$ of the whole lot at d as. per dozen, and the rest at $\frac{1}{4}a$ as. per dozen, find his profit. If the profit is Rs. 4-6-0 when $p = 12$, $q = 5$, $r = 13$, $a = 20$, $b = 12$, $c = 10$, find d.

10. Solve $2x + 3y = 12$, $0{\cdot}5x - y = 4$.

Compare your answer with results obtained graphically.

11. Construct a quadrilateral $ABCD$ having

$AB = AD = 4$ cm., $AC = 8$ cm., and $CB = CD = 6$ cm.

Measure BD and the angles B and D.

12. Draw an equilateral triangle and a regular hexagon so as to lie between the same parallels and compare the areas of the two figures.

1917.

N. B.—Full marks will be given for answering about five-sixths of the Paper.

1. (1) The H. C. F. of three numbers is 7 and their L. C. M. is 12012. Two of the numbers are 861 and 84. Find the least value of the third number.

(2) You are given that $461010 = 77 \times 5987\frac{1}{7}$. Show how you can write down (correct to five places of decimals) the quotient when 0·046101 is divided by $(14 \times 0{\cdot}55 \times 0{\cdot}001)$ without actually going through the ordinary process of division.

2. The following statement gives the figures under each head of contingent charges, both estimated and actual, for the year 1913-14 :—

Items.	Estimated.	Actual.
	Rs. a. p.	Rs. a. p.
Printing	15,650 0 0	10,595 6 0
Stationery	12,550 0 0	6,610 2 0
Travelling expenses ...	10,630 0 0	12,939 7 0
Postal and other charges ...	6,670 0 0	7,483 9 7
Examination charges ...	9,450 0 0	8,362 10 1
Contribution to the Library ...	5,550 0 0	5,550 0 0
Cost of University Lectures, &c.	6,600 0 0	6,434 8 0
Total		

Fill in the row of totals, and find by what percentage (correct to one decimal place) the total of actuals has fallen short of the total of the estimated charges.

3. Two partners invest Rs. 2,700 and Rs. 1,500 respectively in their business, and arrange that 40 per cent. of the profits should be divided equally between them and the remaining profits treated as interest on capital. If one partner's share of the profits is Rs. 90 more than the other's share, what was the whole amount of the profits?

4. A man buys a field for Rs. 3,750 and lets it at a rent of Rs. 150 payable at the end of each year. The rent is invested at 5 per cent. compound interest, per annum. At the close of three years he sells the field for Rs. 4,000 and draws out the accumulated rents and interest. Find the total he will have and the annual percentage (simple interest) he makes by the whole transaction.

5. A railway track starts from station A with a gradient of 1 in 200, *i.e.*, a vertical rise of 1 yard in 200 yards measured along the track. The gradient remains the same for $\frac{1}{2}$ mile, and is followed by rising gradients of 1 in 150 for $\frac{3}{4}$ mile, 1 in 120 for $\frac{1}{4}$ mile level for 1 mile, and falling gradients of 1 in 150 for $\frac{1}{8}$ mile and 1 in 330 for $\frac{3}{4}$ mile, when station *B* is reached. If station *A* is 179 feet above sea-level, find to the nearest foot the elevation of station *B*.

6. The average heights of a man and a woman and their corresponding weights are given in the following table:—

Height.	5'0"	5'2"	5'4"	5'6"	5'8"	5'9"	5'10"	6'0"
Average weight of a man in pounds.	...	126	139	145	155	162	169	178
Average weight of a woman in pounds.	103	112	126	139	152	156	...	...

Draw graphs in one and the same figure showing the relation between the average height and weight for each sex. Find from your graphs the difference between the average heights of a man and a woman who each weighs 10 stones, and the difference in weight if they are each 5'6½" in height.

7. A cylindrical oil tank is 12 feet in diameter and has ends which are hemispheres of the same diameter. Its total length is 40 feet. If it is lying in a horizontal position, how many gallons of oil will be required to fill it to a depth of 6 feet? (1 gallon is equivalent to 277½ cubic inches nearly.)

8. A child is instructed as follows: 'Think of a number, double it, add 11 to the result, multiply the sum by 3, subtract 12, divide the remainder by 3, give me the answer.'

Find a simple rule by which the number first thought of can be readily deduced from the answer given by the child.

9. *A* and *B* engaged in play: when *A* had lost Rs. 20 he had only one-third of the money which *B* had; but by continuing to play he not only won back his Rs. 20, but also Rs. 50 more with it, and then found he possessed half as much again as *B*. With what sum did they respectively begin?

Reduce the conditions of the problem to simultaneous equations in two unknowns, and solve the problem.

10. A is the centre of a circle of 5 cm. radius, and B is the centre of a circle of 2 cm. radius. *CD* is a direct common tangent to the two circles and touches them at *C* and *D* respectively. If *CD* is 4 cm. long, find the distance between the centres. Draw a neat figure to scale.

11. Divide a straight line AB, 6·4 cm. long, internally at X and externally at Y, in the ratio of 5: 8. Verify the result of your construction by measuring the lengths of the segments.

Also show that they satisfy the formula

$$2/AB = 1/AX + 1/AY.$$

12. Show by means of a carefully drawn diagram how you will verify by paper-cutting the truth of the proposition that the square on the hypotenuse is equal to the sum of the squares on the sides of a right-angled triangle.

1918.

N.B.—Full marks will be given for answering three-fourths of the Paper.

1. The fee income from the secondary department of a school in a certain month is given in the adjoining table:—

Form.	Strength.	Amount. Rs.	As.	Ps.
VI	79	414	0	0
V	80	404	4	0
IV	78	388	8	0
III	76	240	9	0
II	74	191	8	6
I	125	326	8	0

Find

(1) the average fee paid by each boy in the whole department, and

(2) the percentage of the I Form collections to the whole income.

2. A merchant bought 40 candies of paddy at Narsapur at Rs. 38 a candy and sent the same to Tanjore, paying Rs. 10 towards boat hire as far as Nidadavole, and 17 times as much for railway charges from there to Tanjore. His agent sold the whole quantity at Tanjore at Rs. 2 4 as. a kalam. Find the amount of gain or loss to the Narsapur merchant if he paid his agent a commission of 5 per cent. on the sale-proceeds.

[A kalam = 24 Madras measures. A candy = 800 seers, each of which is $\frac{1}{8}$ of a Madras measure.]

3. The following is a rough extract from a Railway Guide :—

Trains from Madura.

Distance from Madras in miles.	Stations.	Boat Mail to Ceylon.
348	Madura *dep.*	11-45
448	Rameswaram *arr.*	16-30

Trains to Madura.

Distance to Madras in miles.	Stations.	Boat Mail from Ceylon.	Rameswaram Express.
448	Rameswaram *dep.*	9-45	12-30
348	Madura *arr.*	14-30	19-30

[NOTE.—Time is calculated for 24 hours from midnight to midnight.]

Represent the motion of the three trains graphically. From the graph find when they are 40 miles from Madura, and when and at what distance from Madura the Boat Mail from Madura meets the other two trains.

4. On each page of an Atlas the border lines are $7\frac{1}{2}$ in. lengthwise and 6 in. breadthwise. The map of the Indian Empire is drawn to a scale of 300 miles to an inch and occupies $\frac{4}{9}$ of the space within the border lines. Find the area of the Empire in square miles.

On another page of the same Atlas, with similar border lines, there is the map of the Madras Presidency, whose area is 158,000 square miles, excluding the Native States occupying $\frac{1}{7\frac{1}{2}}$ of the space enclosed by the border lines. Find the scale to which this map is drawn.

5. In the Post Office, War Loan Certificates were issued on the following terms :—For a payment of Rs. 7 12 as., a certificate was issued which entitled the holder to get Rs. 8 1 a., Rs. 8 14 as., and Rs. 10 at the end of one, three, and five years respectively. What rate per annum simple interest does it come to in each of the three cases?

Given also that the certificate-holder is entitled to receive Rs. 8 7 as., and Rs. 9 6 as., at the end of two and four years respectively, show graphically how the money grows during the five years.

6. A cylindrical granary with a conical top is 14 ft. in diameter on the inside. The wall is 11 ft. high, and the total height to the top is 14 ft. It was full of grain. Half of it was sold at Rs. 5 12 as. a bag, and the other half at Rs. 5 4 as. a bag. Find the amount realized, given that a bag of grain measures 8·25 c. ft.

7. Describe a regular octagon on a straight line 2 cm. long, and find its area by any method.

8. At a public examination in the Madras Presidency, x candidates who had taken up Optional Mathematics scored in Elementary Mathematics an average of m marks, and the remaining y candidates for the examination scored an average of n marks. What was the Presidency average in the subject?

If $x = 2,500$, $y = 6,428$, $m = 53$, and $= 80$, find the Presidency average.

9. A village postman attached to the Post Office at P has to go to five villages, A, B, C, D and E, and return to P. A is 5 miles due north of P; B, C and D are on the eastern side of the road from P to A; B is 8 miles from P and 6 miles from A; C and D are 5 and 6 miles respectively from P on a straight road PCD which is at right angles to PB, E is 3 miles from P, its position being to the west of the road PA, and the roads PE and AE being at right angles to each other. Draw a diagram [scale, 1 cm.

= 1 mile] and indicate the shortest route for the postman. What is the length of his circuit?

10. (1) Construct a quadrilateral $ABCD$, given that $AB = 3$ cm., $DA = 4$ cm., $A = 120°$, $C = 70°$, and AB is parallel to CD. Measure BC.

(2) The middle points of the sides of a scalene triangle are joined so as to form a new triangle. What fraction is this of the given triangle? Explain from a figure how your answer can be shown to be true experimentally by superposition.

11. The distance from Madras to Ootacamund by railway is 353 miles, part of which is ordinary railway and the other part mountain railway. The second class fare over the former is 9 ps. per mile, and over the latter 1 a. 9 ps. per mile. If the total fare be Rs. 18-10-6, find the length of the mountain railway.

12. A reservoir which supplies water to a town has two inlets whose diameters are 30 in. and 16 in. respectively, through which water pours in throughout day and night. Water is to be taken into the town through six pipes, all of uniform diameter. But they will remain open only from 5 a.m. to 9 p.m. daily. What should be the diameter of each of the latter, if the quantity conveyed to the town every day is to be equal to the daily supply into the reservoir, assuming that the rate of flow is the same through all the eight pipes?

1919.

N.B.—Full marks will be given for answering about three-fourths of the Paper.

[$\pi = 3\frac{1}{7}$ *throughout.*]

1. In Form 1 of a certain school there are four divisions.

Division	Strength	Total Marks.
A	34	1372
B	35	1389
C	36	1489
D	37	1275

The adjoining table gives the strength and the total marks gained by the students of each division in a certain subject.

Find, correct to one decimal place, the average marks for the whole Form, and ascertain which division gets the highest average and which the lowest.

2. A merchant cleared his goods by selling them at reduced prices. He sold certain goods whose catalogued sale price was Rs. 760 at a reduction of 15 per cent., another set of goods whose sale price was Rs. 880 at a reduction of $12\frac{1}{2}$ per cent., and the remaining goods whose sale price was Rs. 480 at a reduction of 25 per cent. He, however, gained in all 11 per cent. on his original cost price. Find the original cost price, and also what percentage he would have gained had he not made any reduction in the catalogued sale prices.

3. $ABCD$ is a quadrilateral field in which $AB = 198$ yards, $BC = 264$ yards, and $AC = 330$ yards. D is equidistant from A and C, and at a distance of 214·5 yards from the diagonal AC. (1) Draw a plan of the field to a scale of 1 cm. = 33 yards. (2) Find the area of the field in acres.

4. The distance from P to Q along a road is 65 miles. A person A starts from P at 7 a. m. and travels to Q on his motor-cycle at the rate of 10 miles an hour, taking rest for 10 minutes at the end of every hour of cycling. B starts from P at 8-45 a. m. and goes to Q on his motor-car at a uniform rate of 15 miles an hour. Represent their journeys graphically.

Find from the graph (1) when each of them will reach Q, (2) when and where B will overtake A, and (3) by what distance they will be apart at 9-15 a.m.

5. A person borrows Rs. 1,600 at 8 per cent. per annum, simple interest, and forthwith lends it as follows: (1) Rs. 800 to A at $10\frac{3}{8}$ per cent. per annum simple interest, (2) Rs. 600 to B at 10 per cent. per annum compound interest; and (3) the remaining sum to C at $12\frac{1}{2}$ per cent. per annum during the first year, and 12 per cent. per annum during the second, the interest being compounded with the principal at the close of the first year. At the end of two years he collects the amounts due to him from A, B and C, and pays his dues to his creditor. How much does he gain?

6. (1) Show by a geometrical construction how a length which is $\sqrt{3}$ times a given length may be found.

(2) Describe a regular hexagon on a straight line 1·5 inches long and find its area.

7. A stone road-roller should be 4 ft. long, and should weigh 4 tons 8 cwt. What should be its diameter, given that the weight of the stone is 441 lb. per cubic foot?

8. There are some beggars consisting of men and boys. If the former be given 5 as. 6 ps. each, and the latter 3 as. 6 ps. each, a sum of Rs. 12 10 as. will be required. If each man if given 6 ps. less and each boy 6 ps. more, the total sum required is 2 as. less. Reduce the above conditions to equations, and find the number of men and of boys.

Represent the equations graphically, and verify the result from the graphs.

9. A garden is 240 yards long and 180 yards broad. A moat 5 yards wide and 2 yards deep has to be dug close to it on the outside all round, and the earth dug out spread evenly over the garden.

(1) Find in inches, correct to one decimal place, the height by which the level of the garden will be raised thereby.

(2) A certain contractor undertook to finish the work in 30 days and employed 96 men. At the end of 10 days, only 1 ft. 6 in. of the depth of the moat had been dug all round. How many additional men should he employ from the 11th day to finish the work in the stipulated time?

10. A certain boy took a circular piece of card-board having a radius of 17·5 inches, and made an angle of 144° at the centre. He then cut out and removed from it the sector piece containing the angle 144°, and made the bounding radii of the remaining piece coincide. What shape did he get? Calculate its height.

11. A merchant buys m maunds of coffee seeds at Rs. a per viss., and n maunds at Rs. b per viss. and mixes them. He sells the mixture at a uniform price, and gains c per cent. on his total outlay. Find the selling price of one viss of the mixture.

If $m = 4$, $n = 5$, $a = 1\frac{3}{4}$, $b = 1\frac{1}{4}$, and $c = 12\frac{1}{2}$, find the selling price of the mixture per viss.

12. In a certain meadow there is a hut whose ground-plan is a square each side of which is 28 ft. A cow is tethered to a post at one of its corners by a rope 35 ft. long. Find the area that the cow has for grazing.

1920.

N. B.—Full marks will be given for answering about three-fourths of the Paper.

1. The sea-borne trade of the Madras Presidency for the year 1918-19 shows that the value of foreign imports under Tobacco, raw and manufactured, was Rx. 188,652 for 1918-19 and Rx. 118,000 for 1917-18 the average for the preceding four years up to 1916-17 inclusive being Rx. 49,394. Find (1) the increase per cent. in the value of the imports in 1918-19 over that in 1917-18, and (2) by how many rupees the value of the imports in 1918-19 was in excess of the average value for the preceding five years, *i.e.*, up to 1917-18 inclusive, [1 Rx. = 10 rupees.]

2. The cross-section of the canal running across the Godavari at the Gannavaram Aqueduct is a trapezium (a quadrilateral with two of its sides parallel) 24 ft. at the top and 19 ft. at the bottom, the depth being 6 ft. If the slopes are of equal inclination, draw the cross-section to a scale of 1 cm. = 2 ft. Measure the angle at which the sides are inclined to the bottom of the canal

Given that the length of the Aqueduct is 810 yards, find in cubic yards the maximum quantity of water that can be contained in the whole length of the Aqueduct, and also in how many minutes a boat 15 yards long and going down the canal at $9\frac{3}{4}$ miles an hour will pass the Aqueduct.

3. In the catalogue of a certain merchant the selling prices are given as shown in the accompanying table. He usually allows a discount of 5 per cent. on silks and 10 per cent. on checks. Owing to the war conditions he has raised his prices on all his goods by 20 per cent. but allows still the same rates of discount on the increased prices. A customer buys 6 yards of Tassur silk, 7 yards of Nugur silk, 8 yards of Turban silk, 14 yards of Calicut check, and 16 yards of Holland check. What sum has he to pay the merchant?

Goods.	Price per yard.		
	Rs.	As.	Ps.
Tassur silk	4	11	0
Nugur silk	8	2	0
Turban silk	2	5	6
Calicut check	1	9	0
Holland check	0	12	6

4. To find the height of a certain tower a surveyor fixes a pole vertically at a distance of 80 yards from it, the height of the pole above the ground being 15 feet. On going 5 yards from the pole and away from the tower, he sees the top of the pole and of the tower in the same straight line with his eye. Find the height of the tower, given that the height of his eye is 5 feet from the ground.

5. A person bought two horses for different prices. By selling the first at a profit of 10 per cent. and the second at a profit of 15 per cent. he gained 13 per cent. of the total cost price. Had he sold the first at a profit of 15 per cent. and the second at a profit of 10 per cent. he would have got Rs. 6 less on the whole. Find the cost price of each of the horses.

6. A road runs from P to S passing through Q and R. PQ is $10\frac{1}{2}$ miles and QR 7 miles. A starts from P at 6 a.m. and walks to S at a certain uniform rate, halting at Q for half an hour and at R for one hour. B starts from P at 7 a.m. on his cycle, his rate of cycling being 7 miles an hour. He passes A at 7-45 a.m. and goes on to R, where he stops one hour. He returns to Q, where business detains him for an hour and a half. He then goes straight to S, which he reaches at 4 p.m. Find the distance from P to S, A's rate of walking, and the time when he reaches S.

Represent the above graphically, and find from the graph when and where A and B will be together.

7. Describe an acute-angled triangle ABC, given that $AB=3{\cdot}2$ in., $BC=2{\cdot}6$ in., and angle $A=50°$. Measure AC and the angle C. Find the area of the triangle in square inches by any method.

If it represents a field drawn to a scale of 1 in. = 1 furlong, find the area of the field in acres.

8. When performing a certain experiment in Science the temperature at the end of 10, 20, 30, 40, 50 and 60 seconds was found to be 15°, 30°, 48°, 56°, 58° and 45° respectively, it being observed also that 60° was the maximum reached. Draw a smooth curve, showing the changes in the temperature, and find from it the temperature at the end of 25 seconds, and also the time when the temperature was 50°.

9. If m men can do $\frac{1}{n}$ of a piece of work in p days, find an

expression for the number of men required to do the whole work in q days.

Find that number when $m = 15$, $n = 4$, $p = 12\frac{1}{2}$, $q = 20$.

10. When a certain weight was cast it was found to weigh 1·85 lb. more than was intended. Thereupon a cube, with an edge of 1·5 in., was scooped out from the bottom of it. But thereby the weight was diminished only by 1·08 lb. It is proposed to diminish the rest by scooping out a cone with a bottom circumference of $5\frac{1}{2}$ in. What should be the depth of the cone? ($\pi = 3\frac{1}{7}$.)

11. One litre of water weighs 1 kilogramme (kg.), and one litre of pure milk 1·032 kg. A person bought 12 litres of milk, and found it weighed only 12·288 kg. Find in cubic cm. the quantity of water in the milk. [1 litre = 1 cubic decimetre.]

12. A War-Savings Certificate costs 15*s.* 6*d.*, and is worth at the end of the fifth year £1. After that period 1*d.* is added to it every month, and at the end of the tenth year from the commencement a bonus of 1*s.* is added to the total value of the Certificate. What rate of simple interest per annum does this work out for the whole period of ten years?

Forty-eight Englishmen employed in India saved each a sum of Rs. 2 8 as. 4 ps. daily from March 1 to September 2, both inclusive and with the total sum thus saved arranged to buy War Savings Certificates in England. How many Certificates did they buy in all, and what total sum would they get in England by converting the certificates into cash at the end of ten years after their purchase? [Re. 1 = 1*s.* 4*d.*]

1921.

N.B.—(1) *Full marks will be given for answering three-fourths of the Paper.*

(2) *Decimal answers should be given correct to one decimal place, unless otherwise indicated.*

(3) *In all graphical work the scale should be indicated.*

(4) *Geometrical constructions should invariably be stated.*

1. A metre is a ten-millionth part of the arc of a meridian of the Earth from the Equator to the Pole. The Earth's semi-circumference measured along a meridian is 10,800 nautical miles. Find the number of feet in a nautical mile, if 1 metre = 39·370 inches.

2. How far is the following statement correct?—Three measurements are necessary and sufficient to prepare an exact copy of a given triangle.

Measure the sides correct to the nearest tenth of an inch, and the angles correct to the nearest degree, of the given triangle *ABC*.

Construct a triangle *PQR* of the same shape as *ABC*, but of perimeter 9·6 inches.

N.B.—The data required for the construction of triangle *PQR* should all be obtained geometrically.

3. *AB* is a diameter of circle, whose centre is *C*. The circle is folded so that the points *A*, *B* are brought together at the centre *C*, thus causing two creases. Show that the points *A*, *B*, and the four ends of the two creases are the corners of a regular hexagon.

Calculate the area of the hexagon to the nearest hundredth of a square inch, if the radius of the circle is exactly 2·4 inches.

Check by drawing an accurate figure.

4. Here is an extract from a statement of Educational Progress in the Madras Presidency for three years :—

	1914-15	1915-16	1916-17
Total population ...	41,404,405	No change.	No change
Total scholars in all institutions.	1,542,955	1,615,159	1,661,012
Grand total of expenditure in thousands of rupees.	19,353	20,652	21,688

Find from this statement :—

(1) The ratio of the number of scholars to the total population for 1915-16 expressed as a percentage.

(2) The increase per cent. in the number of scholars in 1916-17 over that of 1914-15.

(3) The increase in the average expenditure per scholar for 1916-17 over that for 1914-15. Answer to the nearest anna.

5. When sovereigns were selling at Rs. 18 12 as., a gentleman purchased a watch-chain made of 12 sovereigns (including a wastage of 6¼ per cent). The making charges were 14 as. per sovereign. A few days later he had to sell it, when the price of the sovereign fell to Rs. 16 4 as. The purchaser would take the chain at its net weight only, and would pay only 75 per cent. of the making charges per sovereign on the net weight of the chain. What did the gentleman lose by the transaction ?

6. A High School Co-operative Society was supplied by a Bombay Firm with Mathematical books at a discount of 33⅓ per cent., and History books at a discount of 40 per cent. off the published prices. Subsequently it was found that the Society would have paid 2½ per cent. of the published prices less, if all the books had been purchased from a Madras book-seller, who offered a uniform discount of 37½ per cent. What percentage of the purchases, calculated at the published prices, was of Mathematical books ?

7. The Secretary of a Chit Fund was authorized to buy a cash box. Every year of use was supposed to decrease the value of the box by 12½ per cent. of what it was worth at the beginning of that year. At the end of two years, the Fund was dissolved, and the box was sold by auction for half its original value. By how much per cent. was this less than its value, as calculated at the above rate of depreciation ?

8. At the time of the German reverse (1914), a body of Germans began their retreat at the rate of 24 kilometres a day, from a place 33 kilometres distant from the Allies' Battle Front. The Allies began their pursuit a day later at the rate of 32 kilometres a day. After three days' marching they were delayed in their pursuit for a day to construct a bridge across a river : they then continued their march for three days more and were again delayed for half a day to repair another bridge blown up by the enemy. After how many days from the beginning of the retreat were the Germans overtaken ?

Draw a graph to illustrate the movements of the armies. Find from the graph—

(1) How far the armies were apart seven days after the beginning of the retreat.

(2) The time when the armies were 82 kilometres from one another.

9. A circus tent is in the form of a cylinder surmounted by a cone. The cylindrical portion of the tent is of diameter 96 feet, the height of the top of the tent is 86 feet from the ground, and the height of the cylindrical part 16 feet. Find the total area of the canvas required for the tent. Answer to the nearest square-foot. [$\pi = 3{\cdot}1416$.]

10. The alloy of which the rupee is made consists of silver and copper melted together in the ratio of $r : 1$. The alloy is worth a rupees per lb., and pure silver s rupees per lb. Express the value of copper per lb. in terms of a, s, and r. What does copper cost per lb., if $a = 36\frac{3}{4}$, $s = 40$, and $r = 11$?

11. For an entertainment to be given to the Principal of a College on the eve of his transfer, the students of the B. A. classes agreed to raise among them a certain sum, levying a uniform rate of subscription. The Intermediate students joining them subsequently, the subscription per student was reduced in the ratio of 7 : 4, although the estimated expenses were increased by 50 per cent. Compare the strengths of the B. A. and Intermediate classes.

From your result find the strength of the College, given that the number of students in the Intermediate classes was 182.

12. The following table gives a series of soundings (measures of depth) taken across a river 42 yards broad at a ford:—

Distance in yards from a bank	0	5	8	10	14	16	19	23	26	30	32	35	38	42
Corresponding depth in feet	0	2·8	4·4	5·0	7·2	8·0	10·2	9·8	8·4	7·3	6·(	1·8	3·1	0

Draw a diagram to represent a section of the river at the ford. From it find—

(1) The probable depth in the middle of the stream.

(2) How far a man, who will be safe up to a depth of 5 ft. 2 in., can venture into the river from either bank without danger.

1922.

N.B.—(1) *Full marks will be given for answering about three-fourths of the Paper.*

(2) *In all graphical work the scale should be clearly indicated.*

(3) *Geometrical constructions should invariably be stated.*

I. Here is an extract from Indian Railway Statistics :—

Year.	Miles of State Railways.	Gross earnings in Rupees.
1909	21,181	36,00,82,035
1912	25,126	61,65,07,005

Calculate—

(1) The earning capacity of the Railways per mile in the two cases, correct to the nearest rupee.

(2) The percentage of increase in that capacity for the period 1903—12, correct to the first decimal place.

II. Two persons standing at P and Q, 300 yards apart on a straight seashore, wish to find the distance of a boat B from the shore. They observe the angles BPQ and BQP to be 50° and 70° respectively. Obtain the distance of the boat from the shore.

III. Assuming that 15 centimetres = 5·9 inches, draw a graph to convert inches into centimetres and *vice versa*. From the graph, get (1) the number of inches (to the nearest tenth) in (i) 4 cm., (ii) 10 cm.; and (2) the number of centimetres (to the nearest mm.) in (iii) 2·8 inches, (iv) 4·5 inches.

IV. The following is one of the rules of a Sangeeta Sabha :—

'Each member shall pay Rs. 1-4-0 every time he attends a performance, *or* pay an advance of annual subscription of Rs. 15 and also 10 annas at each performance'—(performances being weekly).

A member X elects the first method of payment, and another member Y the second method. Both attend the performances regularly. After how many performances does Y gain advantage over X?

Verify the solution of the question by drawing two graphs (in the same figure) to show the amounts paid by X and Y.

V. In connection with a street improvement scheme, a Municipal Corporation purchased the whole block of houses on one side of the street and the ground (708 feet by 108 feet) on which they stood. The houses alone cost Rs. 254,000, and the land cost Rs. 45 per *kuli* (=144 square feet). The houses were pulled down and the materials auctioned away for Rs. 95,000. The street (708 feet long) was then widened by 12 feet, and the remaining land was sold at Rs. 185 per *kuli*. Find the net cost of the improvement scheme.

VI. From a rectangular sheet of brass 32 inches long and 20 inches broad, a circular piece of diameter 20 inches is cut out, and the remaining part is melted and beaten into a rectangular piece of the same shape as the original sheet and of the same thickness. Find its length and breadth to the nearest inch. [$\pi = 3{\cdot}1416$.]

VII. Pure gold is said to be 24 carats fine. Mint gold contains 22 parts of pure gold to every 2 parts of alloy, and is hence said to be 22 carts fine. The gold nibs of Waterman Fountain Pens are 14 carats fine. To make such nibs, 89 ounces of pure gold and 48 ounces of mint gold are melted together with a certain quantity of alloy. Find this quantity of alloy.

VIII. At a 'Grand Clearance Sale,' a firm say that they have reduced their catalogue prices 20 °/₀ and also allow their customers 5 °/₀ off their bills for immediate cash payment. If their profits even after such reductions on cash sales come to 14 °/₀, with what rate of profit in view were the original catalogue prices fixed?

IX. I purchased a bungalow two years back for Rs. 50,000, and spent 10 °/₀ of this sum to effect certain alterations and repairs. The bungalow has remained unoccupied till now, and during this interval I reckon 5 °/₀ per annum, compound interest, on my outlay. If I want to realize a profit of 20 °/₀ on the total amount, at what price should I sell the bungalow now?

X. The owner of a lead mine collected 216·15 tons of stuff from which lead was to be extracted. It contained x °/₀ of lead. After washing, in which process no lead was lost, the stuff was found to contain y °/₀ of lead. Show that $\frac{y-x}{y}$ of 216·15 tons was washed away.

Find the quantity of lead obtained from the stuff, if $x = 15{\cdot}9$, $y = 87{\cdot}45$.

XI. Draw a circle of 3 inches radius and divide it into six sectors whose areas are proportional to the areas of the six continents given in the adjoining tabular statement :—

Names of Continents.	Area in square miles to the nearest tenth of a million.
Asia	17·0
Europe	3·8
Africa	11·4
Australia	3·0
North America	8·0
South America	6·8

(1) Indicate in your figure the values of the angles of the sectors, to the nearest degree.

(2) Enter neatly the names of the continents in the corresponding sectors.

XII. Two cylindrical rollers of radii a and b feet lie side by side on a level ground, touching each other along their lengths. Express in terms of a and b the distance between the lines along which the rollers touch the ground. Find the distance, if $a = 2{\cdot}25$, $b = 1{\cdot}69$.

1923.

N. B.—Full marks will be given for answering about three-fourths of the Paper.

1. The following statistics are taken from the Report on Public Instruction in Madras for 1920-21 :—

Year.	Grants from Provincial to		
	Schools under private management.	Local Board schools.	Municipal schools.
	Rs.	Rs.	Rs.
1919-20 ...	40,54,293	38,89,703	5,53,466
1920-21 ...	42,96,756	84,61,824	4,81,880

Find—(1) the grand total of grants-in-aid in 1919-20 and 1920-21;

(2) the increase per cent. in grants for 1920-21 as compared with those for 1919-20 for Local Board schools;

(3) what percentage the grants in 1920-21 for private institutions was of the total grant for the same year.

2. (1) If a piece of iron plate 2·1 metres long, 1·13 metres broad and 2·3 cms. thick weighs 425 Kilogrammes, find to the nearest gramme, the weight of a square metre of sheet iron 2 mms. thick.

(2) Two cyclists start abreast at 8 o'clock to race on a circular track. If one goes round in 3 minutes 12 seconds and the other in 3 minutes 30 seconds, find at what time they will again be riding abreast past the starting point. Find also the number of complete circuits they will then have made.

3. A tradesman marks certain goods 30 per cent. above cost price; he allows cash customers a discount of 12 per cent. and credit customers a discount of 5 per cent; five-eighths of his goods thus marked are sold for cash and the rest on credit. When the whole stock has been sold and paid for, what is his percentage of profit?

4. The Boy Scouts of a High School have a Savings Bank of their own and one of them had a balance of Rs. 6 at the Bank on the 1st January 1922. At the beginning of February and of each alternate month following, he deposited Rs. 3. In the middle of April when the school closed, he withdrew Rs. 5 for his journey. If the interest be allowed at $6\frac{1}{4}$ per cent. per annum, what interest should be added to his account at the end of June 1922, each month being calculated as one-twelfth of a year?

5. A Railway contractor purchased 360 tons of cement at Rs. 95 per ton two years before it was actually required, and in conveying the cement from Madras to the nearest Railway Station, paid a railway freight of Rs. 9-14-0 on each ton. The unloading and conveying charges amounted to 4 annas for a bag of cement (10 bags of cement weighed one ton). On the above outlay he reckoned 5 per cent. compound interest per annum. During the two years the cement lay idle, he had to stock it in a building paying a rent of Rs. 25 per mensem. When after two years the cement was wanted, he charged the Railway a higher rate and thus realized a profit of 60 per cent. on the total amount spent. Find to the nearest rupee the rate he charged for a ton.

6. A contractor employs 20 men working 8 hours a day in order to finish building a house in 25 days. At the end of 14

days only one-third of the work is finished. He engages more men and all his men now work 9 hours a day. What is the least number of additional men that he must engage if the work is to be done by the specified time?

7. Two boys A and B had their heights measured on the first days of six consecutive years and their measurements are given below:—

—	1916.		1917.		1918.		1919.		1920.		1921.	
	FT.	IN.	FT.	IN.	FT.	IN.	FT.	IN.	FT.	IN.	FT.	IN
A	3	0	3	$4\frac{1}{2}$	3	$7\frac{3}{4}$	3	$10\frac{3}{4}$	4	1	4	$2\frac{3}{4}$
B	3	2	3	$3\frac{1}{2}$	3	$5\frac{3}{4}$	3	$8\frac{1}{2}$	3	$11\frac{3}{4}$	4	4

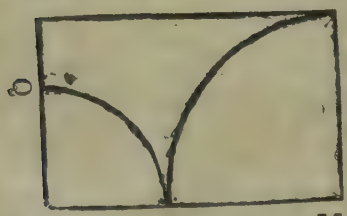
Draw, on one diagram, graphs to illustrate their growth, and find the dates when they were of the same height.

8. If the speed of an aeroplane is v miles per hour. find its average speed from A to B and back, if the wind is blowing all the time from A to B at u miles an hour.

Find the average speed of the aeroplane if $v = 60$ and $u = 5$.

9. I stand on the road six feet from a lamp-post and notice that the end of my shadow just reaches a point at the foot of a wall. Find, by drawing to scale (1 cm. to represent 1 foot), the distance of this point of the wall from the lamp-post, if the light is 10 feet above the ground, and my height is 5 feet 6 inches. Find the distance also by calculation.

10. In the accompanying figure LMNR is a rectangle; a quadrant of a circle is drawn with M as centre and ML as radius meeting NM at P: another quadrant with N as centre and NP as radius, meets RN at Q, P being between N and M, and Q being between R and N. If the curved path LPQ from L to Q is 20 feet less than the path LMNQ and 10 feet less than the path LRQ, find the sides of the rectangle. ($\pi = 3{\cdot}1416$).



11. Draw a right-angled triangle with equal sides and with hypotenuse equal to 2·3 inches. On the other side of the hypotenuse construct a square.

Compare the area of the triangle with that of the square. If the figure thus drawn represents the end view of a hay stack whose breadth is b feet and length l feet, what is the volume of the stack in cubic feet?

Calculate the volume in cubic feet when $b = 20$ and $l = 50$.

12. A boy fills with moist sand a cylindrical bucket which measures internally 12 inches in diameter and 16 inches in height. He empties the bucket and builds a conical mound with the sand on a base whose diameter is 18 inches. Find the height and volume of the mound and the area of its curved surface.

1924

Full marks will be awarded for answering about three-fourths of the paper and five marks will be allowed for neatness and good style.

1. The following statistics taken from the Census of 1921 give the population in thousands of Hindus, Muhammadans and Christians in six districts :—

	Population in thousands.			
	Hindus.	Muhammadans.	Christians.	Total.
Guntur (M) ...	1·514	131	153	
North Arcot ...	1·893	118	36	
Tanjore (M) ...	2·105	180	89	
Trichinopoly (M).	1·746	68	92	
Ramnad ...	1·521	116	85	
Tinnevelly (M) ...	1·600	109	192	

(1) Find the total population for each district (horizontally) and for each community (vertically).

(2) Express the Muhammadan population in each of the districts marked (M) as a percentage of the total population of

that district, correct to two significant figures and state which of them has the highest percentage.

2. (1) Find, in pounds and ounces (correct to the nearest tenth of an ounce), the weight of a cubic foot of water, given that 1 metre = 39·371 inches, 1 kilogram = 2·2046 lb. and 1 cubic metre of water weighs 1,000 kilograms.

(2) A railway company increased its railway fare by one-half and again increased it by $\frac{1}{4}$ of its new value. Find the original cost, to the nearest anna of a ticket which after the two increases costs Rs. 19-4-0.

3. (*a*) The capital of a Co-operative Publishing House is fifteen thousand rupees; one-third of it is borrowed from a Bank at 6 per cent. and the remainder is held in shares. The working expenses of the House are 40 per cent. of the gross receipts in a year. Find what the gross receipts must be in order to pay the shareholders 9 per cent. at the end of the year.

(*b*) Three vessels, each capable of holding 32 gallons, contain mixtures of wine and water. In the first, which is half full, the quantities of wine and water are equal; in the second which is three-quarters full, there are 9 gallons of wine; and in the third, which is full, there are only 4 gallons of wine. If the second is now filled up from the third, and then the first filled from the second, how many gallons of wine and water will be found in the first vessel?

4. For a certain football tournament, twelve teams (each consisting of six players) entered, paying an entrance fee of Rs. 10 each. During each of the first four days of the tournament the spectators who numbered 500 on an average were charged 2 annas each, and on the fifth day, when the finals were played, the attendance was half as much again as on any previous day and tickets were sold at 4 annas each. The Committee in charge spent 30 per cent. of the total income in fencing the ground, 25 per cent. of the remainder for ground-rent and Rs. 30-7-0 on miscellaneous charges. They further set apart Rs. 200 for Reserve Fund after purchasing a silver cup for the winning team and six silver medals for the runners-up, the cost of each medal being one-twelfth of that of the cup. Determine the cost of the cup and of a medal.

5. A dealer offers to sell a motor-car for Rs. 5,000 cash or for twelve monthly instalments of Rs. 500 each, the first being payable at once. Calculate the rate of simple interest charged on the supposition that the Rs. 5000 investad at that rate for 11 moaths gives the same amount as the total of the amounts of the instalments invested at the same rate,

6. Among the elected members in a certain Legislative Council, the ratio of the Ministers' party to the rest was 5 : 2. Eighteen members left the ministerial party and joined the opposition and consequently the ratio was reduced to 26 : 23. Find the total number of elected members.

7. The following table gives the times of sunrise and sunset at a certain place from July to December 1923:—

Date.	July 1.	July 15.	Aug. 1.	Aug. 15.	Sep. 1.	Sep. 15.	Oct. 1.	Oct. 15.	Nov. 1.	Nov. 15.	Dec. 1.	Dec. 15.	Dec. 31.
	H. M	H. M	H. M	H. M	H. M	H. M	H. M	H. M	H. M	H. M	H. M	H. M	H. M
Sunrise	5 47	5 50	5 54	5 56	5 58	5 58	5 58	5 59	6 2	6 8	6 16	6 23	6 31
Sunset	6 39	6 39	6 36	6 31	6 21	6 12	6 0	5 51	5 44	5 39	5 40	5 45	5 53

(1) Plot graphs (in two different diagrams) showing for the given period (*a*) the time of sunrise, (*b*) the duration of sunshine on successive days.

(2) Deduce from the graphs the time of sunrise on November 22, and the duration of sunshine on August 8.

8. A man standing 12 yards from a river bank observes a post on the further bank directly opposite to him. He walks 30 yards parallel to the bank and then walks directly towards the post, reaching the bank after walking 15 yards. Find, by drawing to scale (1 c.m. to represent 3 yards), the breadth of the river, assuming the banks to be straight. Find this distance also by calculation.

9. A surveyor gives the following measurements of a quadrilateral field ABCD (having no re-entrant angle): AB = 1,980 links, BC = 920 links, AD = 1,700 links: $\angle BAC = 35°$, $\angle BAD = 61°$, C and D being on the same side of AB. Draw a plan of the field to a scale of 1 inch for 400 links stating the steps of your

construction. Find the area of the field in acres, given 100 links = 22 yards.

10. A shop-keeper uses a badly made balance so that a body really weighs $a + b$ Q lb. when the balance shows only Q lb. a and b being numbers which are the same for all weights. If the balance shows 1 lb. for a real weight of 1·1 lb., and 2 lb. for a real weight of 1·9 lb., what are the values of a and b and what will the balance show for a real weight of 2 lb.? Draw a graph to show the relation between the real weights and the weights shown by the balance.

11. A canal whose cross-section is a trapezium is 8 feet deep, 18 feet wide at the surface and 12 feet wide at the bottom. Find the area of the cross-section and the number of gallons of water that pass any spot in an hour when the canal is full and water flows at 2 miles per hour. [Given one cubic foot of water = 6·25 gallons.]

12. A right circular cylindrical log of teak is 12 feet long and of diameter 14 inches. Find its volume and curved surface.

If it is planed down to the least amount necessary to form a prism whose cross-section is a regular hexagon, what would be the volume of the prism in cubic feet, given $\pi = \frac{22}{7}$ and $\sqrt{3} = 1{\cdot}73$.

What percentage (correct to an integer) of the whole log would be wasted in planing down.

1925.

N. B.—Full marks will be awarded for answering about three-fourths of the Paper.

1. The following table shows the population in thousands of five Indian provinces according to the Census taken in the years 1911 and 1921 :—

	Population in thousands.	
	1911.	1921.
Bengal	45,483	46,696
Madras	41,405	42,319
Bombay	19,626	19,948
United Provinces	46,807	45,876
Punjab	19,579	20,685

Find, correct to one decimal place—

(i) the percentage of increase or decrease of the population in each of the first three provinces.

(ii) the percentage of increase in all the five provinces taken together.

2. Three men, *A*, *B* and C, have together completed a piece of work for which they receive Rs. 64-14-0. *A* has worked for 4 days of 10½ hours, *B* for 11 days of 7 hours, and *C* for 6 days of 9 hours. How much money ought each to receive?

3. A goldsmith undertook to make a jewel casket out of certain quantities of gold and silver, together weighing 104 ounces and costing £230-4-0. Instead of using the quantities of gold and silver agreed upon, he interchanges them. If gold costs £3-17-6 and silver 5*s*. 6*d*. per ounce, find by how much the value of the casket has been reduced

4. A clerk's salary was £80 for each of the first three years. Then it was raised at the rate of £10 a year for 12 years, after which it remained stationary, When he retired from work he found that his average salary per year had been £174-10-0. How many yeard did he work?

5. Exactly three years ago a man borrowed Rs. 1,890 from a Bank at 6¼ per cent. per annum. At the end of a year he paid the interest for that year and a part of the loan, amounting altogether to Rs. 500. Similarly he paid Rs. 700 at the end of the second year. What sum must he now pay to clear off the debt? (Answer to the nearest anna).

6. Three cyclists, *A*, *B* and *C*, ride round a circular track at 8, 9 and 10 miles per hour respectively. They start together from the same point, but C rides in an opposite direction to *A* and *B*, and meets *A* one minute after passing *B*. What is the length of the track?

7. If the manufacturer of a machine sells 86 machines to a wholesale dealer at 25 per cent. above cost price, and gives one extra machine free; and if the wholesale dealer sells each machine to a shopkeeper at 5 per cent. more than what it cost him, taking into account the machine given free; and if the shopkeeper sells it to a customer for £34-18-0, making a profit of 10 per cent., what did it cost to make?

8. A rectangular metal plate has a circular hole in it. The length of the plate is 1·3 metres, the width is 80 centimetres and the thickness is 2 millimetres. If the plate weighs 32·25 lbs., find the diameter of the hole correct to a centimetre, given 1 c.cm. of the metal weighs 8 grams and 1 lb. = 453·6 grams.

9 The following table shows the number of grams of alum which can be dissolved in 100 grams of water at different temperatures :—

Temperature in degrees C.	0°	10°	20°	30°	40°	50°	60°	70°
Grams soluble in 100 grams of water.	3·9	9·5	15·1	22·0	30·9	44·1	66·6	90·7

Draw a graph and find at what temperature 50 grams of alum could be dissolved in 100 grams of water.

10. A straight rod AB, 2 feet 8 inches long, is held under water, A being 2 feet 6 inches and B 1 foot 2 inches below the surface. Find, by drawing to scale, the distance below the surface of a point C on the rod which divides AB so that AC $= \frac{3}{5}$ BC. Find also the angle which the rod makes with the surface of the water.

11. The cost of making a certain machine consists of an initial expense plus a fixed sum for each machine. To make 25 machines costs £885; to make 40 machines costs £1,335. Find the initial expense and the fixed sum.

12. ABCD is a quadrilateral field in which the diagonals AC and BD are at right angles. AB, BC, CD and AC are 600, 700, 800 and 1 000 links respectively. Draw the quadrilateral to scale and hence find the area of the field in acres. (100 links = 22 yards).

13. A vessel in the form of a hollow right circular cone, whose height is 1 foot 6 inches and radius of base 6 inches, is placed with its axis vertical and vertex downwards. Find how many gallons of water it will hold (1 cubic foot = 6·24 gallons). Find also the cost of tinning the inner surface of the vessel at 4 annas per square foot.

APPENDIX I.

The following tables of money, weights, measures, etc., are worth remembering :—

Indian Money.

12 pies = 1 Anna
16 annas = 1 Rupee

Besides, there are currency notes (Paper-money) issued to the value of Rs. 5, Rs. 10, Rs. 50, Rs. 100 etc

Rupees, half-rupees, quarter-rupees, and two-anna pieces are of silver. The one-anna piece is issued in nickel. Two-anna, and four-anna pieces are also issued in nickel. Pies and quarter anna pieces are of bronze.

The weight of a Re. = 1 tola.

Re. 1 = 192 pies.

English Money.

4 farthings = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£) or sovereign.

The sovereign and the half-sovereign are of gold.

Besides there are silver coins like

a crown (= 5 shillings)
a half-crown (= 2*s.* 6*d.*)
a florin (= 2*s.* = $\frac{1}{10}$ of a £)

and bronze coins like a half-penny.

Other coins are alao mentined :—

1 groat	=	$4d.$
1 guinea	=	$21s.$
1 moidore	=	$27s.$
1 mark	=	$£\frac{2}{3}$

Foreign coins with British equivalents :—

United States :—1 dollar = 100 cents = $50d.$ (nearly.)

France :—1 franc = 100 centimes
($£1 = 25$ francs nearly)

Germany :—1 mark = $1s.$ (nearly)

A Japanese yen = $£\frac{1}{10}$ (nearly)

Table of Number.

12 articles	=	1 dozen
12 dozens	=	1 gross
20 articles	=	1 score

Table for Counting Paper.

24 sheets	=	1 quire
20 quires	=	1 ream
10 reams	=	1 bale

Avoirdupois or Grocer's Weight.

16 ounces (oz.)	=	1 pound (lb.)
28 lbs.	=	1 quarter (qr.)
4 qrs.	=	1 hundredweight (cwt.)
20 cwt.	=	1 ton.

[1 oz. = 16 drams ; 1 cwt. = 112 lbs. ; 1 stone = 14 lbs. 'lb.' should be read as 'pound weight' and '£' as 'pound money.'

Troy Weight

for weighing Gold, Silver and Jewellery.

24 grains = 1 pennyweight (dwt.)
20 dwts. = 1 oz.
12 oz. = 1 pound

[1 pound Troy = 5760 grains
1 pound Avoirdupois = 7000 grains]

Madras Government Weights.

3 tolas = 1 palam
40 palams = 1 viss
8 visses = 1 maund (md.)
20 mds. = 1 candy or baram

[1 tola = the weight of a rupee = 180 grains
1 viss = 5 seers ; 1 seer = 8 palams ;
1 maund = 25 lbs. Avoirdupois]

Madras Measures of Capacity.

8 ollocks = 1 measure (padi)
8 measures = 1 marcal
12 marcals = 1 kalam

(1 parah = 5 marcals ; 1 garce = 400 marcals = 80 parahs)

(1 quota = 21 marcals
1 pucka = 1½ measures.

British Measures of Capacity.

2 pints = 1 quart
4 quarts = 1 gallon
} For liquids and dry goods.

2 gallons = 1 peck
4 pecks = 1 bushel
} For dry goods only.

British Measurement of Lengths.

12 inches (in.) = 1 foot (1 ft.)
3 feet. (ft.) = 1 yard (yd.)
22 yds. = 1 chain (ch.)
10 chains = 1 furlong
8 furlongs = 1 mile

(220 yds. = 1 furlong. 1760 yds. = 1 mile.
A pole = $5\frac{1}{2}$ yds. 1 chain = 100 links).

British Measures of Area.

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 sq. ft. = 1 sq. yd.
484 sq. yds. = 1 sq. chain
10 sq. chains = 1 acre
640 acres = 1 sq. mile

(1 rood = $\frac{1}{4}$ acre; 1 acre = 4840 sq. yds. = 100 cents)

1 cawnie = 1·32 acres.
1 manai = 60′ × 40′
24 manais = 1 cawnie
1 mah = 1600 sq. yds.
1 veli = $6\frac{74}{121}$ acres = 5 cawnies = 20 mahs.
1 sq. mile = 640 acres.

British Table of Cubic Measure.

12 × 12 × 12 or 1728 cubic inches = 1 cubic foot
(cub. in.) (cub. ft.)
3 × 3 × 3 or 27 cub. ft. = 1 cubic yard
(cub. yd.)

(1 gallon = 277·274 c. in.)
(a c. ft. of water weighs 1000 oz.)
(1 c. ft. = 6·25 gallons).

Table of Time.

60 vinadis = 1 naliga
60 naligas = 1 day
(1 jamam = $7\frac{1}{2}$ naligas
1 hour = $2\frac{1}{2}$ naligas)

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year.

1 year has 12 calendar months. The following lines give the number of days in each month :—

Thirty days hath September,
April, June and November.
February hath twenty-eight alone
And all the rest have thirty-one.
But leap-year coming once in four
February then hath one day more.

NOTE :—

1 year = 365·242218 days.
∴ 4 years = 3 years of 365 days each
+ 1 year of 365·968872 days.

∴ every fourth year is generally taken to consist 366 days and is called a *leap year*, *e.g.*, 1892 is exactly divisible by 4 and hence has 366 days. In that year February has 29 days.

[But in so taking 366 days instead of 365·968872 days for every fourth year called a leap year, an error of an excess of ·031128 day is committed. The error amounts to 3·1128 days in every 400 years. To cancel this mistake, years that complete a century, *e.g.*, 1700, 1500, etc., are not considered as leap years unless they are exactly divisible by 400. Thus of the years 1300, 1400, 1500, 1600, the first three are not leap years and 1600 alone is a leap year and so on.]

NOTE :—365 days = 52 weeks 1 day.

∴ If 1-1-17 falls on a Monday, 1-1-18 should be a Tuesday and since 1916 is a leap year, 1-1-16 should be a Saturday].

Useful formulæ to be memorised.

(i) *Area of a rectangle or of a* $\|^m$ } = *base* × *height.*

(ii) *Area of a triangle* = *half the base* × *the height.*

(iii) *Area of a trapezium* = *half the sum of the* $\|$ *sides* × *the distance between them.*

(iv) *The Oce of a* ⊙ = π × *the diameter.*

(v) *Area of a* ⊙ = π × *the square of the radius.*

(vi) *Volume of a cuboid* = *length* × *width* × *height.*

(vii) *Volume of a prism* = *Area of the base* × *height.*

(viii) *Volume of a cylinder* = $\pi r^2 h$ *where* 'h' *is its height and* 'r' *the radius of the circular end.*

(ix) *Volume of a cone* = $\frac{1}{3}\pi r^2 h$ *where* 'r' *the radius of the base and* 'h' *its height.*

(x) *Area of the curved surface of a cylinder* } = $2\pi rh$.

(xi) *Area of the curved surface of a cone* } = $\pi r \sqrt{h^2 + r^2}$.

(xii) *Volume of a hollow cylinder.* } = $\pi \times (R+r) \times (R-r) \times h$ *where* 'R,' 'r' *are the external and internal radii and* 'h' *its height.*

(xiii) $(x + a)(x + b) = x^2 + x(a + b) + ab.$

(xiv) $(x - a)(x - b) = x^2 - x(a + b) + ab.$

(xv) $(a + b)^2 = a^2 + 2ab + b^2.$

(xvi) $(a - b)^2 = a^2 - 2ab + b^2.$

(xvii) $(a + b)(a - b) = a^2 - b^2.$

(xviii) Simple Interest Formulæ :

$$I = \frac{P \times n \times r}{100}; \quad A = P\left(1 + \frac{nr}{100}\right).$$

(xix) Comp. Int. Formulæ :

$$A = P\left(1 + \frac{r}{100}\right)^n$$

ANSWERS.

ANSWERS.

CHAPTER I.

Section I. Pages 6-7.

1. 60; 3000, 50 times. The place values of 3 and 2 will be increased tenfold and those of 6 and 4 will be unaffected.

2. The gratest No. = 94,320. The least No. = 20,349.

3. (*a*) Rs. 11-13-3. (*b*) 1422. (*c*) 149. (*d*) Rs. 380-14-4.

4. (*a*) 152,100; 76,000; 380,000; 43. (*b*) 2,677,584.
(2) 1,277 597,503: 173,732; 34,959,180; 9,344,244;
q = 393, R = 49. (*d*) 1 025,000; 207; 8800.
(*e*) 98,260,173.

5. $x = 348$; The quotient = 350 and the remainder = 84.

6. 102,459; 14,688; 25,704; 102,528.

7. The quotient is 72 and the remainder is 822.

8. The quotient is 226 and the dividend is 87643.

9. 7563; 1890. Remainder 11. **11.** 6 pies. **12.** Rs. 290-14-0.

13 3 cucumbers **14.** Rs. 28-9-0.

CHAPTER I.

Section II. Pages 11—15.

1. 47, 67 and 97.

3. 2, 3, 4, 6, 9, 12 are all factors of 118764. 5 is a factor of 90,245, 2, 4, 5, 7, 8 are factors of 4,837,000.

4. (*a*) 2. (*b*) 1. (*c*) 6.

5. $y = 9$; $x = 0$; $y = 0$ *or* 9, $x = 8$.

6. $a = 3$, $b = 4$ *or* $a = 7$, $b = 8$.

8. 1 ft. 6 ins. **9.** 3; 48; 7. **10.** 47; 23; 188.

11. Multiplicand 1425, Multiplier 76. Second partial product 8550; Quotient 41, Divisor 35. **12.** 32 pieces. **14.** 943 and 851.

15. 18; 43 heaps. **16.** 6, 7, 12, 14, 21, 28, 84; 84. **17.** 6 stones.

18. 12 marakals. **19.** Rs. 1200. **20.** 22 times. **21.** 35', 40'.

22. 119 pupils. **23.** 4 times. **24.** after 140 days on 18-1-18.

25. 141; 188. **26.** 90,090.

CHAPTER I.

Section III. Pages 20—24.

2. 3; 72; 50; 7. **4.** $29\frac{159}{160}$, $25\frac{7}{8}$.

6. $\frac{9}{10}$, $\frac{80}{47}$, $\frac{8}{14}$ $\frac{10}{30}$. (Each fraction is greater than $\frac{1}{2}$. Hence the sum should be greater than 2.)

7. 47 miles nearly (46 miles 4 fur. 191 yds. 0 ft. 9 ins.)

8. 750 acres. **9.** Rs 4. **10.** 18. **11.** Read: that above ground $= \frac{1}{5}$ of the pole $+ 3''$; $37\frac{1}{2}^{11}$, $\frac{15}{7}$. **12.** $\frac{5}{9}$.

13. 300 passengers. **14.** QS = 3 miles, TS = 12 miles.

17. (*a*) 200 $\frac{2}{4}\frac{2}{4}\frac{19}{20}$. (*b*) $\frac{8}{7}\frac{8}{4}\frac{76}{01}$. (*c*) $53\frac{1}{7}$. (*d*) $8\frac{7}{8}$. (*e*) $10\frac{3}{4}$. (*f*) $219\frac{5}{9}\frac{5}{6}$.

18. (*a*) Rs. 4-14-0. (*b*) Rs. 337-2-8, (*c*) Rs. 1283-14-6. (*d*) £3384-6-8. (*e*) Rs. 11744-12-8. (*f*) Rs. 2608-11-8. (*g*) £601-2-6.

19. £12466-13-4.

20. Rs 21527-8-10. **21.** £1697351-4-3.

CHAPTER I.

Section IV. Pages 32—35.

2. 5·64 litres; 506,827 grams; 50,682,700 c. grams; 506 827,000 m. grams.

4. 95·392. **5.** (*a*) 627·0286. (*b*) 9555·772.

7. $g \times g = \cdot 6$, $f \times f = 2 \cdot 8$. $e \times e = 3 \cdot 6$, $d \times d = \cdot 7$, $c \times c = 2$. $b \times b = 1 \cdot 8$, $a \times a = 1$.

8. 34,364·653; 95·122; 42,74·943; 10·991: 671·502; ·158.

9. 75,550; 1638·0945. **10.** Rs. 6,648-7-6.

11. 3·287: 80,100; 317·08; 100·82; 3287: ·0801; 317,080; 10·082.

12. 379·4. **13.** ·01304. **14.** 19 men's weight.

15. 100 nails; ·096. **16.** 78·375; 3·375; 1532·8125; 1·09; 23·2.

17. $5 \cdot 72^{n}$. **18.** ·0$\dot{9}$; ·8; ·$\dot{7}$; ·96; ·$\dot{1}$42857$\dot{1}$; ·$\dot{4}$2857$\dot{1}$; $\dot{6}$; ·25; ·625; ·875.

19. 8 hrs. 15 min. **20.** ·4818; Rs. 96,360.

21. (*a*) 2·729; 5·725; 3·542; 5·275.

(*b*) £8-17-7*d*. £7-17-1*d*; £12-1-1$\frac{1}{2}$*d*.

CHAPTER II.

Section I. Pages 37-38.

1. (*a*) $ab=ba$; (*b*) $\frac{a}{b}=\frac{ma}{mb}$.

2. $\frac{192\,p+12\,q+r}{192}$.

3. (*a*) $C=\frac{b}{64}+\frac{g}{32}+\frac{3p}{64}+\frac{v}{16}$.

(*b*) $P=36d-ne$. (*c*) $S=r+(n-1)\,i$. (*d*) $v=u-$ ft.

(*e*) $S=md-nr$.

4. Rs. $x\,y\,r$. 5. $c=8+(w-12)\,\frac{1}{2}$ when 'w' is greater than 12.

6. (1) $(240\,X+12\,Y+Z)\,d$. (2) $\text{Rs}\,(X+\frac{Y}{20}+\frac{X}{240})$.

7. $x=\frac{22}{15}y$; $\frac{3m}{88t}$ miles per hr.

8. $100\,z+10\,y+x$.

9. Rs. $\frac{12\,lp+12\,mq+rt+48\,x}{192}$.

CHAPTER II.

Section II. Pages 43—55.

10. $+1$; -10 ; 0 ; 1 ; $-14\cdot5$. 11. 577 years. 12. 44 centuries.
13. $x=-5$; $y=-1$; $z=-20$. 14. (*a*) $-5x$. (*b*) $+12y$.
(*c*) $+976$. (*d*) -35. (*e*) -6.
15. -20. 17. $-5°a$. 18. 5 houses up the street.
19. $+5x$; $5a$, -18 ; $+5$; -3 ; $+12$; $+24$; $+13$; -19.
20. It is cooled through $35°c$. 21. $3x°c$.
22. $+(6z-3x)$; he will be a loser to an extent of Rs 9000.
23. $+(x-2y)$; 90 yds. to the south of the gate.
24. $-x=$ (x miles to the south of the station). 25. -825.
26. $+26$. 27. $-(m-n+x-y)$; 21 yds. to the east.
28. $(2x-v)$ ft. above A;—$(b-x)$ ft. below C ; T is in the same level : T is 4 ft. below C. 29. (*a*) $x+12y+3z$;
(*b*) $-(2x+4z)$; (*c*) $7p-q+2r+1$;
(*d*) $6l-m-3n$; (*e*) $-x-2y-4z$.
30. $m-2n$; $2b$; $-r-l$; $-x+6y-7z$.

32. $(q + r) ; (c-p) ; (4-3) ; (3-4)$.

33. 37, 25, 15, 7, 1, $-1\frac{1}{4}$, -3, 267 ; -24, -12, -4, 0, 0, $-1\frac{1}{2}$, -4, -760 ;

21, 13, 7, 3, 1, $\frac{3}{4}$, 1, 343 ; 82, 17, 2, 1, 2; $6\frac{1}{16}$, 17, 160,001 ; -11, -8, -5, -2, 1, $2\frac{1}{2}$, 4, 58 ; $-2{\cdot}2$, $-1{\cdot}6$, -1 $-{\cdot}4$, $2{\cdot}2$, ${\cdot}5$, -8, $11{\cdot}6$;

19, $19\frac{1}{3}$, $19\frac{2}{3}$, 20, $20\frac{1}{3}$, $20\frac{1}{2}$, $20\frac{2}{3}$, $26\frac{2}{3}$.

34. $+4$ or -4, $\frac{1}{4}$ or $-\frac{1}{4}$. **35.** $a^2 b^2 c$; $+x^9$; p^{10} ; 1 ; 0, 9, 4, 1, -6. **36.** $-a^3$; $-37x$.

37. $A = +2p^3q^4$, $B = +2pq^3$ or $A = -2p^3q^4$, $B = -2pq^3$. **38.** $\frac{a^4b}{c^4}$, $\frac{1}{8}$. **39.** 10^{-13} ; 10^{-6} ; 10^3 ; 10^{-8}, 10^5,

40. ·73125 ; ·00012 ; ·11264.

CHAPTER II.

Section III. Pages 59-60.

42. $x^2 + x - 6$; $x^2 + 5x + 6$; $x^2 - x - 6$; $x^2 - 5x + 6$.

43. (1) 995,000. (2) 3, (3) Rs. 25, (4) 89,400 ; 9,600 ; 994,000.

44. $8 ; \frac{a^3}{2} ; \frac{2}{b} ; a-2b ; \frac{c}{b} ; p^2 + q^2$.

45. x^2+3x+2 ; x^2+x-2 ; x^2-3x+2 ; x^2+x-1 ; $4a^2-b^2$; a^2-4b^2 ; $a^4+2a^2b^2+b^4$; $a^4-2a^2b^2+b^4$; x^6-y^6 ; x^2+5x+6 ; $x^2-11x+30$; x^2-25 ; $a^3+3a^2b+3ab^2+b^3$; $x^3-6x^2+11x-6$; $ab^2+a^2c+bc^2-b^2c-ac^2-a^2b$; $x^3+8x^2+19x+12$; $x^2+7x+10$.

46. $2(a+b)$; $a(a+b)$; $a(a-c)$; $p(a+b+c)$; $(5x+4y)(5x-4y)$; $b(2a+b)$; $(1+\frac{r}{100})\frac{r}{100}$; $a(1+x)$; $(1+x)^2$; $(1-x)^2$.

47. 3 ; 5 ; 14 ; -336 ; $-7\frac{35}{48}$. **48.** 55 ; 120 ; $\frac{(n-10)(n-9)}{2}$.

49. $2x$; $2y$; $2x^2+2y^2$; $4xy$; $(x+y+1)(x-y+2)$.

50. Rs. $(a+b)(a-2b)$.

51. 1, 0, 0, $\frac{1}{37}$, $\frac{5}{14}$; -1 $-\frac{2}{5}$, $\frac{1}{7}$, $\frac{7}{15}$, $\frac{3}{5}$; $\frac{1}{6}$, 0, 0, 6, $3\frac{1}{6}$.

CHAPTER III. Pages 64—70.

1. $x = 2$; $x = 1\frac{9}{17}$; $x = 16$; $x = 6$; $x = 1\frac{1}{9}$; $x = 14$; $x = 13$; $x = 16$; $x = 7\frac{1}{2}$;

2. $2x + 20 = 24$; 2.

3. $x = -\frac{1}{4}$; $x = 9$; $x = 4$; $x = -6$; $x = 5$; $x = 16\frac{1}{4}$.

4. A = Rs. 415, B = Rs. 830, C = Rs. 980. 5. 175.

6. 5 furlongs; 35 lakhs of rupees. 7. Rs. 120,000.

8. assets Rs. 16,500; debts Rs. 22,000. 9. 8 as.

10. 16. 17, 18. 11. 16, 18, 20. 12. 10 a m.

13. Length of the bridge = 220 ft.
Length of the train = 176 ft. 14. 11.

15. $x = 2{,}240$; $x = 5$; $x = 18{,}000$; $x = 87\frac{3}{11}$; $x = 6$; $x = \frac{1}{3}$; $x = 54\frac{6}{17}$; $x = \frac{1}{5}$; $x = 1\frac{1}{8}\frac{9}{5}$; $x = 1$. 16. 55 doz.; 168 doz.

17. Rs. 2. 18. 6 days. 19. F = 100,000; G = 200,000. 20. 20 yds × 60 yds, 21. 28 ft. by 25 ft,

22. My friend weighed 120 lbs. and I weighed 135 lbs.

23. A, 40 years: B, 25 yrs. 24. 5 sides,

25. 9 cm, 56. 24 miles. 27. 12, 15 miles.

28. Rupee coins 13; half-rupees 26; quarter-rupees 52: quarter-annas 832.

29. 84, 30. $A = \frac{q + r - p}{2}$, $B = \frac{p + r - q}{2}$.

31. 192 coins. 32 (1) $49\frac{1}{11}$ in. (2) $32\frac{8}{11}$ in.

33. 25 Europeans. 34. Q, 50; P, 100.

35. $m = \frac{b(c + d - 1)}{a}$ 36. $n = \frac{b + ac}{bc - a}$;

$n = \frac{l - bd}{ad + c}$ $n = -4\,(S + \frac{1}{5})$.

37. $W = \frac{dGP}{h - Gd}$; 240. 38. $m = 2\,(S_1 - 60)$;

$n = \frac{5 - 2S_2}{S_2}$; 120; $\frac{1}{2}$. 39. $x = 2{\cdot}84$; $n = 1$; $x = 77_9$;

$m = 4\frac{4}{15}$; $n = \frac{n}{4}$; $n = 4\frac{5}{6}$; $n = 15$; $n = 10_{\infty}$

40. Rs. 1,380. 41. 8 yrs. 42. 1280.

CHAPTER IV. Pages 75—96

13. 6·9 cm.; 1 cm. nearly. **19.** 51°; 129°; 56°; 71°; 53°.
20. (*a*) $31\frac{1}{2}°$; $37\frac{1}{2}°$. (*b*) 90°; 90°. (*c*) 108°; 72°.
23. (*a*) 107°, 117°, 101°, 100°, 115°. (*b*) 90°, 92°, 99°, 79°,
(*c*) 63°, 44°, 73°. (*d*) 111°, 109°, 89°, 51°.
25. 6·1 miles to the nearest ·1 mile.
31. (*a*) $\angle P = 101°$; $\angle S = 93°$; $\angle Q = 65°$; $\angle R = 101°$.
(*b*) $\angle QPS = 93°$; $\angle PSR = 63°$ and $\angle QRS = 104°$.
(*c*) PQ = 2·6 cm. nearly, QR = 6 cm., $\angle QPS = 102°$.
(*d*) PS = 2·9 cm., RS = 5·3 cm., $\angle QPS = 107°$, $\angle RSP = 81°$.
(*e*) $\angle PSR = 90°$, $\angle QRS = 84°$, $\angle RQP = 67°$, $\angle QPS = 119°$, QR = 5 cm.
(*f*) PS = 4·7 cm., RQ = 7·7 cm., RS = 5·1 cm., $\angle RSP = 109°$ and $\angle QRS = 69°$.
(*g*) PQ = 5·5 cm.; RS = 3·8 cm.; PS = 3·7 cm. $\angle QPS = 78°$; $\angle RSP = 106°$.
(*h*) $\angle QPS = 98°$; $\angle RSP = 92°$; PQ = 1·2 cm.; PS = 1·75 cm.; CS = 1·65 cm.
33. 390 yds. **34.** 60°.
40. PA = 4·89 miles, PB = 6·45 miles.
41. 42·1 miles per hour. **45.** ·6 cm.
61. Each side is 3·6 in. nearly; 67° and 143°.

CHAPTER V. Pages 101—108.

3. $x = 15$ tons $x =$ £2-3s-6d. **4.** $2\frac{3}{4}$ mds.; Rs. 25-15 as.
5. $\frac{(6+2)}{(5+2)}$; $\frac{6}{5}$; $\frac{(6-2)}{(5-2)}$; $\frac{(5-2)}{(6-2)}$; $\frac{5}{6}$; $\frac{(5+2)}{(5+2)}$.
6. 5 hrs. 20 min. **7.** $13\frac{1}{3}$ miles per hr.
8. Rs. 26-6-6. **9.** 28 ft. $10\frac{1}{2}$ ins. 128 ft.
10. Rs. 1,600. **11.** No.
12. 11-25 A.M. **13.** Rs. 19-8 as
14. 1 hr. **15.** 7 hrs. 30 min.
16. 4 sovereigns; 18 bangles. **17.** 15 pieces.
19. 2 hrs. 6 min. **20.** 100 cows.
21. Rs. 15,600. **22.** 66 days.
23. Rs. 202-8 as. **24.** 3 weeks.

25. 1 vise ; 2 vis. 20 pals.
26. Rs. 4-2 as.
27. 5 as. 8 ps. per md.
28. 12 ft.
29. Rs. 2,64,000.
30. 4 in. and 5·6 in.
31. $\frac{8}{13}$.
32. Rs. 2,200 ; Rs. 1,650.
33. 916,666⅔ mohurs' weight.
34. Rs. 472-8 as.; Rs. 562-8 as.
35. $2\frac{4}{5}$ tons.
36. $78\frac{1}{6}$ yds. ; $46\frac{7}{8}$ yds.
37. 180·5 miles ; 722 miles and 361 miles.
38. 4 cm.; 6 cm. 10 cm ; 16 cm.
39. Copper 19 lbs. 11 oz, Zinc 10 lbs. 15 oz., Lead 4 lbs, 6 oz.
40. 18 ; 6 ; 9 ; 12 ; 15.
41. 16 seers of sugar ; 82 seers of wheat ; 4 seers of ghee.
42. A, Rs. 2,700 ; B. Rs. 3,600 ; C. Rs. 4,500.
43. Passengers, Rs. 130,800 ; Goods, Rs. 129,200.
44. Chauffeur, Rs. 1,760 ; Repairs, Rs. 660 ; Petrol, Rs. 440.
45. A, Rs. 15,000 ; B, Rs. 10,000 ; C, Rs. 20,000.
49. $\frac{19}{77}$ measure.
50. $1\frac{1}{24}$ measures.
51. 66 ; 17 ; 13.
52. 53 ; 45.
53. B = Rs. 3,600.
S = Rs 19,000.
N = Rs. 3.600.
54. Stracey, Rs. 35,000 and David, Rs 70,000.
55. 207825.

CHAPTER VI. Pages 110—114.

1. (*a*) $x = 19$. (*b*) 1·8 in.
2. 65·5 ; 67 ; 66·8. To Ranganathan.
3. 53; three are above the average and three below the average.
4. 57.
5. 42.
6. $5\frac{1}{2}$ hrs. a day.
7. Rs. 3-1-1 nearly ; Rs 818-5-4.
8. Rs. $5\frac{1}{2}$ p. month.
9. 3185 ; 455.
10. 53.
11. 45.
12. A = 531 ; B = 596 ; C = 227, B is the most thickly-populated Taluk.
13. 6 pies to the nearest pie.
14. Rs. 9-9-9 to the nearest pie.
15. Rs. $\frac{12z}{x-y}$; Rs 20.
16. $9\frac{8}{16}$ kalams of paddy.

17. Rs. 21,500. 18. $\left(14+\frac{n}{4}\right)$ miles.

19. Rs. 261-12-11 to the nearest pie.

20. $+2$; $-3\frac{1}{2}$.

22. No. 148 P. $11\frac{7}{11}\frac{8}{7}$ miles per hr. ;
No. 60 P. $15\frac{1}{9}$ miles per hr. ;
No. 76 P. $14\frac{2}{4}\frac{0}{7}$ miles per hr.

CHAPTER VII. Pages 116—121.

1. 70 %. 2. $x = 83\frac{1}{3}$. 3. 4 °/₀. 4. 7·88 °/₀.
5. 46·8 °/₀. 6. 75 °/₀. 7. The money-lender.
8. in (*b*). 9. in Trichinopoly District.
10. $1\frac{1}{3}$ °/₀ ; $1\frac{1}{9}$ °/₀ ; the first boy. 11. My friend's weight,
12. Rs. 35. 13. 300 °/₀. 14. 400. 15. by $33\frac{1}{3}$ °/₀; to $66\frac{2}{3}$ °/₀. 16. (*a*) Rs. 12-12 as. (*b*) Rs, 20
17. $51\frac{1}{8}\frac{1}{8}$ °/₀. 18. Rs. 125. 19. Rs 25.
20. 1st line :—45334 ; 43. 2nd line :—105055, 7 3rd line :—52725. 21. Raw cotton, 1,530,230 cwt. ; Wheat, 7,323,845 cwt.; Rice, 1,933 072 cwt ; Linseed 1,738,240 cwt.; Jute, 271,045 cwt.; Tea, 50,981 cwt. (answer expressed to the nearest cwt. in each case). 22. Rs. 6,00,000. 23. (*a*) Rs. 5 250 (*b*) Rs. 200.
24. 700,000. 25. Rice 60 °/₀ ; house-rent, 25 °/₀ ; Oil $6\frac{1}{4}$ °/₀ ; Milk, 5 °/₀ ; other expenses $28\frac{3}{4}$ °/₀ ; 20 °/₀ of his expenditure.
26 6 °/₀. 27. $35\frac{5}{1}\frac{5}{2}$ °/₀. 28. 25 °/₀. 29. $24\frac{8}{9}$ °/₀.
30. $16\frac{2}{3}$ °/₀ ; 50 °/₀. 31. $42\frac{6}{7}$ °/₀.
32. To last for the same time, the sums to be invested in the Kashmere and German shawls are in the ratio $\frac{111\frac{1}{4}}{100} : \frac{133\frac{1}{3}}{100}$.
Hence it is more economical to buy the Kashmere shawl.
33. Rs. 509-4 as. 34. $20\frac{10}{27}$ °/₀.
35. $\frac{100\,(t-p)}{p}$ °/₀ ; $\frac{100}{(100-q)}$.
36. Best in Vernacular ; Worst in English.
37. 25 °/₀. 38. $16\frac{2}{3}$ °/₀.

CHAPTER VIII. Pages 123—142.

1. 375 sq. ft. **2.** 246 sq. ft. ; 10 chains ; 11 feet ; 16 chains ; 10 feet ; $60\frac{1}{2}$ yards. **3.** $98\frac{1}{8}$ acres.

5. 121 sq. ins. **6.** $5\frac{1}{8}$ sq. ins.

9. (*a*) 39,600 sq. ft. (*b*) 4,900 sq. ft. (*c*) 1575 sq. ft.

10. 9955 sq. ft. **11.** 25,900 sq. ft. ; 70 ft. ; 120 ft. ; Rs. 964.

12. Rs. 96. **13.** 22·1 sq. ft. **14.** 144. **15.** 144.

16. 21. **17.** Rs. 9. **18.** Rs. 108. **19.** Rs. 27.

20. 200 mats ; 2,568 sq. ft. **21.** 36 yards ; Rs 18.

22. Rs. 31-8 as. **23.** 89,900 sq. ft.

24. 234 sq. yds. **25.** $\left(xy - 2x - \frac{6y}{5} + 2\frac{2}{5} \right)$ sq. ins.

26. 2640 sq. yds. ; $6746\frac{2}{3}$ sq. yds.

27. $4880\frac{1}{4}$ sq. ft. ; $7529\frac{15}{16}$ sq. ft. **28.** 10 ft **29.** $18\frac{3}{4}$ ft

30. 50 yds. **31.** Rs. 9-5-0.

32. (*a*) 660 sq. ft. ; 270 sq. ft. (*b*) 16 ft ; 224 sq. ft. (*c*) 14 ft. ; 9 ft. (*d*) 25 ft. ; 1200 sq ft.

33. $h = 8$; $l = 25$; $b = 22$; and $A = 1200$.

34. Rs. 12-3 as. **35.** Rs. 19 **36.** 1524 sq ft. ; Rs 95-4 as.

37. 8 yds. broad ; 4 yds high **38.** Rs. 45.

39. Rs. $[2 + \frac{1}{12}(ab + ac + bc)]$. **40.** Rs 6.

41. (*a*) 7·25 sq. ins. ; (*b*) 12 48 sq. yds. ; (*c*) 22 acres.

42. 13·69 sq. inches ; 12·88 sq. inches. **43.** 1 : ·866.

46. 6 cm. **48.** 1560 sq. yds. ; HB = 50 yds. ; GC = 102 yds.

50. 1559 sq. ft. to the nearest sq. ft. **51.** $2\frac{7}{8}$ ins.

52. 15·70 acres nearly.

54. 2·6″ nearly ; 6·5 sq. ins. nearly.

55. Their bases are in the ratio of 4 : 5. **56.** 2 chains.

60. 2 sq. ins. nearly. **62.** (1) 3·9 sq. ins. nearly. (2) 29·9 sq cm. nearly.

63. 3·68 sq. cm. ; 7·6 sq. cm. ; 3·96 sq. cm ; 8·53 sq. cm. 6·53 sq. cm.

65. $b = 4$; $d = 2\frac{2}{3}$; $A = 96$; $a = 29$.

66. 998 acres nearly. **67.** 50 links. **68.** Rs. 4687-8-0.

CHAPTER IX. Pages 148—151.

4. (+ 5, — 5). **8.** 8·6 miles nearly.
16. (+ 7, + 15), (— 3, — 15), (0, 0). **18.** 44 8.
19. (— 3·50), (0, + 3·2); 2·1; 2 3.
20. The island is 70 miles from Cochin and is $37\frac{1}{2}°$ N of E from it. The island is 104 miles from Calicut and is 21° S of W from it.
23. 116 sq. units.

CHAPTER X. Pages 154—158.

1. (*a*) Rs. 20; Rs. 270. (*b*) Rs. 30; Rs. 230.
(*c*) £61-17-6; £611-17-6. (*d*) Rs. 145; Rs. 895.
(*e*) £632-15-0; £4858-15-0.

2. $P = \frac{1 \times 100}{nr}$; $n = \frac{1 \times 100}{Pr}$; $r = \frac{1 \times 100}{Pn}$; $P = \frac{100 \times A}{100 + nr}$;

$n = \frac{100\,(A - P)}{Pr}$; $r = \frac{100\,(A - P)}{Pn}$.

3. (*a*) P = Rs. 400; $n = \frac{2}{3}\frac{5}{4}$. (*b*) P = Rs. 3212; A = Rs. 3971.
(*c*) P = Rs. 60; I = Rs. 15. (*d*) $r = 3\frac{1}{3}$; A = Rs. 2560.
(*e*) $r = 8$; I = Rs. 48. (*f*) P = Rs. 550; $n = \frac{10}{11}$.
(*g*) $n = 1\frac{1}{2}$; A = Rs. 472. (*h*) I = Rs. 105; A = Rs 855.
(*i*) P = Rs 500; $r = 8$. (*j*) P = Rs. 1160; A = Rs. 1334.
(*k*) P = Rs. 695; A = Rs 729-12-0.

4. 2 pies per rupee per month. **5.** Loss = Rs. 143-12-0.

6. House 7·8 °/₀; Garden 9·75 °/₀. The garden pays a higher rate.

7. 3 years. **8.** 25 months.
9. Rs. 480. **10.** Rs 1524-11-4 nearly.
11. Rs. 600. **12.** 20 %.
13. $12\frac{8}{4}\frac{6}{7}$ °/₀. **14.** 12 °/₀.
15. Due on 22-10-15. **16** $10\frac{10}{29}$ °/₀; 20 °/₀.
17. Rs. 1632.
18. Rs. 768-4-7 to the nearest pie.
20. $6\frac{88}{117}$ °/₀. **21.** $3\frac{10}{19}$ °/₀.
22. Rs. 1407; 8 °/₀. **23.** $5\frac{9}{5}\frac{8}{1}$ °/₀.
24. $19\frac{18}{68}$ °/₀. **25.** 6·4 °/₀.

CHAPTER XI. Pages 161—176.

1. 2·0 lbs. unit chosen ·1 lb.;
420 lbs. „ 10 lbs.;
£ 032, „ £·001;
Rs. 60. „ Re. 1;
·48 cwt., „ ·01 cwt.;
48,000 sq. miles. „ 1000 sq. miles;
4,000 gallons. „ 100 gallons;

2. A's, Rs. 864,800; Rs. 865,000; Rs. 860,000
correct to 4, 3, 2 significant figures respectively.
B's Rs. 786,500; Rs. 786 000; Rs. 790,000
correct to 4, 3, 2 significant figures respectively.

3. 20,00000; 2040000; 2041000; 2040800; 2040790.

4. 20,000 miles, 25,000 miles, 24 900 miles,

5. 1 gram = ·04 oz. **6.** 1 cub. cm. = ·002 pint.

7. (*a*) 2333·22; 5·77, (*b*) 1049176; 7·5.
(*c*) 2; 3.
(*d*) 1230 units; 1230×10^3; 1067×10^6; ·2319.

8. ·64; ·94; ·77; ·34; ·72; ·22.

9. 1·0. **10.** (*a*) ·1429.
(*b*) 1·7183. (*c*) 5493.
(*d*) ·683.

11. 10·76 sq. ft.; 32·2 ft. **12.** 92,160,000 miles.

13. 931 days. **14.** 10 lbs. **15.** 12 million sq. metres.

16. 180,000. **17.** £55-10*s*.-9*d*. *to the nearest penny.*

18. 5021×10^5 maunds. **19.** Rs. 405000.

20. 148,800,000 sq. miles. **21.** Rs. 1213.6 as.-7 ps.

22. £692,000,000.

23. 786; 880; 1·414; 1·732; ·3183; 15·03; 1·0882.

24. 11*s*.-10*d*. **25.** 75·1 per cent.

26. $\frac{1}{p} = ·318$; $\frac{1}{e} = ·368$; $p + \frac{1}{p} = 3·46$; $e + \frac{1}{e} = 3·09$;

$$\left(p - \frac{1}{p}\right)\left(e - \frac{1}{e}\right) = 6·64;$$

$e^2 + p^2 = 17·3$; $(p + e)(p - e) = 2·48$; $\frac{1}{2}\left(p - \frac{1}{p}\right) = 1·41$.

27. $x = y \times ·4343$; $x = ·3010$; $y = 1·946$, ·1655, ·05827.

28. 12·91. **29.** 10·5. **30.** Rs. 15·035. **31.** 14 per cent.

32. 18·1 per cent. **33.** 252 to the sq. mile; Rs. 2-5-9 per head; 51 °/₀; Rs. 1165, **34.** 3417.

35. 1100 miles per hour. **36.** 196; 6·027; 83·4; 862×10^9; ·523; 25·6; ·345; ·243; 1·57.

37. 11·225; 16·892; ·472; 8·368; 5·659.

38. (*a*) 5·8 miles per hour. (*b*) 12000 tons per sq. mile, the unit taken being 1000 tons.

39. M = 27·32 **40.** 1 cm. = ·0328 ft.; 1 gram = ·00220 lb.; 10546 killograms per sq. metre. **41.** 1728 yds.; 13·2 ins.

42. 69 miles. **43.** 353,100,000. **44.** 13 900 acres.

45. ·074 °/₀. **46.** A fathom = 6 09 ft. **47.** 356×10^5 miles

48. 189. **49** Earth 510×10^{12} sq. metres.
Moon 380×10^{11} sq. metres.

50. 22 years. **51.** 14·60 lbs. **52.** Perimeter = 14" to the nearest inch; Area = 12 sq. ins to the nearest sq. inch.

53. AC = 2·02"; BC = 2 29"; Area of △ 2 19 sq ins. ∠A = 1 030 radians, ∠B = ·8552; radians, ∠C = 1·257 radians.

54. G = 981. **55.** + ·073 °/₀; — ·32 °/₀; ·4216 sq ins. in excess.

CHAPTER XII. Pages 182—186.

[The answers given below are to the nearest unit pie or penny.]

1. (*a*) Rs. 874-12-10. (*b*) £1011-4-10. (*c*) Rs. 1672-15-6. (*d*) £868-4-4½*d*, (*e*) Rs. 1504-11-10.

2. $n = 2$; $r = 5$; P = 1000. **3.** Rs. 2000.

4. Rs. 4-10-3. **5.** 60,500; Rs 41322-5-0.

6. Rs. 2391-9-4. **0** From Krishnalal 4 as. 1 pie.

8. Rs. 40 000 **9.** 3431. **10.** Rs. 14,400; Rs 6944-7-1.

11. Rs. 900. **12.** Rs. 185-3-0; Rs 121-8-0.

13. 828.262; 811,821. **14.** 1·23". **15.** 5·499.

16. Rs. 5967-13-8. **17.** 87·04 °/₀. **18.** Rs 1909-1-5.

19. Rs. 89966-1-7. **20.** Rs. 662-4-7.

21. Rs. 1379-12-0. **22.** 5 years.

24. Rs. 855-10-6. **25.** Rs. 5339-15-7.

26. Rs. 202,600. **27.** Rs. 32-8-2.

CHAPTER XIII. Pages 188—196.

1. (*a*) 90 c. in.
(*b*) 16500 c. cm.
(*c*) 375 c. ft.

4. 3240 c. ft. ; 108 c. ft. **5.** 106·92 measures.

6. 277·2 c. ins. ; 4 gallons nearly.

7. $1620\frac{2}{3}$ lbs. **8.** 12 xyt c. in. ; 567 c. in.

9. 1,260,000 gallons.

10. Height = 10 ft. ; floor area = 760 sq. ft.

11. $2\frac{1}{2}$ hours. **12.** 9801 c. ft. of water.

13. 17,424,000,000 c. ft. of water. **14.** ·05 ins.

15. 6,600 cart-loads.

16. 645 hours, 20 minutes. **17.** 63 c. ft.

18. 200 c cm. **19.** ·3 ins.

20. 2 sq. cm. **21.** $\frac{5}{12}\,lwhr$ c. ft.

22. $\frac{3}{16}$ cm. **23.** $h + \frac{243n}{5120xy}$ ft

24. 4830 c. ins. ; 930 c. ins. ; 465 oz.

25. 4824 c. ins ; 18216 c. ins. **26.** 1·02 c. ins.

27. $1831\frac{1}{2}$ lbs. **29.** 4 c. ft, 483 c. ins. ; 1245 c. ins. of wood.

30. 7·94 grams nearly ; 1406·7 c. cm. nearly.

31. $\{(a+2t)(b+2t)(c+2t)-abc\}$ c. ins. ; Rs. $2\{(a+2t).(b+2t)(c+2t)-abc\}\}$; 14·211 c. ins ; Rs. 28-6 as. 6 p. (nearly)

32. (*a*) (i) 1000 c. ins. ; 600 sq. ins. ; (ii) 4320 c. ins. 1632 sq. ins.
(*b*) $251\frac{1}{3}$ c. ft. ; $435\frac{1}{9}$ sq. ft.
(*c*) 164 c ins. ; 210 sq. ins.
(*d*) 80 c. ins. ; 244 sq. ins.

33. Rs. 22-12-5 ps. nearly. **34.** Rs. 12-0-3 ps. nearly.

35. 216 : 27 : 8. **36.** 70 c. ft. — 1608 c. ins.

37. 4 : 1 : 8 : 1 **41.** 5·96″. **42.** 16 ft.

43. 7,761,600 c. ft. **44.** $\frac{xd}{12y}$ sq. ins.

46. 12,160 lbs. **47.** $\frac{lbh-pn}{lb}$ inches

48. $\frac{12c\,(ab+xy)}{d}$ inches ; 24 ins.

CHAPTER XIV. Pages 200—211.

1. $y = \frac{15-x}{2}$; $y = \frac{x-15}{2}$; $y = \frac{15-2x}{2}$; $y = \frac{x}{15} - 10$;

$y = 5x + 100$; $y = 15x + 150$; $y = \frac{3x}{2} - 18$; $= \frac{25x+1}{20}$.

2. $x = 5 - 2y$; $x = \frac{5-3y}{5}$; $x = \frac{15-4y}{6}$; $x = 45y - 18$;

$x = \frac{8}{2}y - 12$; $x = \frac{20-5y}{4}$.

3. (*a*) $x = 1$. $y = 5$; $x = 2$, $y = 8$;
$x = 3$, $y = 11$; $x = -\frac{6}{11}$; $y = \frac{4}{11}$
$x = 4\frac{3}{8}$, $y = 15\frac{1}{8}$; $x = -1$. $y = -1$;
$x = -3\frac{3}{8}$. $y = 8\frac{4}{1}$.
(*b*) $x = -3$. $y = 2\frac{3}{4}$; $x = 6\frac{1}{4}$. $y = -1\frac{7}{8}$;
$x = 3\frac{9}{17}$. $y = -\frac{4}{9}$;
$x = 1\frac{1}{2}$, $y = \frac{1}{3}$; $x = 6\frac{9}{10}$; $y = -2\frac{1}{6}$.

4. (*a*) $x = 6$, $y = 2$. (*b*) $x = 8$, $y = 2$. (*c*) $x = 1$, $y = 5$.
(*d*) $x = 3$, $y = 4$. (*e*) $x = 3$, $= -2$. (*f*) $x = 100$, $y = 99$.
(*g*) $x = 1\frac{4}{18}$, $y = \frac{7}{81}$. (*h*) $x = -\frac{5}{7}$, $y = -\frac{1}{14}$.
(*i*) $x = 3$, $y = \frac{1}{3}$. (*j*) $x = 2$, $y = 3$. (*k*) $x = 20$, $y = 8$.
(*l*) $x = 10$, $y = -5$. (*m*) $x = 5$, $y = 2$ (*n*) $x = 2$, $y = -1$.
(*o*) $x = 2$, $y = 1$. (*p*) $x = 4$, $y = 1$. (*q*) $x = 6$ $y = 8$.
(*r*) $x = 4000$, $y = 3000$. (*s*) $x = 500$, $y = 225$.
(*t*) $x = 5$, $y = 2$. (*u*) $x = \frac{1}{2}$, $y = \frac{3}{4}$. (*v*) $x = 1$, $y = \frac{1}{2}$.
(*w*) $l = 2\frac{10}{21}$. $b = -2\frac{2}{41}$. (*x*) A = 1, B = 1.
(*y*) $m = 24$, $n = 20$. (*z*) $x = -\frac{q}{b^2(b+1)}$; $= -\frac{q}{b(b+1)}$.

5. (*a*) $x = 5$, $y = 5$. (*b*) $x = 5$, $y = 6$. (*c*) $x = 12$, $y = 24$.
(*d*) $x = 50$, $y = 36$. (*e*) $x = 20$, $y = -10$.
(*f*) $x = 55$, $y = 65$ (*g*) $x = 1$, $y = 5$. (*h*) $x = 4$, $y = 3\frac{1}{3}$.
(*i*) $x = 1\frac{1}{2}$, $y = 1\frac{1}{8}$ (*j*) $x = 2$, $y = 3$. (*k*) $x = -1$, $y = 2$.
(*l*) $x = 16$, $y = 2$. (*m*) $x = -\frac{1}{2}$, $y = -2$,
(*n*) $x = 30$, $y = 70$. (*o*) $x = 117$, $y = 78$ (*p*) $x = 2$, $y = 3$.
(*q*) $x = 10\frac{4}{7}$, $y = 9\frac{6}{7}$. (*r*) $x = 2$, $y = 3$, (*s*) $x = 2$, $y = 3$.
(*t*) $x = 1$, $y = 3$. (*u*) $x = 4$, $y = 6$. (*v*) $x = 1\frac{5}{7}$, $y = 3\frac{1}{7}$.
(*w*) $x = 15\frac{22}{49}$, $y = 11\frac{87}{49}$. (*x*) $x = \frac{ac-bd}{a^2-b^2}$, $y = \frac{ad-bc}{a^2-b^2}$.
(*y*) $a = 8\frac{3}{4}$, $b = 3$. (*x*) $x = 4\frac{4}{31}$, $y = 3\frac{3}{5}$.

6. 19; 11. **7.** a chair Rs. 6; a table Rs. 10.
8. A, 23 marbles; B, 8 marbles.
9. Trunk Rs. 14; Lock Rs. 2. **10.** Curd 4 as.; rice 6 as.
11. 30; 4. **12.** 18; 10.
13. Trained teachers 7; Untrained teachers 4.
14. Men 42,000; Women 65,000.
15. Bench Rs. 10; chair Rs. 7-8 as.
16. $l = 40$ yds, $b = 36$ yds.; $l = 60$ yds., $b = 24$ yds.; $l = 48$ yds., $b = 30$ yds.
17. Original $l = 900$ ft.; Original $b = 400$ ft.; Present $l = b = 600$ ft.
18. 1st field $l = 30$ yds, $b = 24$ yds.
2nd field $l = 36$ yds., $b = 20$ yds.
3rd field $l = b = 30$ yds.
19. 13 cm. **20** Bullock cart, 4 as.; cycle 2 as.
21. Wet lands 10 acres, Dry lands 100 acres.
22. $x = \frac{1}{2}$. $y = \frac{1}{3}$. **23.** A = 2. B = — 3. C = $1\frac{1}{2}$; $36\frac{1}{2}$.
24. — 27. **25.** $m = \frac{2}{5}$; $c = 4$. **26.** $a = 3\frac{1}{2}$; $b = -1\frac{1}{2}$.
27. $\frac{3}{6}\frac{8}{2}\frac{6}{5}$. **28.** 357.
29. (*a*) A = $-1\frac{7}{8}$, B = $2\frac{1}{8}$. (*b*) A = 3, B = 2.
30. A = — 1; B = 2. **31.** l=6; m=·009: R=9·6 and 28·4.
32. Yes. **33.** $x = \frac{d-r}{p-q}$; $y = \frac{pd-qr}{p-q}$.
34. $a = 2$; $b = 3$. **35.** $a = 1\frac{1}{3}$; $b = 3$.
38. k = 11·2, **39.** $\frac{7}{11}$.
40 10 seers frem the first bend and 15 seers from the 2nd.
41. 5 : 4. **42.** Gold 22 tolas; silver 38 tolas.
43. A = Rs. 60,000; B = Rs. 75,000.
44. Movable property Rs 40,000; Immovabla property Rs. 117.000.
45 Gold 3·85 tolas; Copper 4·45 tolas; 2nd Bangle 9·625 tolas; Gold 4·45 tolas.
46. $\frac{1}{2}$ anna per seer above 10 seers. **47.** 15 seers; Rs. 2 per maund. **58.** Train 15 miles per hour; Motor 10 miles per hour.
49. 20 mins.; 30 mins.
50. Uphill 12 miles; Down hill 6 miles; Level 20 miles.
51. PQ = 15 cm.; QR = 18 cm. PR = 10 cm.

52. 8 miles.

53. 15 miles; 18 miles; 20 miles; 45 miles.

54. Rs. 500; Rs. 600.

55. Bandy Rs 50; Bull Rs. 125.

56. Ordinary cycle Rs. 100; Motor cycle Rs. 50.

57. $2\frac{2}{3}$ yds per second; $2\frac{1}{2}$ yds per second.

58. *I Year.* Income Rs. 900; Expenditure Rs 800; Savings Rs. 100.

II Year. Income Rs. 900; Expenditure Rs. 600; Savings Rs. 300.

59. Single-crop lands Rs. 9 per acre; Double-crop lands Rs. 22-8-0 per acre.

60. Speed of the current 2 miles per hr.; Rate of rowing in still-water 4 milas per hr.

61. 15 miles per hr.; 20 miles per hr.; 120 miles from A.

62. Lsngth of the journey 100 miles; rate of the cycle $7\frac{1}{2}$ miles per hr.

63. 31 : 13.

64. I class tickets 140;
II class ,, 200;
III class ,, 400;
Total collection Rs. 920.

65. Cart fare per mile 1 anna; charge by goods train per md. 8 as.

66. 30 members : 80 members.

67. 12 miles per hr.; B 10 miles per hr.

68. $t = 4$; $p = 3$.

69. 10 members; 12 months.

70. 1st class 10 pupils; 2nd class 40 pupils : 3rd class 120 pupils.

CHAPTER XV. Pages 214—224.

1. $x = -25, -7, -8, -1\frac{1}{3}$; $y = 10, 7, 11, 14$.

The value of y increases by 2 units, 1 unit, 2 units and 3 units, respectively.

2. 9, 13·5, $11\frac{1}{4}$, $10\frac{1}{2}$, 17, 10, $12\frac{1}{3}$, 14, 41.

$\pi = 2{\cdot}25, 1{\cdot}5, 1{\cdot}8, 2{\cdot}5, 1{\cdot}4$ 1·8, 1·8, -25, $\frac{1}{18}$.

The value increases by 2 units, 3 units, $\frac{5}{3}$ units, 8 units, 4 units, 2 units, 3 units, 2 units and 8 units respectively.

3. 45° in each case.

4. (*a*) $y = 2x + c$, $y = 3x + c$ where 'c' is any constant.

6. $y = x + 2$, **12.** 49, 26, $12\frac{1}{3}$, $-5\frac{1}{3}$, 13, -23, 100, 50; $x = 2{\cdot}2$, 3·6, 8·4, $-10\frac{2}{3}$, 8, $-3{\cdot}2$, 3·2, 4·3.

17. (*a*) $x = 1$; $y = 1$. (*b*) $x = 3$; $y = 1$.
(*c*) $x = 4{\cdot}6$; $y = 1{\cdot}5$. (*d*) $x = +3{\cdot}2$ (nearly).
$y = -{\cdot}9$ (nearly). (*e*) $x = 1$. (*f*) $x = 2$.
(*g*) $x = {\cdot}07$ (nearly): $y = -{\cdot}03$ (nearly).

18. 4·5 Kilograms; 11 lbs. **19.** 7 ft. per second (nearly); 15 miles per hour. **23.** 3·2 (nearly); 4.

26. (0, 8), (— 1·5, 0); (0, 5), (— 1·7, 0).
(0, 3), (— 1·8, 0).

27. $\frac{8}{10}$, $\frac{7}{8}$, $\frac{5}{7}$.

28. 180 pies; 90 miles; 18 miles.

29. 3·5 viss; Rs. 17-12-0 (including the Gunny bag).

30. — 12°C.; 4·4°C.; 14°F.; 53·6°F.

31. (*a*) Yes, (*b*) Yes. Read (+2, + ·2) as the co-ordinates of the 4th point. $y = {\cdot}6x - 1$. **32.** $k = {\cdot}6\frac{2}{3}$.

33. $m = 20$; $c = -50$. **34.** Yes.

36. $y = 3{\cdot}6x + 1{\cdot}4$. **38.** $y = 1{\cdot}7$; $\frac{5}{3}$

39. $y = \frac{6}{5}x + 1{\cdot}8$.

40. (i) $+4$; $\frac{1}{2}$; (ii) $+4$; $\frac{2}{3}$; (iii) $+5$; $-\frac{1}{3}$.
(iv) $+4$; $-\frac{2}{3}$. **41.** -22; $+12$; -30.

42. (i) $x = -1$; $y = -1$. (ii) $x = 3$; $y = 9$.
(iii) $x = -2$; $y = 4$.

43. (*a*) $x = 4$; (*b*) $x = -5$; (*c*) $x = -2$; (*d*) $x = +11$.

44. Yes. The three equations have a common solution $x = -2$ $y = -3$.

45. 18·5°; 71·5°, 161·5°, 108·5°, 53°, 90°, 143°, 53°, 37°, 90°, 37°, 53° 53°, 90°, 143°. **46.** 90°.

47. a Kerosine light 15 as.; a Castor oil lamp 8 as.

50. After 7 hrs nearly.

51. AB = 40 ft in 55 seconds. CD = 16 ft in 55 seconds.
EF = 20 ft. in 55 seconds. OH = 35 ft. in 55 seconds.

52. 10 miles in 44 mins. is the fastest.
$9\frac{1}{2}$ miles in 43 mins. is the slowest.

53. 81 ft.; 11th second nearly.

54. (i) 1·7 hours nearly after starting, 8 miles (nearly) from Q.

(ii) at the end of the 1st hour and 2·3 hrs. after starting.
(iii) at the end of the 1st hour and third hour.

55. $7\frac{1}{2}$ hours, $26\frac{1}{2}$ ft. $17\frac{1}{2}$ ft.

56. (i) A, 20 gallons, B, $17\frac{1}{2}$ gallons.
(ii) after 20 minutes. (iii) after 40 minutes.
(iv) after 50 minutes and 1 hour 20 minutes.

57. Rs. 8 (nearly). Rs. 3·3 (nearly).

58. Re. 1-2 as. 42 words. **59.** 8 °/₀.

60. Rs. 12 56 miles. **61.** Rs 575 60 members.

62. Rs 2,500. Rs. 100. **63.** 167·1°F. **64.** $13\frac{1}{2}$°.

65. Rs. 200; 11 months. **66.** 1300 ft

CHAPTER XVI. Pages 234—247.

1. (*a*) 14, 25, 76, 220, 75, ab^2, bc^3, ab^3c^3.
(*b*) 10, 12, 8, 14, ab^2c, $x^2y^3z^6$, x^2y^2.
(*c*) 9, 5, 10.

2 3 10, 14, 2, 2, 206.

4 (*a*) 12″. 30′, 13 chs., 15 yds. **5.** $v = 30$. **6.** 256. 256.

7. (*a*) 451, 459 131, 89, 1035, 1234, 4321, 1369, 338, 305, 550. 42, 999.

(*b*) 18·04, 56 25, 12 35, 45·45, 678·75, 5·99, 43·001.

(*c*) $18\frac{4}{7}$, $5\frac{11}{15}$, $15\frac{15}{16}$, $5\frac{2}{3}$, $5\frac{113}{116}$, $1\frac{2}{115}$, $\frac{15}{16}$, $1\frac{1}{67}$.

8. 4·123 : 4·243 : 1·237 : 2 646 ; ·141 ; ·192.

9. 70 yds. **10.** (*a*) 14 yds (*c*) 34 yds. **11.** 25 : 182.

12 Length = 52 yds., Width = 26 yds. **13.** 5677.

14 139 yds. to the nearest yd.

15. 37 rows; 37 trees in each row.

16 125 subscribers, Rs 375. **17.** $5\frac{1}{2}$ minutes. **18** Rs 288.

19 (*a*) $x = 11·7$. (*b*) $x = 11·7$. **20.** $x = 9$.

21 ·875″ **22** 75 ft. **23.** 14·7 ft. to the nearest ft.

24. 6·4 to the nearest tenth of a unit.

25. From A to B. 6 miles.
„ C to D. $5\frac{1}{2}$ miles.
„ B to C. 5 miles.
„ A to C. 3·6 miles
„ A to D. 3·2 miles } to the nearest tenth of a mile.
„ D to B. 4·7 miles

26. 25·3 ft to the nearest foot. **27.** 101. **29.** $v = ·28$.

30. 272·5 to the nearest tenth. **31.** $v = 44$ to the nearest unit.

32 (*a*) Velocity = 46·5 miles. (*b*) The velocity of the cyclonic wind = 70 miles per hour.

33. 228 days' supply of water. **34.** 7·9 seconds to the nearest tenth of a second.

37. 2·06. **38.** 70·7. **39.** 4·9.

40. $a = 7{\cdot}07''$, diagonal $= 7{\cdot}07''$. **41.** ·577, ·544, 2·618, 8·2434.

42. 8". **43.** 18·3"; 36·6". **44.** 53° 8'. **45.** 1·32 ft.

46. 20 ft. to the nearest foot. **47.** 8 ps.

CHAPTER XVII. Pages 255—269.

18. 3 chains nearly. **19.** 3·3 cm.; 105°. **20.** 2·6 cm.

21. 12; No. **22.** 2·6" nearly, 2·1" nearly, 9" nearly.

26. 3·9 sq. ins., 5·8 sq. ins., 10·9 sq. ins., and 17·3 sq. ins.

27. 5·1 sq. cm. nearly 13 sq cm. nearly, 18·4 sq. cm. nearly.

29. 5·2 sq. ins. nearly.

31. 17·1 yds. **32.** 72·4 sq. chains nearly. **39.** 1·5 cm.

40. 4 such circles. **42.** 2·25 miles.

43. 2·5" from B 2·63" from C; 1·9" from B and 1·84" from C or 1·19" from B and 3·17" from C.

45. ·6"; 1·04", 1·47". **46.** ·75".

47. ·9 mile. **48.** 3·5" nearly.

49. 22·4 ft nearly. **50.** 200 ft.; 90 ft.

51 10·4 miles from A, B and C 3 miles.

52. 4·6 cm., 3·5 cm., 1·5 cm., 3·4 cm. and 4 cm., to the nearest tenth of a cm.

56. The same sort of figures.

58 AD = 3·6 cm. nearly; BC = 2·5 cm nearly; $\angle BAD = 69\frac{1}{2}°$; $\angle BCD = 110\frac{1}{2}°$.

59 Yes; No. **60.** 3·9 furlongs nearly.

61. 8·11 miles nearly.

62. $\angle LEK = 90°$; $\angle FEB = 75°$; $\angle FEC = 105°$; $\angle BEL = \angle LEF = 37{\cdot}5°$; $\angle CEK = \angle KEK = 52\frac{1}{2}°$; $\angle AFE = 105°$; $\angle EKC = 37\frac{1}{2}°$; $\angle DKE = 142{\cdot}5°$ and $\angle DLE = 37\frac{1}{2}°$.

CHAPTER XVIII. Pages 571—280

2. 5·03". **3.** 3·5 ft. **4.** 900.

5. (*a*) Diameter = $\frac{7}{3\frac{1}{2}}$ chain : Radius = $\frac{7}{4\frac{4}{4}}$ chain.
(*b*) ○ce $6\frac{3}{5}$ chains; Diameter = 2 chains.
(*c*) Radius = 3' 3" ; ○ce = $20\frac{3}{7}$'.

6. 27·5". **7.** ·598".
8. ·11". **9.** 7924 miles.
10. 6". **11.** 9350 ft. ; 7480 ft.
12. Minute-hand 237·6" : hour-hand 15·4".
13. $r = 3\frac{177}{1250}$. **14.** 3·8", nearly.
15. 6·6". **16.** $3\frac{29}{226}$". **17.** $\frac{1}{7}$
18. $12\frac{4}{7}$ sq ft. : $28\frac{3}{7}$ sq. ins : $78\frac{4}{7}$ sq. cms. : $113\frac{1}{7}$ sq. cms. : 173 03 sq. yds.
19. 3222 sq ft.
20. Oce 44 yds ; radius = 7 yds.
21. 18 pieces ; $210\frac{6}{7}$ sq ins. **22.** $\frac{17}{1900}$.
23. $3\frac{109}{900}$ **25.** $101107\frac{2}{7}$ sq. ft.
26 $28\frac{3}{7}$ sq. miles.
27. Minute-hand 54·31 sq ins. ; hour hand $3\frac{1}{7}$ sq ins.
28. 16·5 sq. ins. **29** 4576 yds. ; 231·34 sq. yds.
30. 4 sq cm.; 12 sq cm. ; 5 sq. ins ; 14 sq. ins. ; 18·9 sq. ins.
31. B's = $\frac{78}{852}$ x sq miles C's = $\frac{231}{852}$ x sq. miles D's = $\frac{165}{852}$ x sq. miles E's = $\frac{285}{852}$ x sq miles
32. 440 yds. **33.** ·83 sq in. ; ·42 sq. in.
34. 4·8 sq. cm. **35.** 1·12 sq. miles.
36. (*a*) 1·1 sq ins. ; (*b*) 5·82 sq. ins.
37. 30° East Longitude.
38. 2 99 sq ins. to the nearest hundredth of a sq. in.
39. 19·15 feet.
40. 10·9 ; 1 nearly.
41. 5 5 ft. nearly. **43.** 1·445 sq. cm. nearly.
44. 17 ft nearly. **45.** 825 acres nearly.
46. 893776 : 802625.

CHAPTER XIX. Pages 283–298.

1. (*a*) 5·625 c. in.; 19·5 sq. in.
   (*c*) ·75 c. in.; 6·5 sq. in. (*d*) 1440 c. in.; 1500 sq. in.
   (*e*) 6·79 c. in.; 35 29 sq. in. (*f*) 99·87 c. in.; 161 28 sq. in.
   (*g*) 109·8 c. in.; 163 45 sq in (*h*) 2848·5 c. in; 921 sq. in.
2. (*a*) 2 : 1; 1100 : 953. 3. 4489·344 c. ft. of water.
4. 5120 c. ft.
5. Gabled end=200 sq. ft.; rect. end=554 sq. ft.
6. 2112 c. ft. 7. 38·28 c. inches.
8. 1 c. ft. 912 c. in.; $11\frac{8}{9}$ sq. ft 9. $\frac{3a^2h}{2}$ c. inches.
10. 3·897 c ft. 11. 1140480 c. ft of water.
12. 210 sq. ft. nearly; 26 students. 13. 119699·25 c. ft.
14. 27·5 oz. 15. 1154 c. ft. 16. 2216·96 c. ft.
17. (*a*) 352 sq in.; 502·85 c. in. (*b*) 660 sq. in.; 1257·14 c. in.
   (*c*) 1673·57 sq in.; 4950 c. in. (*d*) 132·59 sq. ft.; 116·21 c. ft.
   (*e*) 66·78 sq. ft.; 34·375 c. ft. 18. Measure 5·09″ nearly.
   Ollock 2·35″ ,,
   Marcal 9·5″ ,,
19. $h=6''$; $d=2{\cdot}55''$ nearly. 20. 1570·8 c. in.; 785·4 c. in.
21. 1837·5 lbs. 22. 1767·86 c. ft. nearly.
23. 721·875 gallons. 24 456·5 lbs.
25. 3 c. ft.—1039 c. in. nearly. 26. 14 : 15 : 24.
   28 : 45; 96.
27. 17·35 nearly. 28. 308 c. in
29. ·334 lb. nearly. 30. 14·95 c. ft. nearly.
31. 6·35 in. nearly.
32. 2·7 c. in. nearly. 33. 23·8″ nearly.
34. Rs. 757-15-6. 35. 225·08 c. ft.
36. $bo\left(h+\frac{11b}{28}\right)$; 13714·29 nearly.
37. 26142·9 c. ft. nearly. 38. 548·03 c. ft. nearly.
39. $l=\frac{1400x}{11d^2}$.

$$d=10\sqrt{\frac{14x}{11l}}.$$

40. $\frac{d^2h+c^2l}{4v}$. **41.** $\frac{\pi l''}{4}\left(d^2-\frac{n}{36}\right)$ c. ft.

42. ·94 cm. nearly.

43. The single pipe would discharge twice as much as the double pipe ; single pipe ; 408365·7 c. ft. nearly.

44. 6·29 c. in. nearly ; 2·4 oz nearly.

45. $264\frac{12}{35}$ c. ins. ; 1209 oz. to the nearest oz.

46. 8·9 oz. nearly ; 163·625 oz. **47.** 2262·9 c. in nearly; 92·1 % nearly.

48. 2·93 cm. nearly; ·112 cm. nearly. **49.** 6135·36 lbs.

50. ·53 cm. nearly. **51.** 3200 : 147.

52. $l=\frac{4V}{\pi(D^2-d^2)}$;

$l = 2·8$.

53. 35·4816 lbs. **54.** 4·7 cm. ; 18·9 sq cm. 301·7 c. m. ; 301·7 sq. cm., 50·3 c ft.; 113·1 sq ft. ; 110·88 c. yds ; 151·8 sq. yds.

55. 13·6 cm. nearly ; 24·9 c. cm. nearly. **56.** 28·875 c. in.

57. 5 : 12. **58.** 75·4 c. cm. nearly.

59. $h=\sqrt{\frac{s-\pi r^2}{\pi r}-r^2}$;

$h=\frac{3V}{\pi r^2}$.

(1) V = 57·75 ; S = 97·218. (2) r = 5·25 ; S = 303·3.

(3) r = 3 ; V = 37·7. (4) h = 8 ; V = 301·7 nearly.

60. h = 4·33 cm. V = 28·35 c cm. nearly. **61.** $3\frac{1}{3}$ c. ins.

62. 26·565 sq ins. **63.** V = 137·314 c. in. nearly. Surface Area = 143·82 sq. ins. nearly.

64. 93·89 sq. yds. nearly. **65.** 279·4 c. ft. nearly : 230·8 sq ft.

66. 67·05 c ins. : 121·19 sq. ins. **67.** ·08 c in. nearly.

68. 1·81 c ins nearly. **69.** 6776 c. ft.

70. 8904·8 c. ft.

71. $\frac{11}{21}d^2h$ c ins. **72** h = $12\frac{1}{9}$ cm.

73. (1) 261·9 c. ins. (2) 154 c. ins.

74. $V=\frac{7A^2\sqrt{484r^4-49A^2}}{1452r^3}$.

75. 51·5 marcals nearly. **76.** 12·95 sq. in nearly.

77. 101·83 c. ins. nearly ; Rs. 13-2-0. **78.** 8 as. 9 ps. nearly.

CHAPTER XX. Pages 301—314.

1. (*a*) 11·2 ft. ; 11·3 ft. ; 10 95 ft. ; 7·95 ft. 6·5 ft.

(*b*) (i) 3-12 A.M. 5-54 A.M. (ii) after 8 A.M. (iii) between 2 A.M. and 6-36 A.M.

(*c*) the depth was greatest at 4-42 A.M. and was least at 11-22 A.M.

3. 1 A.M. and 6 A.M.

4. Interchange ' 2 ft.' and ' 3 ft ' in the third line, 2·4 ft. ;

3-45 A.M. and 2-40 P.M. **9.** 57·5 grams ; 28°C.

10. 19 grams, 59°C. : 52 grams. 89°C.

11. 115 lbs. ; 15 lbs. ; $1\frac{1}{2}$". **12.** 1·1 ft.

15. 50 cm.

16. 75°. **17.** Rs 47-14 as.

19. 44·7 miles (nearly). Rs. 52-14 as.

20. 160 tons. **22.** V = 665 (nearly).

P = 200 (nearly).

23. At Chingleput at 10-30 A.M.

24. 56·5 minutes after the first pipe was opened.

25. 4-33 P.M. ; 8 miles from the starting place.

26. at 9-15 at Q.

27. 3·4 hrs. after starting at 6 1 miles from the inn A leaves.

28. 5·96 miles from Madras Beach, 7 seconds after 9-19 ; 1·6 miles.

29. 3·6 miles ; St. Thomas Mount.

30. 1·4 hours (nearly). **31.** 1 : 2.

32. (i) Just after 3-16 P.M.

(ii) At 3 P.M, and just before 3-33 P.M.

(iii) 3-55 P.M. (nearly) ; 3-27 P.M.

(iv) 3-49 P.M. (nearly).

CHAPTER XXI. Pages 315—351.

1. In 1919—Friday.

3. 31 ft. long, 28 ft. wide, and 17 ft. high. 4. 35.

6. ·0013 nearly ; ·0099 sq. ins. (nearly). 7. $f = \frac{1}{25}$.

8. $t\left(1 - \frac{p}{q}\right)\left(1 - \frac{l}{m}\right)$ tons ; $\frac{t}{\left(1 - \frac{p}{q}\right)\left(1 - \frac{l}{m}\right)}$ tons.

9. 72000 sq. ft. 10. 15·22878 ; 1·934 (to three places of decimals). 11. 80 ; $3\frac{1}{7}$.

12. — 1° Longitude. 13. 16·37805 lbs.

14.
$$\begin{array}{r} 749 \\ 182 \\ \hline 749 \\ 5992 \\ 1498 \\ \hline 136318 \end{array}$$

15. 38 mangoes. 17. Weight of the empty bottle = 180 grains ; 4 : 5. 18. 30 men.

19. 2·414 ; ·414 ; $\frac{707}{1207}$; $\frac{793}{207}$; $\frac{500}{707}$; $\frac{207}{1207}$.

20 £32 4*s*. 6*d*. (to the nearest penny).

21. (*a*) 10 as. in the Re. (*b*) Rs. 25,600. (*c*) Rs. 2,187·8 as. (*d*) 7*s*. 6*d*. in the £. (*e*) £1,246 13*s*. 4*d*.

22. 188·456. 23 300·3005 miles.

24. 26973 ; 270072·91. 25. 391·314752 tons.

26. 25 : 36 ; 3 : 5. 27. 16 seconds.

28. $x = \frac{1}{3}$; hypotenuse = 13 cm.

29. $x = 20$; 85° ; 106° ; 132°, 37°.

30. Silver 108 $\frac{12}{11}$ grams ; brass 51 $\frac{99}{110}$ grams

31. (*a*) 105·5 yds. ; 189·4 and 44·2 yds.
(*b*) Rs 853, (*c*) Gain 6$\frac{1}{4}$°/₀ ; (*d*) Rs 840 ; Rs. 720 ; Rs. 480.
(*e*) Rs. 3·225.

33. The length of the shortest path = 31·6 ft. ; the length of the thread = 23·8 ft. 34. 37·7°/₀ nearly.

36. The areas are equal. 37. 2 : 1.

42. $m = 3\frac{1}{2} : n = 4$; the areas are in the ratio 9 ; 4.

43. $x = 3$. **46.** (i) 1·782 sq. ins. (ii) ·97 sq. in. (iii) 2·90 sq. ins. **47.** (1) 1·98 sq. ins. (2) 6·93 sq. ins.

48. (1) 3·90 sq. ins. (2) 29·93 sq. cm. **49.** 144 sq. ins.

50. A, 15 biscuits; B, 25 biscuits. **51.** 98 lbs.; 70 lbs.

52. 112·2 °/₀. **53.** 16 more men.

54. 100 coolies; Rs. 1,200. **55.** Rs. 17,550.

56. 20 years. **57.** 4 as.

58. $a = 2$; $b = 4$. **60.** 6435 sq. ins.

61. 5 P.M.; 30 miles from P. **62.** $10\frac{1}{8}$ secds.; $25\frac{5}{16}$ secds.

63. $2\frac{14}{25}$ chains; $\frac{2}{3}$ acre; $1\frac{1}{3}$ acres.

64. $3\frac{1}{3}$ days. **65.** $\frac{1}{4}$ of the cistern.

66. 1 : 5 : 2. **67.** BQ = 1 cm.; AR = 2·5 cm.

68. 4 days. **69.** pathway $(3\,lw + 3\,w - 9w^2)$ sq. ft.; each bed $\frac{1}{4}\,(lb + 9w^2 - 3lw - 3bw)$ sq. ft.

70. ·5 sq. cm. **71.** $14\frac{1}{11}$ ft.

72. ·11453146 of a gallon. **73.** ·2207 of a gallon.

74. (a) 23·4 °/₀; 23·8 °/₀. (b) U.P. = $3\frac{1}{1}\frac{1}{4}\frac{8}{2}$; Bengal $\frac{1}{8}\frac{7}{1}\frac{2}{5}$; Madras, $7\frac{4}{15}\frac{3}{2}$. Madras is the most litigous province of the three.

75. $10\frac{14}{11}$ °/₀. **76.** AC : BC = $\sqrt{5}$: $\sqrt{3}$.

77. 2,234,674 rupees to the nearest rupee. **78.** The first.

79. 999·2 oz. to the nearest tenth of an oz. **81.** 120°.

85. $10\,(r - 1) = l$. **86.** 155 : 49. **87.** 7 : 8.

88. $x : y = 9 : 11$; $x : y : z = 2 : 3 : 4$. **89.** S = $43\frac{3}{4}$.

90. 173·2 yds. **91.** 561·118 c. ins. of butter milk.

93. Equal. **94.** Equal. **98.** 16 ft. to the nearest foot.

99. Sister 4 years; father 30 years.

100. Father 60 years old; Grandfather 80 years old.

101. Copper, $542\frac{2}{10}$ lbs. per c. ft.; zinc, $439\frac{9}{10}$ lbs. per c. ft.

102. 20 miles an hour: 20 miles.

103. 6 oz. from the 1st and 5 oz. from the 2nd.

105. 724 yds. the nearest yard.

106. $40\frac{1}{2}$ ft. **107.** 36·2 ft. to the nearest tenth of a foot. **108.** 346 ft. to the nearest foot.

111. The areas are equal. **112.** 4·8 cm.

113. W = 100 to the nearest unit; v=1462 to the nearest unit.

114. C = 22°; 98°; B = 52°, 128°.

115. $66\frac{52}{57}$ c. cm. **116.** 22·8″.

117. 448 lbs.; 7·168.

118. $\frac{(2x+m-n)}{(x+m)\ (x-n)}$ hours.

119. 5 minutes after starting when the former remains in the house; 750 yds. from the place of starting. **120.** 9000 c. ft.

121. Rs. 210. **122.** 126·8 yards; 84·5 yds.

123. Triplicane $\frac{p-mr}{q-r}$ sq. miles; the remaining portion of Madras $\frac{mq-p}{q-r}$ sq. miles.

124. 16°/₀ correct to two significant figures.

125. 3″, 1·7″, 55° and 125° nearly

126. 5 years. **127.** 3 : 4.

128. $\frac{4}{5}$ of a sq. inch. **129.** $46\frac{2}{7}$°/₀.

130. $16\frac{2}{3}$°/₀. **131.** $33\frac{1}{3}$°/₀; $46\frac{2}{3}$°/₀.

132. When the longest side is 3 cm. long, the sides are in the ratio 2; 1·7 : 1·3 (very nearly).

133. 1st July of the year **135.** Rs. 150; $6\frac{2}{3}$°/₀; Rs 20.

136. Rs. 2,814 to the nearest rupee.

137. $32\frac{2}{9}$°/₀. **139.** 259·8 sq. cm.; 10·39 sq. ins. 23·38 sq. ft.; 4·4 cm. nearly. **140.** 2·83¹ nearly; 56·7 cm. nearly.

141. 7·3 ins. **142.** 1·46 sq. ins. 4·6 cm.

143. $3503\frac{6}{11}$ c. ins.; $2027\frac{101}{176}$ oz.

144. The wages of a man, 6 as. The wages of a woman, 4 as.

145. 1056 ft per second.

146. Rs. 31-9-11 ps.; Rs. 10-8-8 ps.; Rs. 15-15-0 ps.; Rs. 20-9-1 p.; Rs 9-12-3 ps Rs 11-9-1 p. to the nearest pie.

148. 2·3 ins. nearly ; 97° and 83°. **150.** 35 per cent.

154. $\frac{100x\,(pt + pq + kt)}{(ptr + pqr + ktr - 100pq)}$; 8,080,000.

155. (i) $x = -\frac{5}{7}$, $y = -\frac{1}{14}$, (ii) $x = 3\frac{1}{3}$, $v = 7\frac{1}{3}$.
(iii) $x = 8$, $y = 5$.

156. 9; 45; 90; 423. **157.** 27 to the nearest unit.

158. $4\frac{13}{18}$ °/₀ ; $5\frac{10}{19}$°/₀ ; $8\frac{8}{51}$ °/₀ ; the railway in India.

160. They will both meet at the starting point 22·7 mins. after first leaving it.

161. C would be 12 yards behind A and B 25·6 yards behind A; A. 10 yds. in 3 seconds, B. 18 yds, in 7 seconds, and C, 28 yds. in 13 seconds.

162. 570 c. ft. **163.** 73 : 500 nearly.

164. $m = 1{\cdot}2\ c = 1092{\cdot}4$. **165.** 32·23 c. ins.; 8·64 lbs.

166. 6·51 sq. ft.; 18·48 sq. ft.; 4·27 sq. ft. to the nearest hundredth of a sq. ft. **168.** $5\frac{1}{2}$ ft.

169. 10 miles per hour. **170.** 1·0025; 1·00005; ·9995; ·99995; ·9997.

171. 124 ft. nearly.

172. $\frac{1}{1676}$ °/₀ too much ; $\frac{1}{6721}$ °/₀ too small.

173. $8\frac{1}{3}$ sq. chains. **175.** A Rs. 9,600 ; B, Rs. 4,800.
C. Rs. 19,200. **176.** 9·9 as. per rupee.

177. 3 4 °/₀. **178.** 3·3 miles per hour.

179. $233\frac{1}{3}$ °/₀, $265\frac{5}{7}$ °/₀. **180.** 72 miles.

181. 4 acres 11 cents (nearly).

182. 2 as. $3\frac{1}{7}$ ps.; 13 as. $8\frac{4}{7}$ ps.

183. 40 marks nearly. **184.** 3293 ; 48·38°/₀

185. Rs. 45,000. **186.** P = $7\frac{1}{2}$, Q = 2·5.

187. $14\frac{3}{7}$ °/₀. **188.** 54045.

189. Rs 192,901-3-3·11 ps. to the nearest pie; Rs. 277,777-12-5 ps. to the nearest pie. **190.** 53·36 sq. ft.

191. 33 : 40; 1089 : 1600. **192.** $6254\frac{1}{2}$ c. ft.; $2941\frac{17}{64}$ c. ft.

$(lb - lt - bt + t^2) \times \frac{h}{4}$ c. ft; $(lb - lt - bt + t^2) \times \frac{(b-l)}{8}$ c. ft.

193. $8xy+4y^2 = x^3$. 194. Rs. 1 996 in 1900; Rs. 2,000 in 1901.

195. $11\frac{1}{35}$ miles per hour ; $2\frac{23}{35}$ miles per hour,

196. Re. 1-4 as in the cheaper firm and Rs. 2 in the other.

197. 10 as. in the rupee. 198. Rs. 100,000 ; 10%.

199. 200 miles ; 25 miles each hour. 200. Rs. $558\frac{1}{2}$.

201. 37·5%.

202. $(1-\frac{y}{100})^8 : \left[1-\left(1-\frac{p}{100}\right)\right]^8$ 203. $16\frac{1}{2}$".

204. 70 sq. ins.

205. Rs. 2,600 ; Rs. 20.000 : Rs. 6,000 ; 13% ; 57%.

206. 30 per cent. Rs. 20. 207. Rs. 12-8 as. Rs. 13-5-4 ps.

209. 3·8", 2·9", 5 4", 3·8",

210. Read '1·4 cm.' instead of '4 cm.' in the second line and 'ACBD' instead of 'ABCD' in the third line.

$\angle CBD = 70°$, $\angle ADB = \angle ACB = 90°$, AC = ·9 cm., CB = 1·85 cm. and AD = 1·5 cm.

211. $\frac{637\frac{1}{2}}{x}$ where 'x' is the price in Rs. for a ticket of the last class ; 2550.

212. $\left(y = x+\frac{xy}{100}\right)$ per cent. decrease ; 12 per cent. decrease.

213. Rs. 9, Rs. 8, Rs 7. 214. $7\frac{1}{3}$ miles per hour.

215. 9 : 8. 216. 36 2 ft.

217. 5 : 6. 218. 7·8 sq. ft : 2 sq. ft.

219. Re. 1-8 as. (second) ; Rs 2-4-0 (first).

220. 8 ps. per mango. 221. 4225 sq. yds.

223. 2·3193 chains. 225. $11\frac{11}{35}$ lbs $47\frac{1}{9}$ sq ins.

226. 9 ; 7 at the beginning ; 2 : 5 at the end.

227. (i) $21\frac{203}{495}$. (ii) ·000628 (iii) 5·6576. (iv) $1\frac{1}{2}$.

229. Add the words 'and, that the road is 20 ft. wide' at the end, 3849120 c. ft nearly. 231. 383 ft. (to the nearest foot) ; 13 52 miles (correct to the hundredth of a mile).

232. 12 1 : 29·05 ; 37·3.

238. The trains pass each other at a distance of 5·8 furlongs from M at 19-17·6.

234. $2\frac{1}{2}$ mins. later.

235. $660\frac{80}{808}$.

236. 167 more coolies.

237. 3·04.

238. 196 yards.

239. 92·8125°/₀.

240. ·5 inch nearly.

241. Read 'R' instead of 'Q,' PS = $2\frac{3}{4}$ miles, RS = 4 miles. At 6-22 and 6-34.

242. $6(a+b+c)^2 : (a+2b+3c)(6a+3b+2c)$; 40 grains of the 1st mixture and 15 grains of the 2nd mixture.

246. $22\frac{2}{9}$ °/₀.

247. $6\frac{1}{4}$ as. ; $10\frac{5}{8}$ as.

248. Total surface area of the 1st = 2002 sq. ins. Volume of the 1st = 5931 c. ins. Total surface area of the 2nd = 1278·8 sq. ins. Volume of the 2nd = 2301·6 c. ins.

249. 12557 oz. nearly.

250. $\frac{8}{11}\frac{45}{82}\frac{}{78}$, $\frac{78125}{463024}$.

251. 3072 : 1771.

252. 1·3 ft. (nearly).

253. 21 °/₀ in 1918 ; 19 °/₀ in 1917 ; 18 °/₀ in 1916 ; in the year, 1917. (Omit one of the 9's in the number in the 1st line).

CHAPTER XXII. Pages 353—355.

1. (*a*) £1260, (*b*) £1575. **2.** 12 °/₀, Rs. 26-4-0, Rs. 625, and Rs. 1000. **3.** $100y = x(100 - r)$ Rs. 32-13-0, Rs. 7-14-0, Rs. 15-2-3, Rs. 34-4-7. **4.** $9\frac{3}{8}$ °/₀, **5.** 23·9 °/₀, cost Rs 625, List price Rs. 774-3-1. **6.** 30·2 °/₀, **7.** 75·8 dollars **8.** The second offer by $6\frac{2}{3}$ °/₀. **9.** Rs. 5009-2-3. **10** Rs. 40-12-11. **11.** $12\frac{1}{2}$ °/₀, Rs. 77-0-0. **12.** 24·03 °/₀. **13.** 31·5 °/₀ ; £ 480. **14.** Rs. 60-15-6. **15.** 6·9 °/₀. **16.** Publisher Rs. 604-12-10, author Rs. 3193-3-2, Bookseller Rs. 2952. **17.** $16\frac{2}{3}$ °/₀.

Pages 356-357.

1. (*a*) Rs. 4849-4-3. (*b*) Rs. 3596-4-2 (*c*) £ 100. **2.** Rs. 5081-2-8. **3.** Rs. 42-12-0. **4.** Rs. 17092-0-4. **5.** 4·9 °/₀. **6.** Rs. 5458-11-6. **7.** 1 year 6 months 25 days. **8.** Rs. 18750.

Page 363.

1. Rs 5456-4-9. **2.** Rs 762-9-0. **3.** Rs. 180-7-0. **4.** Rs. 233-5-4; 5 years. **5.** Rs. 1534-0-5 ; Rs. 28-7-3, **6.** (*a*) Rs. 10-15-7, (*b*) £125-3-1. (*c*) Rs 270-15-8. **7.** 4 months. **8.** 6 months.

Page 365.

A. 1. Rs 1036-3-5. **2.** Rs 3571-6-10. **3.** Rs. 3003-1-4. **4.** Rs 788 nearly. **5.** Rs. 56310.

B. 1. 139 francs. 97·1 centimes. **2** 11760 marks and 87 pfennig. **3.** 441 roubles, and **4.** 315 krones and 14 ores.

Pages 370-371.

1 £ 542-1-10. **2.** Rs. 30540. **3.** Rs. 69498-7-3. **4.** 39*d*. **5.** 7398 francs and 90 centimes. **6.** 56·84. **7.** £ 1 = 4·54 milreis. **8.** a gain of £ 1-18-4. **9.** Rs. 455-10-6.

Page 372.

1. 12*s*. 7*d*. **2.** 258 8 roubles. **3.** Rs, 348-6-2.

Page 374.

1. $10\frac{3}{7}$ months. **2.** June 7th.—

CHAPTER XXIII.

Page 382.

1. Rs. 2725, Rs 2220. **2.** 50 shares, Rs. 7500. **3.** 320, Rs. 480 **4.** $4\frac{4}{5}$°/₀ Rs. 43-2. **5.** Rs 144. **6.** Rs. 6750 **7.** £37-12-1. **8.** £ 2750, £ 500. **9.** £6-14-0 a share, £32-3-2·4*d*. **10.** Rs. 900 and $7\frac{3}{11}$°/₀.

Pages 387-388.

1. £4887·5. **2.** Rs. 3000. **3.** £5321-13-1½. **4.** Rs. 1255. **5.** Rs. 2660. **6** Rs 8300 **7.** Rs 4000. **8.** 80. **9.** Rs. 425-11-3. **10.** Rs. 27. **11.** £462. **12.** Rs. 8400, Rs. 8862, **13.** The 6 °/₀ at 125. **14.** 3·7 °/₀, Rs. 90. **15.** £3840.

Pages 392—398.

1. Rs. 6000, 112½. **2.** 111. **3.** 96. **4.** Rs. 1000, an increase of Rs. 55. **5.** £500. **6.** $4\frac{6}{21}$ years **7.** Rs. 50-10-0. **8.** Rs, 165, 4·4°/₀. **9.** $83\frac{1}{3}$. **10.** Rs. 58736-4-0. **11.** £2000. **12.** 145¼. **13.** $102\frac{2}{3}\frac{2}{3}$. **14.** The 6°/₀ stock, Rs. 22227-7-0. **15.** gains at 78°/₀. **16.** 17199 in $3\frac{1}{2}$°/₀ and Rs. 10192 in the $7\frac{1}{2}$ per cents. **17.** £2500. **18.** Rs. 17454-8-9. **19.** 112·73. **20.** $5\frac{9}{11}$°/₀. **21.** £127-14-3. **22.** Rs. 495-0-3. **23.** 3·64°/₀. **24.** The latter investment 5·6°/₀. **25.** Rs 450,000. **26.** 65·7°/₀.

27. On application Rs. 1500.
On allotment Rs. 2000.
1st July Rs. 2000.
1st October Rs. 2500.
1st December Rs. 2000.

28. Rs $6607\frac{1}{7}$ **29.** $5\frac{8}{9}$%. **30.** Rs. 1281-12-3 **31.** 96, Rs. 53866-10-8. **32.** 86·6. **33.** Rs. 3114-0-6 ; read 'anna' for pence. Rs. 8-0-0 per share. **34.** 5323-8-0. **35.** $4\frac{1}{2}$% stock by Rs $113\frac{7}{17}$. **36** $3\frac{1}{6}$%. **37.** Gains by Rs 536. **38.** £1341-1-0. **39.** Rs 4837-4-0. (correct to the nearest anna) in the 8% and the remainder in the 5%. **40.** Sum to be placed in the hands of the broker £8553-8-9 ; the broker gets for his services £24-11-3 ; I derive an annual income of £437-5-0 from the investments.

Pages 405-406.

1. 9% stock at 150. **2.** 8% consols. **3.** The first two shares. **4.** $3\frac{6}{7}$% stock at 90 and $4\frac{1}{14}$% stock at 115 **5.** $4\frac{11}{16}$% stock at 75 and $8\frac{1}{8}$% stock at 130. **6.** (1) Rs. 531-10-8. (2) Rs. 15. **7.** £362-10-0. **8.** £300 in the 5% and the remainder in the 7%. **9.** 40 in the $4\frac{1}{2}$% and 60 in the 6%. **10.** 65 in the $6\frac{1}{4}$% and 35 in the 5%.

CHAPTER XXIV. Pages 412—418.

1. $\left(\frac{p}{q} - 2\right)(q + 5)$ miles. **2.** 416 miles. **3.** The new speed must be $\frac{(68 - a)}{(56 - a)}$ of the old speed ; numerical result :— the new speed must be $\frac{3}{17}$ of the former. **4.** 20 miles and 10 miles respectively. **5.** 43 miles per hour. **6** 4·8 miles per hour. **7.** $33\frac{1}{3}$ miles per hour, $48\frac{1}{3}$ miles. Read the detention to be 15 min. and not 45 min. **8** The rate of the carriage 10 miles an hour **9.** The break-down must have occurred 45 miles from the destination. **10.** 60 miles per hour. **11.** 14 min. **12** $8\frac{3}{4}$ miles **13.** 3·2 miles nearly. **14.** 60 miles. **15.** $12\frac{3}{5}\frac{7}{2}$ hours. **16** $42\frac{3}{7}$ miles per hour. **17.** 3 miles per hour; the shortest distance from A to B is $8\frac{1}{2}$ miles. **18.** πx ft., BC = $235\frac{5}{7}$ ft **19.** 44 miles north. **20.** They meet for the first time at 1-20 p m. at a place $33\frac{1}{3}$ miles from A. **21.** $36\frac{12}{23}$ miles. **22.** They will meet next 1 hr. 21 min. after their first meeting

when the faster man has travelled $\frac{3}{5}$ of the circuit. **23.** They meet at 7-40 a.m at a place $39\frac{1}{3}$ miles from A. **24.** 10-6. The rate of the first train is $23\frac{1}{3}$ miles per hour. **25.** $\frac{1}{3}(16 - x) + \sqrt{x^2 + 49}$. **26.** The distance from the foot to the top of the mountain = 5 miles. A's rate = $1\frac{2}{5}$ miles per hour; B's rate = $1\frac{20}{31}$ miles per hour. **27.** Rates are $\frac{20}{37}$ miles and $3\frac{3}{37}$ miles. PQ = 74 miles. **28.** 1 hr. $12\frac{8}{7}$ min. after my starting at a place $4\frac{6}{7}$ miles from Triplicane. **29.** $58\frac{3}{4}$ miles **30.** A's course is $\frac{10}{9}$ as long as B's. A has now increased his speed by 50% **31.** 18 miles. **32.** Read "If they meet at a place $10\frac{1}{2}$ miles from Chingleput." The distance from Egmore to Chingleput is $37\frac{1}{2}$ miles. **33.** $8 - 10\frac{10}{11}$ p.m. **34** Distance = 16 miles. **35.** The waggon's rate is twice the person's rate, **36.** $3\frac{3}{4}$ miles. $32\frac{2}{9}$ min. **37.** 45 miles, A's speed 15 miles an hour, B's speed 18 miles an hour and C's speed 20 miles an hour. **38.** 3096 miles. **39.** 5 min fast, **40.** 1000 ft. per second. **41.** 40 ft. per second.

Pages 419-420.

3. $10\frac{17}{28}$ miles. **4.** Rate of the stream $\frac{1}{2}$ mile per hour and the rate of the man 5 miles per hour **5.** 3 miles = rate of the stream. The rate of the man disappears from the equation and hence the answer is independent of the rate of the man. **6.** $p = q(t - c)$, $c = \frac{1}{2}(t - e)$, $t = 12\frac{5}{6}$ $c = 5\frac{5}{6}$. **7.** Time when there is no current: time when there is the current :: $x^2 - r^2 : xr$ where x is the rate of the current. Since the answer required is only a ratio, it does not depend upon the distance.

Pages 422—424.

1. A wins the race by $\frac{80}{85}$ miles. **2.** 3 min. $16\frac{7}{11}$ sec. after the hound saw the hare at a place 5108 ft from that place. **3.** Racing distance $1347\frac{1}{2}$ yds. rate of the winning boat 8085 yds. per hour and that of the losing boat 7725 yds. per hour. **4.** B can give C a start of $7\frac{1}{3}$ yds. **5.** Racing distance 275 yds. **6.** $a = \frac{7}{6}$ **7.** The length of the race = 1600 yds., Rama takes 15 min. and Krishna 17 min. over the distance. **8.** A's rate = $\frac{9q}{5p - 15q - 3}$ ft. per min.; B's rate = $\frac{9q(p - 3q)}{p}$ ft. per

min. Racing distance $\frac{72q}{5p+5q-3}+p$. 9. $9\frac{17}{18}$ miles. 10. Myself by $12\frac{1}{3}\frac{1}{3}$ yds. 11. Rama by 6 min. 40 sec. 12. Read 'a two mile race' instead 'of a mile race.' The speed required is 44 yds. in 7 seconds. 13. Govind will win the race by $\frac{1}{75}$ of the racing distance. The racing distance is 1200 yds. 14. 1320 yds. 15 $56\frac{2}{3}$ min. from the start. 16 A takes $101\frac{1}{4}$ sec., B $104\frac{14}{17}$ sec. and C 110 seconds to run the quarter of a mile. 17. $\frac{88bc}{c-b}$ ft.

Pages 428—430.

1. $293\frac{1}{3}$ ft. and $366\frac{2}{3}$ ft. 50 miles and $62\frac{1}{2}$ miles per hour. 2. $281\frac{3}{8}$ ft. 3. 450 miles. 4. $377\frac{1}{7}$ ft. 5. 198 yds. 7. lengths are 150 yds. and 180 yds. Rates $30\frac{1}{2}\frac{1}{1}$ miles and $36\frac{9}{1}\frac{9}{1}$ miles per hour. 8. $3\frac{3}{4}$ sec. and 9 ft. (9) $19\frac{7}{8}$ miles, $25\frac{1}{8}$ miles per hour. 10. B is travelling in the opposite direction at 19 miles per hour. 11. $26\frac{1}{4}$ min. 12. $10\frac{1}{2}$ min. 13. My rate 140 ft. per min. Car's rate = 110 yds. per min. 14. $\frac{p}{r}-\frac{p}{r+s}=n$, $r=12$. 15. $\frac{(3a+1)\,b}{9}$, yes

CHAPTER XXV. Pages 431-442.

1. 4·2 miles. 2. 4 hrs after A's starting at a place 36 miles from X. 3. 36 miles. 4. $16\frac{2}{3}$ yds. 5. 11 min. after the first man's start at a distance of 400 yds. 6. At 3.20 p.m. 7. 5·7 miles. 8. Yes, at 5.36 a.m., at a place $11\frac{1}{3}$ miles from home 9. 1 min. $52\frac{1}{2}$ sec after the dog gave chase. 10 The tortoise wins the race. 12. 26 min in all. 14. Q passes P at 3.50 p.m., at a place 40 miles off. 15. B, overtakes A 15 min after his starting at a place 2 miles away. 16. B's rate of walking is 2·7 miles per hour. 17. B's rate of walking is 5·8 miles per hour. 18. $13\frac{1}{3}$ miles per hour. 19. 35 miles at the rate of 10 miles an hour and the remaining 45 miles at the rate of 18 miles an hour 25. They are $12\frac{1}{2}$ miles apart at 1.30 p.m. They are first at a distance of 6 miles from each other at 2.48 p.m. 26. B will reach P at about 5.9 p.m. 27. 48 miles from P.

CHAPTER XXVI. Pages 445—447.

1. Rs. 35, Rs. 140 and Rs. 315. **2.** Rs. 4103-3-6 and Rs. 1221-12-6. **3.** A Rs. 22652 and B Rs. 15101-9-0. **4.** A gets Rs 1841-6-3 **5.** A gets Rs. 4600, B Rs. 4200. **6.** A Rs 4200, B Rs 6300, and C Rs. 3500. **7.** A gets Rs. 780, and B Rs. 1140. **8.** A's capital Rs 22441-8-3 and B's capital Rs, 21418-2-1. **9.** Rs 898-1-2, Rs 2586-7-1, Rs. 1767-15-9. **10.** $9\frac{7}{9}$ months after the commencement of the business. **11.** Rs. 33-12-0, Rs, 93-12-0, Rs 183-12-0. **12.** $ap^2q^2 : bp^4 : cq^2r^2$; 375 : 324 : 50 **13.** A Rs. 15-12-0, B Rs. 28-14-0, C. Rs, 20-4-0 **14.** A in addition to the remaining sum of Rs. 23-6-8, should receive from B Re. 1-0-0 and from C Rs 39-14-8 **15.** One man gets $\frac{ps}{mp+wq}$ annas and one woman gets $\frac{qs}{mp+wq}$ annas.

CHAPTER XXVII. Pages 453—459.

1 30 days. **2.** hrs 3, 45 mins. **3.** 1 woman takes $\frac{bcd}{c-d}$ days to complete the work, 120 days. **4.** $\frac{pqr}{pq+qr+rp}$ days. **5.** $\frac{3pq-48m}{3q+16}$, 12 days. **6,** 1 man will reap $\frac{x-y}{a-c}$ acres in 1 week. ; $\frac{z(a-c)}{d(x-y)}$ wks., $\frac{1}{4}$ acre, 2 wks. **7.** B takes $\frac{apn}{pn-m(a-b-p)}$ days to complete the work, 54 days. **8.** $x=\frac{ad+bd-c}{b}$, 32 days **9.** $a=\frac{6}{7}$. **10.** $\frac{mnx}{2,613,600}$ hrs. **11.** 45 men. **12.** The contractor gains Rs. 48,229-11-0. **13.** The coolies turn out 6 times as much work as the soldiers. **14.** A saving of Rs. 250 in the latter case. **15.** 12 men. **16.** Each would have taken 72 days to complete the work. **17.** The work will be on hand for 3·7 days more. **18.** Rs. 12-2-0. **19.** $\frac{xyz}{xz+xy-yz}$ hrs. Assume that all the three taps are working. **20.** B alone would take 120 days for doing the whole work. **21** No ; 75 more men. **22.** 960 more men must be employed. **23.** 11 cows. **24.** 83 oxen. **25** 39 men. **26.** Rs. 750 Rs. 900. **27.** The Indian workmen, Rs. 2794. **28.** 12 men, Rs. 1863, **29.** Higher paid men :

lower paid men as 1·3 : 1. **30.** Read 150 min. in place of 45 min The well will be empty in $41\frac{1}{9}$ min. after the smaller pump recommences work. **31.** 18 workmen. **32.** $41\frac{4}{7}$. **33.** Rs. 149-6-6. **34.** $5\frac{1}{3}$ and $9\frac{1}{2}$ hrs. **35.** A should get Rs. 9-14-7; B Rs. 10-5-1; and C. Rs 7-8-4. **36.** Read, "The wall in all takes 35 days" Rama must have worked at it for 15 days, **37.** Rs. 118-2-0. If boys alone are employed, the cost would be Rs. 252; if men alone were employed, the cost would come to Rs. 283-8-0. **38.** $4\frac{9}{12}$ days. **39.** Rs. 169 (to the nearest rupee). **40.** $11\frac{1}{9}$ mins and $9\frac{1}{11}$ mins. **41.** 13. **42.** 5 hours.

Pages 464-465.

1. 6·7 days. **2.** 28 days. **3.** $2\frac{1}{2}$ hrs. **4.** 2·1 hrs. **5.** 6 days. **8.** 2·7 hrs. **9.** 2 days.

CHAPTER XXVIII. Pages 468—476.

1. 81·3 °/₀. **2.** Re 1-9-0. **3.** $24\frac{1}{3}$ °/₀. **4.** Ordinary cycle Rs. 100 and motor cycle Rs 500. **5.** Rs. 26-0-8 per thousand. **6.** Rs. 450; and Rs. 540. **7.** Gain Rs 152-6-5. **8.** Rs. 1,500. **9.** 38·6°/₀. **10.** at 271·2°/₀. profit. **11.** 25°/₀. **12.** Assume that the repairs and the necessary outgoings are fixed and are at 15 °/₀ of the former rent $3\frac{9}{11}$ °/₀. **13.** £60. **14.** 8·3°/₀; a decrease of Rs 1,100 in the net annual profit. **16.** Best sort : 2nd sort :: 2 : 9. **17.** 1389 copies. **18.** 57·9 per cent. **19.** Tea 4*s* 2*d*. per lb., coffee $9\frac{1}{5}d$. per lb. **20.** $18\frac{3}{4}$°/₀. **21.** Rs 143-4-3, 48·5 °/₀. **22.** Rs. 188-4-2. **23.** The member who waits till the last chit is the gainer, **24.** Read "If a profit is made of 5 °/₀ (and not 10 °/₀) by the sale." Rs. 55,714 $\frac{2}{7}$. **25.** Rs 15,208 and Rs. 45,624. **26** Rs. 50. **27.** Rs. 15-7-6 and Rs. 15-10-0 **28.** Rs 3,376-5-0. **29.** Rs. 329-8-0. **30.** He was able to sell only 6109 sheets. **31.** Rs 1,040-13-0, Rs 106-3-6. **32** Algebra Rs. 3-12-0 a copy and Geometry Rs 3-4-0 a copy. **33.** Rs 486-14-0. **34.** Read "He assumed that a viss was $\frac{1}{4}$ of a thulam." He thought that he made a profit of $45\frac{5}{11}$ °/₀ but he actually made a profit of $16\frac{4}{11}$ °/₀. **35.** Rs 67-13-7. **36.** £500,000. **37** The armourer loses Rs. 5,618-12-0 by the contract. **38.** Rs. 3-12-0. He will neither gain nor lose. **39.** Rs. 33-0-0. **40.** Buying

price Rs. $222\frac{2}{9}$ and selling price Rs. $263\frac{8}{9}$. **41.** Water added $\frac{(al+p^2)(100+m)}{100t} - a - p$ gallons. 0 gallons. **42** 6 pies. **43.** $\frac{100(r-s)-rs}{100}$ %. **44.** The difference between the buying and the selling price is $\frac{100\,b\,(100-l)-100kl}{b\,(100-l)}$ This is gain or loss according as the numerator is + ve or − ve. **45.** Rs 16. **46.** Profit $Pl^2 + 2Pl - 2Pk$. This will be zero when $l = -1 \pm \sqrt{1+2k}$. **47.** $3000 - 2k$, $1000 + 2k$, 1700, 600; 825 one rupee tickets and 175 three-rupee tickets. **48.** 100 000 **49.** 500 tickets; 185·7 %. **50.** My monthly salary Rs. 140-10-0. **51.** 2*s*. 4*d*,, $28\frac{4}{7}$ %. **52.** 36 sheets, 8·8 %, **53.** $2\frac{1}{2}$ measures of water. **54.** $114\frac{2}{7}$ %.

S S. L. C. Public Examination,

1916.

1. Non-cultivating landowners :—decrease 11·8 p. c. : decrease 17·4 p. c.
Cultivating landowners : —increase 11·0 p. c.; increase 13·6 p. c.
Non-cultivating tenants :— decrease 66·7 p. c : decrease 75 p. c.
Cultivating tenants ;— decrease 25·8 p. c. ; decrease 26·5 p. c.
Labourers—increase 1·8 p. c. ; increase 1·5 p. c.

2. Rs 9-4-6 p. **3.** Rs. 20, Bank shares the most profitable. **4.** 8 balls. **5.** 17 p. c. : 4 as. **6.** 17 c ft. 411·4 c. in nearly. **7.** 1·64 inches.

8. 212 ; 213. **9.** $(p+q+r)\left(\frac{2}{3}d+\frac{1}{6}a\right)-(ap+b+cr)$. $d = 2$ **10.** $x=6\frac{6}{7}$; $y=\frac{4}{7}$. **11.** 5·8 cm. ; 105° ; 105°. **12.** 2 : 3 or 8 : 9.

1917.

1. (1) 77. (2) 5·93714. **2.** Estimated Rs. 61,100. Actual Rs. 52,975-10-8 ; 13·3 p. c. **3.** Rs. 525. **4.** Rs. 722-14. About 6·4 p c. **5.** 205 ft. **7.** 12682 gallons. **8.** $y=2x+7$ where x is the number thought of and y represents the answer. **9.** If x and y represent the sums of money A and B started with, the equations are $x - 20 = \frac{1}{3}(y + 20)$; $x + 50 = 2\frac{1}{3}(y - 50)$; whence $x = 70$, $y = 130$. **10.** 5 cm.

1918.

1. (1) Rs. 3-13-3. (2) 16·7. **2.** Gain Rs. 200.
3. 13 -39 ; 12—36 ; 16—42 ; 13 -8, 29 mi ; 14—53, 56 mi.

4. 1,800,000 sq. mi. 1 in. = 120 mi. 5. 4·0 p. c.; 4·8 p. c.; 5·8 p. c. 6. Rs. 1,232. 7. 19·3 sq. cm.

8. $\frac{mx+ny}{x+y}$; 36·4. 9. 29 mi. 10. (1) 3·7 cm. (2) $\frac{1}{2}$. 11. 30 mi. 12. 17 in.

1919.

1. 33·6; C = 41·4; D = 34·5. 2. Rs. 1,600; 32½ %. 3. 12·7 acres. 4. (1) A at 2-30 p.m.; B at 1-5 p. m.; (2) 11-15 a. m.; 37½ mls. from P; (3) 12½ mls. 5. Rs. 112. 6. 5·84 sq. in. 7. 2 ft. 8 in. 8. 24 men, 20 boys. 9. (1) 7·2 in.; (2) 18 men. 10. 14 in. 11. $(ma + nb)(100 + c)/100(m + n)$; Re. 1 9 as. 12. 29645 sq. ft.

1920.

1. 65·9 p. c; Rs. 12,65,363. 2. 112½°, 11610 c. yds. 3 min. 3. Rs. 115 8 as 4. 175 ft. 5. Rs. 240; Rs. 360. 6. 3 mls an hr.; 31½ mls. 6 p.m., 5½ mls. from P. 11-3 a.m. 13·65 mls. 2-30 p.m., 21 mls. 7. AC = 2·92 in.; C = 70½°.

8. 36½°; 35, 57 sec. 9. $\frac{pmn}{q}$; 38 (37½). 10. 3 in. 11. 3000 c. cm. 12. 6·8 p c.; 1935; £2516 16s;

1921.

1. 6075·6 ft. 2. 3·9 %; 7·7 %; 8 as. 5. Rs. 45-4-11. 6. 75 %. 7. 34$\frac{1}{13}$%. 8. 14½ days; 44 Km.; 10½ days. 9. 12667 sq ft. 10. $(r+1)a - rs$; Re. 1. 11. 8 : 13. 20

1922.

23 (1) Rs. 17040, Rs. 24533. (2) 44·0 %. 2. 249 yds. 2 (1) (i) 1·6 in. (ii) 3·9 in. (2) (i) 7·1 cm. (ii) 11·4 cm. 3. 24. 5. Rs. 95,575. 6. 14 in., 23 in. 7. 91 oz. 50 %. 9. Rs 72765. 10. 34·37 tons. 1. 122°, 27°, 82°, 22°, 58°, 49°. 12. $2\sqrt{ab}$.

1923.

(1) Rs. 8047452, Total grants-in-aid in 1919-20.
Rs. 8189960 ,, ,, 1920-21.

(2) Increase % in 1920-21 in the grants to Local Board Schools over that in 1919-20 = 3·6 %.

(3) The grants-in-aid to private institutions in 1920-21 = 52·5 % of the total grants that year.

R.E.I.N.M's
Mr. Sec. School Library

(1) The weight of the blo[illegible]
(2) at 4·52 ; 35 and 32 circuits respectively.

3. 17·8 °/₀.

4. Interest to be added at the end of June, 0-4-3 correct to the nearest pie.

5. Rate per ton = Rs. 250. 6. 26 men.

7. Aug. 1, 1916 and June 1, 1920.

8. $\frac{v^2 - u^2}{v}$ miles per hour. 59·6 miles.

9. 7 ft. 4 in. 10. Length 46·6 ft. and breadth 31·6 ft.

11. △ is $\frac{1}{4}$ of the square and $\frac{5\,b^2 l}{4}$ or 50000 c. ft.

12. Volume = 1810·3 cubic inches.
Height = 21⅓ in.
Area of the curved surface = 650 sq. inches.

1924.

1. (1) 1798 ; 2047 ; 2324 ; 1901 ; 1722 ; 1901.
10379 ; 667 ; 647.
(2) 7·3 ; 5·6 ; 3·3 ; 5·7. Guntur.

2. (1) 62 lbs 15 3 oz. (2) Rs. 7. 5 as.

3. (*a*) Rs. 2000. (*b*) 13 ; 19.

4. Rs. 41. 8 as. ; Rs. 3, 7 as., 4 ps. 5. $54\frac{6}{11}$ °/₀. 6. 98.

7. (1) 6 — 12. (2) 12 hrs. 38'. 8. 28 yds.

9. 0·98 or 1 acre nearly (qn. 980 and not 1980).

10. $a = 0·3$; $b = 0·8$; 2·125 lbs.

11. 120 sq. ft. 7,920,000 gallons.

12. $12\frac{5}{8}$ c. ft. ; 44 sq. ft. ; 10·6 c ft. ; 9 °/₀.

1925.

1. (1) 2·7 °/₀ increase, 2·3 °/₀ increase, 1·4 °/₀ decrease.
(2) ·9 °/₀.

2. Rs 15-12-0, Rs. 28-14-0, Rs. 20-4-0.

3. £ 28-16-0.

4. 40 years. 5. Rs. 958-13-0. 6. 5·6 miles. 7. £ 24.

8. 40 cms. 10. 2 ft. below the surface. 11. Initial expense £ 135, and the fixed sum £ 30. 13. 1·56 gallons, 10 annas (to the nearest anna).

Errata.

Page	Line	For	Read
16	8	(a)	$(a + 3)$
21	28	$\frac{2}{3}$ of the pole + 3″	$\frac{1}{6}$ of the pole + 3″
	8	139·73652 / 1·386	139 73652 / 1·386
1[illegible]	3 from the bottom	985 and 105	·985 and 1·05
1[illegible]	5	128	12·8 miles
[illegible]	4	$x^3 \div 3x^2 + 20$	$x^3 + 3x^2 + 20$
7[illegible]	13	$(x+2)\,(x+3) \times (x-4)^2$	$(x + 2)\,(x + 3) + (x - 4)^2$
[illegible]	[illegible]	36 cm.	3·6 cm.
[illegible]	9	eediny	feeding
[illegible]	8	925 °/₀	92·5 °/₀
19	12	801	80·1
1[illegible]	5	2025	2·025
		42367	423·67
		6046	60·46
68	15	297854	2·97854
175	19	456// by 245″	4·56″ by 2·45″
176	2	$\frac{t^3}{p^3}$	$\frac{t^3}{p^3}$
194	23	$3x^2y\ 3xy^2$	$3x^2y + 3xy^2$
195	6 from the bottom	ad	and
243	28	$254\frac{2}{3\,1\,6}$	$254\frac{1}{3\,5\,1}$
,,	,,	$1\frac{0\,3\,9}{1\,8\,2\,9\,5}$	$1\frac{6\,9\,9}{1\,5\,2\,9\,6}$
,,	,,	$\frac{5\,2\,5}{3\,1\,5}$	$\frac{6\,2\,5}{2\,5\,6}$
277	12	$1\frac{1}{12}$	$1\frac{1}{21}$
284	Fig. 172	CD	DC
		ld	

Page	*Line*	*For*	*Read*
290	7	hnd	and
„	8	lather	latter
292	2 from the bottom	$\frac{4}{3}\pi r^2$	$\frac{4}{3}\pi r^3$
300	23	879°	87·9°
301	14	345″	3·45″
		45″	4·5″
		51″	5·1″
		55″	5·5″
357	19	Rs. 191,011-9-0	Rs. 19101-9-0
365	22	45 milreis	4·5 milreis
366	6	1050	Rs. 1050
392	10	4^1 °/o	$4\frac{1}{6}$ °/o
396	6 from the bottom	pence	anna
413	4	45 min.	15 min.
417	3	If they meet exactly between Egmore and Chingleput	If they meet $10\frac{1}{2}$ miles from Chingleput
457	16	45 min.	150 min.
458	9	25 days	35 days
469	2	be	he
„	28	be	he
471	21	be	he
473	27	$\frac{1}{6}$ of a thulam	$\frac{1}{2}$ of a thulam
475	27	e rupees	one rupee
480	9	660	6690
482	2	internally at	internally at X

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